

crystal. The experimental data agree with the calculated ones [3].

We investigated the spectral characteristics of the output radiation of the PLG with respect to the two parametric frequencies, but only with respect to the signal frequency in the case of the resonator. In the former case the spectrum consists of groups of lines determined by the mode-coincidence conditions [7]. The total width was  $< 8 \text{ \AA}$  and the distance between line groups was  $\Delta\lambda = 2.2 \text{ \AA}$ . The corresponding luminescence line width is  $4 \text{ \AA}$ , and  $\Delta\lambda_{\text{scat}} = 1.9 \text{ \AA}$ . In the scheme with the resonator, with only the signal frequency varied, the spectrum consisted of one line of width  $\delta\lambda = 0.1 \text{ \AA}$ , i.e., of one longitudinal mode, and the intermode distance for our resonator was  $0.25 \text{ \AA}$ . The spectrum was recorded with a Fabry-Perot interferometer. At an appreciable excess of pump power  $P_p$  over the threshold power  $P_{\text{thr}}$ , several modes appeared, or even two groups of lines, with a spacing characteristic of a resonator having two frequencies. Apparently, the reflection of the mirrors at the idling frequency,  $R_1 \sim 20\%$ , came into play. The power conversion coefficient was comparable with the efficiency for a generator using a KDP crystal.

The  $\alpha$ -HIO<sub>3</sub> crystal is sufficiently resistant to radiation damage. Faults in the crystal, in the form of filaments, were produced at  $P_{\text{breakdown}} \geq 55 \text{ MW/cm}^2$  (in the case of the KDP crystal,  $P_{\text{breakdown}} \approx 500 \text{ MW/cm}^2$ ). Thus, the limiting efficiency of  $\alpha$ -HIO<sub>3</sub> crystal is 9 times higher than that of KDP.

- [1] S.K. Kurtz, T.T. Perry, and J.G. Bergman, Appl. Phys. Lett. 12, 186 (1968).
- [2] G. Nath and S. Haussühl, Appl. Phys. Lett. 14, 154 (1969).
- [3] A.J. Campillo and C.L. Tang, Appl. Phys. Lett. 16, 242 (1970).
- [4] F.R. Nash, J.G. Bergman, et al., J. Appl. Phys. 40, 5201 (1969).
- [5] T.I. Freidman, Nonlinear Optics, Proc. 2nd All-union Symposium on Non-linear Optics.
- [6] J.E. Bjorkholm, Appl. Phys. Lett. 13, 53 (1968).
- [7] A.I. Kovrigin and P.B. Nikles, et al., Vestnik, Moscow State Univ. 11, No. 5 (1970).

#### SPECTRUM OF THE FIELD OF MOVING FOCAL REGIONS [1]

V.N. Lugovoi and A.M. Prokhorov  
P.N. Lebedev Physics Institute, USSR Academy of Sciences  
Submitted 7 October 1970  
ZhETF Pis. Red. 12, No. 10, 478 - 483 (20 November 1970)

1. Much attention is being paid in the current literature to theoretical and experimental investigations of an effect discovered by Shimizu [2], of broadening of the spectrum of intense beams that are not stationary in time and pass through a medium with a well-pronounced quadratic Kerr effect (cf., e.g., [3 - 6]). In the cited paper, the theoretical analysis of this question is based on the assumption that waveguide propagation in the form of thin filaments takes place in the light beam. A number of additional assumptions are also employed to explain the main features of the experimentally-observed spectral picture.

At the same time, theoretical investigations of the propagation process itself, reported in [7, 8, 1] (and subsequent experimental investigations [9 - 11]), have shown that the picture of this process differs greatly from that previously proposed. In the case of giant laser pulses, the light beam has a waveguide but a multifocus structure, accompanied by formation of a finite number of individual focal regions on the beam axis. The nonstationary

character of the laser beams leads to the motion of the focal regions along the axis. It is therefore of interest to consider the spectrum of the field  $\mathcal{E}(t)$  produced by one of the moving focal regions of a multifocus structure. Such an analysis is presented below. It shows that the main features of the experimentally observed picture of spectral broadening can be explained in the proposed theory.

2. We consider first a beam stationary in time, incident on the boundary ( $z = 0$ ) of a medium having a refractive index  $n = n_0(1 + (1/2)n_2|E|^2)$ . In the axially symmetrical case the complex amplitude  $E(r, z)$  of the oscillations of the electric field on the beam axis (i.e., at  $r = 0$  and  $z \geq 0$ ) is written in the form [7]

$$E(0, z) = |E(0, z)| \exp \left[ ik \int_0^z b_0(z') dz' + i\phi_0 \right], \quad (1)$$

where

$$b_0(z) = \frac{1}{2} n_2 |E(0, z)|^2 - [ka(z)]^{-2} \quad (2)$$

$$a(z) = \left\{ - \frac{\partial^2}{\partial r^2} \left| \frac{E(r, z)}{E(0, z)} \right|_{r=0} \right\}^{-1/2}$$

is the "radius" of the beam, defined for  $r \rightarrow 0$ ,  $k = \omega_0 n_0 / c$ ,  $\omega_0$  is the frequency of the incident-beam field oscillations,  $c$  the speed of light in vacuum, and  $\phi_0$  an arbitrary initial phase.

According to [8], the power  $P$  "flowing into" any of the focal regions is given by the relation  $P \approx P_{cr}^{(1)}$  ( $P_{cr}^{(1)} = cN_1^2 n_0 / 8n_2 k^2$ ,  $N_1 \approx 2$ ), from which we get, say for a beam with a Gaussian intensity profile,  $(1/2)n_2 |E(0, z)|^2 \approx 2[ka(z)]^{-2}$ . The power  $P$  "flowing out" of each focal region can under real conditions be lower than  $P_{cr}^{(1)}$ , owing to multiphoton absorption, stimulated Raman scattering, etc. Similar factors can play an important role in focal regions, and can limit, in particular, the maximum energy attainable in them. The plot of  $|E(0, z)|^2$  can be asymmetrical about the point  $z = z_f$  (where  $z_f$  is the point of maximum energy density with respect to  $z$  in the focal region under consideration), and in general one cannot exclude the occurrence of more complicated internal structures of the focal regions under the influence of similar factors. Nonetheless, we confine ourselves first only to certain very simple approximations of the functions  $b_0(z)$  and  $|E(0, z)|^2$ , namely, we put

$$b_0(z) = \frac{n_2}{4} |E(0, z)|^2, \quad |E(0, z)|^2 = E_f^2 \left[ 1 + \left( \frac{z - z_f}{\Delta z_x} \right)^2 \right]^{-1}. \quad (3)$$

According to (3), the function  $|E(0, z)|^2$  is symmetrical with respect to the point  $z = z_f$ .

3. The foregoing solution in a nonlinear medium, corresponding for example to the boundary condition  $E(r, 0) = E_0 \exp(-r^2/2a_0^2)$ , is also a function of  $E_0$ , i.e.,  $E = E(r, z, E_0)$ . The solution of the corresponding nonstationary problem, i.e., the solution corresponding to the boundary condition  $\bar{E}(r, 0, t) = \psi(t) \exp(-r^2/2a_0^2)$  is [12]

$$\bar{E}(0, z, t) = E \left[ 0, z, \psi \left( t - \frac{z}{v} \right) \right], \quad v = \frac{c}{n_0}. \quad (4)$$

We denote by  $t_0$  the instant of time at which the considered focal region passed through a cross section  $z$ . Confining ourselves only to the vicinity of this region, we obtain with the aid of (1), (3), and (4) the following expression for the complex amplitude  $\bar{E}(0, z, t)$ :

$$\bar{E}(0, z, t) = \frac{E_f \Delta t_x \exp(i \chi_0)}{\sqrt{(\Delta t_x)^2 + (t - t_0)^2}} \left[ \frac{\Delta t_x + i(t - t_0)}{\Delta t_x - i(t - t_0)} \right]^{a/2}, \quad (5)$$

where

$$\Delta t_x = \frac{\Delta z_x}{v_f}, \quad a = \frac{(-1)^s}{4} k n_2 E_1^2 \Delta z_x, \quad (s = 0, 1), \quad (6)$$

$v_f = |dz_f/dt|$  is the absolute value of the velocity of the focal region at the instant of passing through the section  $z$  under consideration;  $s = 0$  when  $dz_f/dt < 0$  and  $s = 1$  when  $dz_f/dt > 0$ ;  $\chi_0$  is an arbitrary constant.

The Fourier transform

$$\bar{E}(0, z, \omega) = \int_{-\infty}^{\infty} \bar{E}(0, z, t) e^{-i \omega_0 t} e^{i \omega t} dt$$

is determined by the expression

$$\bar{E}(0, z, \omega) = \begin{cases} - \frac{F \exp[i(\omega - \omega_0)t_0]}{\sqrt{\omega_0 - \omega} \Gamma\left(\frac{a+1}{2}\right)} W_{\frac{a}{2}, 0} [2\Delta t_x(\omega_0 - \omega)], & (\omega < \omega_0) \\ \frac{F \exp[i(\omega - \omega_0)t_0]}{\sqrt{\omega - \omega_0} \Gamma\left(\frac{1-a}{2}\right)} W_{-\frac{a}{2}, 0} [2\Delta t_x(\omega - \omega_0)], & (\omega > \omega_0). \end{cases} \quad (7)$$

Here  $F = \pi E_f \sqrt{2\Delta t_x} \exp(i\chi_0)$ ,  $\Gamma(z)$  is the Gamma function, and  $W_{\lambda, \mu}(z)$  is the Whittaker function. If  $\Delta\omega \ll \omega_0$  (see (8)) the quantity  $|\bar{E}(0, z, \omega)|^2 / \sqrt{8\pi}$  gives the spectral distribution of the intensity of the oscillations of the electric field on the beam axis at the point  $z$ .

4. According to (7), when  $a \gg 1$  the spectral intensity distribution is essentially asymmetric with respect to the point  $\omega = \omega_0$ . If  $dz_f/dt < 0$ , i.e., if the focal region moves in a direction opposite to that of the incident beam, then the major part of the energy is in the region  $\omega < \omega_0$  (i.e., the predominant broadening of the spectrum occurs in the Stokes region<sup>1)</sup>). In the

<sup>1)</sup>The fraction of the energy in the anti-Stokes region covers a frequency interval much smaller than the width  $\Delta\omega$  of the spectral distribution in the Stokes region. A spectral asymmetry of this type was observed experimentally (cf., e.g., [3]). At the same time we note that additional assumptions are made in the filament model to explain the spectral asymmetry, but an asymmetry of this type still remains unexplained in this model [3, 4].

opposite case (when  $dz_f/dt > 0$ ), the main fraction of the energy is located at  $\omega > \omega_0$ , i.e., in the anti-Stokes region. The spectral distribution of the intensity has a structure (at  $\omega < \omega_0$  for the first case and at  $\omega > \omega_0$  for the second) due to the oscillatory dependence of  $|\bar{E}|^2$  on  $\omega$ . The values of  $|\bar{E}(0, z, \omega)|^2$  at the minima of the oscillations are zero; the period of the oscillations increases with increasing difference between  $\omega$  and  $\omega_0$ ; the last oscillation occurs at  $\omega_0 - \omega = \Delta\omega$  for the case  $dz_f/dt < 0$  and  $\omega - \omega_0 = \Delta\omega$  for the case  $dz_f/dt > 0$ , the spectral width  $\Delta\omega$  being in both cases

$$\Delta\omega = \frac{\omega_0}{2} \frac{v_f}{c} \Delta n_f, \quad (8)$$

where  $\Delta n_f = (1/2)n_0 n_2 E_f^2$  is the increment of the refractive index at the point of maximum energy density in the focal region under consideration. The total number  $m$  of the indicated oscillations is approximately equal to  $|\alpha|/2$  and accordingly the "average period" of the oscillations,  $\Delta\Omega = \Omega/\pi$  is given by<sup>2)</sup>

$$\Delta\Omega = \frac{2}{\Delta t_x}. \quad (9)$$

According to (8), the spectral width  $\Delta\omega$  is proportional to the velocity of the focal region, and therefore under quasistationary conditions (cf. [1]) and at constant values of  $E_f^2$  this width increases monotonically with increasing distance from the turning point of a given region to the section  $z$  in question<sup>3)</sup>. When  $v_f/c \sim 1$  (see [12])  $\Delta n_f \sim 0.1$  (see [14]), and at a ruby-laser frequency  $\omega_0$ , the width of the spectrum  $\Delta\omega$  is of the order of  $700 \text{ cm}^{-1}$ .

5. We now consider briefly the case when the  $|E(0, z)|^2$  curve is strongly asymmetrical with respect to the point  $z = z_f$ , so that the characteristic interval  $\Delta z_x^{(-)}$  of its variation with respect to  $z$  is much larger when  $z < z_f$  than the corresponding characteristic interval  $\Delta z_x^{(+)}$  when  $z > z_f$ . We put here  $b_0(z) = (n_2/4)|E(0, z)|^2$  (see (3)). Then all the foregoing features of the spectral distribution of the intensity  $|\bar{E}(0, z, \omega)|^2$  remain also in this case. The spectral width is then determined by (8), and in lieu of (9) we get for the average period  $\Delta\Omega$  of the oscillation the relation  $\Delta\Omega \sim 1/\Delta t_x^{(-)}$ , where  $\Delta t_x^{(-)} = \Delta z_x^{(-)}/v_f$ . In particular, if the main factor limiting the energy density in the focal region is the finite time  $\tau$  of establishment of the Kerr effect (see [1]), then  $\Delta t_x \sim \tau$  and  $\Delta\Omega \sim 1/\tau$ .

In conclusion we note that if the function  $b_0(z)$  in a given focal region assumes both positive and negative values (the latter are possible because of

<sup>2)</sup> Thus, the structure of the spectrum (which was observed experimentally [2, 3]) is explained in this theory without additional assumptions. At the same time, to explain this structure in the filament model it is assumed that so-called self-modulation exists (or that picosecond pulses are produced) [3, 4]. The nature of this self-modulation, however, remains unknown and must in turn be explained [3, 5].

<sup>3)</sup> Our results do not agree with the theoretical conclusions of [13], where it was assumed that the spectral width  $\Delta\omega$  due to the moving focal region has a maximum when the distance from the turning point to the output plane of the medium is approximately equal to the longitudinal dimension of the focal region.

the competition between the two terms in formula (2)), then an appreciable broadening of the spectrum, due to motion of this focal region, may be observed on either side of the frequency  $\omega_0$ .

- [1] V.N. Lugovoi and A.M. Prokhorov, ZhETF Pis. Red. 7, 153 (1968) [JETP Lett. 7, 117 (1968)].
- [2] P. Shimizu, Phys. Rev. Lett. 19, 1097 (1967).
- [3] A.C. Cheung, D.M. Rank, R.Y. Chiao, and O.H. Townes, Phys. Rev. Lett. 20, 786 (1968).
- [4] T.K. Gustafson, J.P. Taran, H.A. Haus, J.R. Lifshitz, and P.L. Kelley, Phys. Rev. 177, 306 (1969).
- [5] K. Shimoda, J. Appl. Phys. Japan 8, 1499 (1969).
- [6] N.G. Bondarenko, I.V. Eremina, and B.I. Talanov, ZhETF Pis. Red. 12, 125 (1970) [JETP Lett. 12, 85 (1970)].
- [7] V.N. Lugovoi, Dokl. Akad. Nauk SSSR 176, 58 (1967) [Sov. Phys.-Dokl. 12, 866 (1968)].
- [8] A.L. Dyshko, V.N. Lugovoi, and A.M. Prokhorov, ZhETF Pis. Red. 6, 155 (1967) [JETP Lett. 6, 146 (1967)].
- [9] M.T. Loy and Y.R. Shen, Phys. Rev. Lett. 22, 994 (1969).
- [10] V.B. Korobkin, A.M. Prokhorov, R.B. Serov, and M.Ya. Shelev, ZhETF Pis. Red. 11, 153 (1970) [JETP Lett. 11, 94 (1970)].
- [11] N.I. Lipatov, A.A. Manenkov, and A.M. Prokhorov, *ibid.* 11, 444 (1970) [11, 300 (1970)].
- [12] A.A. Abranov, V.N. Lugovoi, and A.M. Prokhorov, *ibid.* 9, 675 (1969) [9, 419 (1969)].
- [13] M.M. Denariez-Roberge and J.P. Taran, Appl. Phys. Lett. 14, 205 (1969).
- [14] R.G. Brewer and C.H. Townes, Phys. Rev. Lett. 18, 196 (1967).

#### REGENERATION OF $K^0$ MESONS ON A DEUTERON AT HIGH ENERGIES

Z.R. Babaev<sup>1)</sup> and P.I. Margvelashvili

Tbilisi State University

Submitted 9 October 1970

ZhETF Pis. Red. 12, No. 10, 483 - 486 (20 November 1970)

In this paper, postulating dispersion relations for forward scattering of K mesons by deuterons and using isotopic invariance of strong interaction, we calculate the differential cross section and the phase of the amplitude of the coherent regeneration process  $K_2d \rightarrow K_1d$ . The calculations were made either under the assumption that the Pomeranchuk theorem is violated [1] or in the Regge-pole model [2].

Figure 1 shows plots corresponding to two parametrizations of the total-cross-section difference  $\Delta\sigma = \sigma_t(K^-d) - \sigma_t(K^+d)$  above 3 GeV (or equivalently to two parametrizations of the imaginary part of the coherent-regeneration amplitude):

$$\Delta\sigma(w) = \begin{cases} 5.5 + 20/w & \text{I} \\ 6.0 + 11.4 \cdot \exp(-0.2w) & \text{II} \end{cases}, \quad (1)$$

where the constants have the dimensions of cross section and  $w$  is the meson laboratory energy. These parametrizations imply the assumption that the Pomeranchuk theorem is violated.

<sup>1)</sup>Institute of High-energy Physics.