LINEARIZED GRAVITATION THEORY AND THE GRAVITON MASS

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It is shown in the present article that in the linearized gravitation theory there is no continuous transition to the limit with respect to the graviton mass. The solutions of the equations of the linearized gravitation theory [1], obtained for the case of a finite graviton mass μ as μ approaches zero, do not go over into the solution of the linearized equations of general relativity theory (GRT). In particular, if the potential ϕ produced by a massive body with mass m is modified in the following manner

$$\phi \to \phi', \quad \phi = -\frac{\gamma m}{r}, \quad \phi' = -\frac{\gamma m}{r}e^{-\mu r}, \tag{1}$$

where r is the distance to the body and γ is the gravitational constant, then the following inequality is valid for the angle of deflection of the light beam in the field of the massive body:

$$\theta(\mu \neq 0) \leqslant \frac{3}{4}\theta(\mu = 0), \tag{2}$$

where θ (μ = 0) is the angle of deflection of the light beam, calculated within the framework of the GRT.

Thus, a rough measurement of the angle θ suffices to be able to establish, within the framework of the linearized theory, that the mass of the graviton is identically equal to zero. The discussed result is also of interest from the point of view of the possibility of going over to the limit of zero mass for a particle with spin. From this point of view, the Yang-Mills equations were analyzed recently in detail [2 - 5].

Let us proceed to prove the statement formulated above. In the lowest order in the gravitational coupling constant, all the effects of interaction between two particles a and b are described by a diagram with exchange of one graviton. Such a diagram corresponds to the following expression [6, 7]:

$$\gamma \Gamma^a_{ik} D_{ik,\,i'k'} \Gamma^b_{i'k'},$$

$$D_{ik,\,i'k'} = \frac{1}{2k^2}[\delta_{ii'}\delta_{kk'} + \delta_{ik'}\delta_{ki'} - \delta_{ik}\delta_{i'k'}],$$

where k is the graviton momentum, Γ^a_{ik} and $\Gamma^b_{i'k'}$ are the gravitational vertex parts of a and b. The terms proportional to the momentum in the graviton propagator drop out, by virtue of the transversality of the vertex Γ_{ik}, $k_i\Gamma_{ik} = 0$. The transversality of the vertex corresponds to conservation of the energy-momentum tensor.

The form of the graviton propagator can be found by direct quantization of the linearized GRT Lagrantian. It is important to us that the expression (3) describes all the classical effects of the gravitational interaction of the particles (Newton's law, the deflection of a light beam in a gravitational field, etc.). The form of the graviton propagator can be established by starting from the requirement of correspondence of the results with the classical analysis. Our approach is therefore essentially classical, and all the results can also be obtained by analyzing the equations of the classical field. This analysis will be presented in a more detailed paper.

Let us assume now that the graviton has a finite mass. We shall show that in this case, even if $k^2 \gg \mu^2$, the expression corresponding to the exchange of one graviton does not go over into (3). Let us consider first the exchange of a massive particle with spin 2. The propagation function of such a particle is

$$\frac{1}{2(k^2-\mu^2)}\left[\delta_{ii'}\delta_{kk'}+\delta_{ik'}\delta_{ki'}-\frac{2}{3}\delta_{ik}\delta_{i'k'}\right], \tag{4}$$

where we have omitted the terms proportional to the graviton momentum since, as already noted, such terms make no contribution, owing to the transversality of the vertex.

We see that the propagator of a particle with spin 2 does not go over when $k^2 \gg \mu^2$ into the propagator of the graviton, since the coefficient of the last term is different. This corresponds to the fact that the virtual graviton carries a spin not only 2 but also 0 [8]. It is therefore natural to introduce a scalar particle in addition to the tensor one. The central point of the proof is the statement that the exchange of the scalar particle makes the following contribution to the propagator:

$$\frac{a}{2(k^2-\mu^2)}\delta_{ik}\delta_{i'k'}, \tag{5}$$

where a is a positive number. The positiveness of a follows from the positiveness of the energy of the scalar field. Since $a > 0$, exchange of a scalar particle cannot lead to the appearance of a term $-\frac{1}{3}\delta_{ik}\delta_{i'k'}$ in the propagator of the massive graviton, as needed for the existence of a continuous transition to zero mass.

Thus, the most general form of the propagator of the massive graviton is

$$\frac{1}{2(k^2-\mu^2)}\left[b(\delta_{ii'}\delta_{kk'}+\delta_{ik'}\delta_{ki'}-\frac{2}{3}\delta_{i'k'})+a\delta_{ik}\delta_{i'k'}\right], \tag{6}$$

where $a, b > 0$. If we normalize the constants a and b in such a way that the usual Newton's law holds at small distances, $r \ll 1/\mu$, then we obtain for the angle of deflection of the light

$$\theta(\mu\neq 0)=\theta(\mu=0)\left[\frac{3}{4}-\frac{3}{4}a\right]. \tag{7}$$

The question may be raised why an "incorrect" sign of the propagator of the scalar part of the graviton does not lead to difficulties in the case when μ is equal to zero. The answer is that out of the six states possessed by the virtual graviton, only two are radiated, with helicity ± 2. The probability of emitting the four other states is equal to zero. Analogously, in the case of electrodynamics, the probability of emitting timelike and longitudinal photons is equal to zero. The proof that scalar gravitons are not emitted makes use essentially of the transversality of the vertex and of the masslessness of the graviton [8, 9]. If the graviton does have a mass, then the sign in front of the scalar part of the propagator must be positive.

This paper was reported to the Conference on High Energy Physics in Kiev. Professor Zumino told me at this conference that a similar investigation was made by Veltman.

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E R R A T A

In the article by A.Z. Gryasuk et al., Vol. 12, No. 6, page 194, last line, "...we find that $B_g/B > 0$" should be replaced by "...we find that $B_b/B > 9$."

Supplement

to Article by V.G. Serbo, Vol. 12, No. 1

In the article the author uses the constant $\sigma_2 = 0.67\mu b$, obtained in [1]. As reported by the authors of [1], an error was made in the calculation of σ_2. The correct value of this cross section is larger by almost one order of magnitude

$$\sigma_2 \equiv \sigma(\gamma\gamma \to 2e\bar{e}) = \frac{a^4}{36\pi m^2}\,[175\,\xi(3) - 38\,] = 6.44\ \mu b$$

($\xi(3) = 1.202$ is the Reimann Zeta function).

When this is taken into account, the numerical estimates and relations derived in the article must be altered. For colliding beams with energy $2E = 7$ GeV, the cross section is $\sigma(ee \to ee + 2e\bar{e}) = 0.66\ \mu b$. The constant cross section at high energy (obtained by numerical integration) is

$$\sigma(\gamma\gamma \to e\bar{e} + \mu\mu) = 5.7\cdot 10^{-33}\ \text{cm}^2 = 0.88\cdot 10^{-3}\sigma_2 = 38\,\sigma(\gamma\gamma \to 2\mu\bar{\mu}).$$

On the other hand, the formulas obtained in the article remain unchanged.

Reference

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