front duration, and τ the spontaneous lifetime of the upper level. For the realistically attainable quantities $I \approx 10^{23}$ cm^{-2}sec^{-1}, $t \approx 10^{-5}$ sec, and $\sigma \approx 10^{-18}$ cm^2 inversion is possible if $\tau \gtrsim 10^{-5}$ sec, which is attainable for the metastable atoms under consideration.

[1] M.A. Duguay and P.M. Rentzepis, Appl. Phys. Lett. 10, No. 12 (1967).
[2] A.H. Boyd, Planet. Spac. Sci. 12, 729 (1964).
[3] I.I. Sobel'man, Vvedenie v teoriyu atomnykh spektrov (Introduction to the Theory of Atomic Spectra), Fizmatgiz, 1963.
[4] C. Moor, Atomic Energy Levels, Washington, 1949.

RAYLEIGH SOUND FIELD BURSTS IN A METAL IN A MAGNETIC FIELD

A.M. Grishin and O.I. Lyubimov
Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences

Submitted 12 October 1970
ZhETF Pis. Red. 12, No. 10, 489 - 492 (20 November 1970)

Acoustic Rayleigh surface waves in solids are superpositions of transverse and longitudinal volume acoustic oscillations. Usually these oscillations are localized in a thin layer near the sample surface (cf., e.g., [1]). In pure metals at low temperatures, however, the picture of penetration of the acoustic field in a metal in an external magnetic field may differ greatly from the situation of the normal skin effect. We investigate theoretically in this paper the possibility of existence of sharp bursts of field at distances much larger than the depth of penetration of the sound in the absence of a magnetic field.

1. We choose for the metal a model that is acoustically isotropic. A constant and homogeneous magnetic field $\vec{H}$ is parallel to its surface, the Oz axis is parallel to the vector $\vec{H}$, and the Ox axis to the inward normal to the separation boundary. The equations of elasticity theory are

$$\ddot{\mathbf{u}} = s_t^2 \Delta \mathbf{u} + (s_\ell^2 - s_t^2)\nabla \operatorname{div}\mathbf{u} + \frac{1}{\rho} f, \qquad (1)$$

where $\vec{u}(\vec{r}, t)$ is the displacement vector, ρ the metal density, s_ℓ and s_t the velocities of the longitudinal and transverse sound, and f the electron force. According to [2], the force f is expressed in terms of the electron distribution function F and the deformation-potential tensor $\Lambda_{\alpha\beta}$ as follows:

$$f_\alpha = \frac{\partial}{\partial x_\beta} \int d\tau_p \Lambda_{\alpha\beta} F(\mathbf{p}, \mathbf{r}, t); \qquad d\tau_p = \frac{2d^3p}{(2\pi\hbar)^3}.$$

We have confined ourselves here to a direct deformation interaction between the electrons and the sound, and have neglected the resultant electric field, with the intention of dealing subsequently with the case of strong spatial dispersion, when $\kappa_\ell D \gg 1$ (D is the diameter of the electron trajectory, $\kappa_\ell^{-1} = (k^2 - \omega^2/s_\ell^2)^{-1/2}$ is the depth of penetration of the Rayleigh wave into the metal, ω is the sound frequency, and $\vec{k}$ is the planar wave vector with components k_y and k_z).

The interaction of the sound waves with the conduction electrons leads to a renormalization of the three-dimensional elastic moduli. It is particularly easy to calculate in the Froehlich model, when the tensor $\Lambda_{\alpha\beta}$ is isotropic. This model does not take into account the influence of the electrons on the propagation of the transverse sound and leads to a change of the velocity of the longitudinal mode. Claiming only a qualitative description of the effect, we neglect the contribution made to the volume renormalization of the moduli by the electrons colliding with the surface, and use the electron distribution function for an unbounded metal. Then the renormalized longitudinal velocity can be written in the form

$$s_\ell^2 = s_\ell^2(1-\Delta),$$

$$\Delta = \frac{\zeta}{2}\left[1-\sum_{n=-\infty}^{\infty} J_n^2\left(\frac{1}{2}\sqrt{k_x^2+k_y^2}\,D\right)\frac{\omega}{\omega-n\Omega+i\nu}\right], \qquad (2)$$

$\zeta = (\Lambda/\varepsilon_F)^2$ is the dimensionless parameter of electron-phonon interaction, $\nu = 1/\tau$ the collision frequency, Ω the cyclotron frequency, and $J_n(z)$ a Bessel function. Formula (2) was obtained for the model of a metal whose Fermi surface is a circular cylinder with its axis along $\vec{H}$. The electron force $\vec{f}$ can now be eliminated from (1) by assuming the sound velocity to be renormalized.

2. To solve the boundary-value problem the system (1) must be supplemented by the conditions on the free surface x = 0

$$u_{xy} = u_{xz} = 0; \quad u_{xx} + (u_{yy} + u_{zz})\frac{s_\ell^2 - 2s_t^2}{s_\ell^2} = 0.$$

Here $u_{\alpha\beta} = (1/2)(\partial u_\alpha/\partial x_\beta + \partial u_\beta/\partial x_\alpha)$ is the deformation tensor. A solution in the form $\vec{u}(\vec{r}, t) = \vec{u}(x)\exp[i(\vec{k}\cdot\vec{r} - \omega t)]$ for the planar components of the longitudinal acoustic mode, satisfying the system (1) with the boundary conditions, is

$$u_\alpha(x) = u_\alpha(0)\frac{2}{\pi}\int_0^\infty \frac{dy\cos(\kappa_\ell x y)}{y^2+1+\sum_{n=-\infty}^{\infty} G_n J_n^2\left(\frac{1}{2}\kappa_\ell D\sqrt{y^2+\mu^2}\right)}. \qquad (3)$$

Here $u_\alpha(0)$ are the amplitudes of the unperturbed solution, $\mu = k_y/\kappa\ell$, and

$$G_n = \zeta\frac{\omega^2}{(\kappa_\ell s_\ell)^2}\frac{\omega}{\omega-n\Omega+i\nu}.$$

3. Under the conditions of acoustic cyclotron resonance, $\omega \sim n\Omega \gg \nu$, when $G_n \sim \omega\tau \gg 1$, and under conditions of strong spatial dispersion, $\kappa_\ell D \gg 1$, the field distribution of the metal has the following form in the case of longitudinal polarization of the sound wave ($k_y = 0$)

$$u_z(x) = u_z(0)\frac{2\sqrt{2}}{G_n^{1/2}\kappa_\ell D}\sum_{m=m_n}^{\infty} y_m^{1/2}\cos\left(y_m\frac{x}{D}\right)\exp\left[-\frac{x}{D}\left(\frac{2y_m}{G_n}\right)^{1/2}\right], \qquad (4)$$

where $m_n = [1 + (-1)^n]/2$ and $y_m = (-1)^{n+1}\pi/2 + 2\pi m$ are the zeroes of the asymptotic form of the Bessel function. The important terms in the sum over m are those with $m \sim \omega\tau(D/x)^2$. At sufficiently large $\omega\tau$ we can replace the sum for the first few bursts with an integral with respect to m. Then the amplitude of the bursts at the maximum is given by

$$u_z(rD) = u_z(0)\,8\sqrt{2}\,\frac{G_n}{\kappa_\ell D}\,\frac{\cos\frac{\pi}{2}r}{r^3} . \qquad (5)$$

We see therefore that the height of each burst decreases like r^{-3}, and the signs of neighboring maxima alternate. The field at the maximum of the r-th burst is $\omega\tau/r^2$ times larger than the field between them, and attenuates little in the interior of the metal at distances larger than κ_ℓ^{-1}.

It is easy to conclude from (4) that the field bursts at a depth equal to an integer number of electron-trajectory diameters have the form of a packet made up of the waves $\cos(y_m x/D)$. These waves are acoustic oscillations that attenuate weakly in the interior of the metal. This is confirmed by the presence of poles of the integrand of (3) near the zeroes ($y = y_m/\kappa_\ell D$) of the matrix elements of the electron-phonon interaction. This phenomenon is analogous, in essence, to the anomalous penetration of an electromagnetic field into a metal [3, 4].

If $k_y \neq 0$, then the field bursts are formed by a set of non-equidistant series $\cos\{(x/D)[y_m - (k_y D)^2/2y_m]\}$. Such a wave packet spreads out already for the first bursts. The criterion for the stability of the r-th burst follows from the requirement that the phase shift $(x/D)(k_y D)^2/2y_m$ be small compared with π for each harmonic:

$$(k_y D)^2 << \frac{\omega\tau}{r^3} .$$

The appearance of dispersion with $k_y \neq 0$ is connected with the nonlinearity of the dispersion of weakly-damped electron waves.

4. In the case of low acoustic frequencies, $|\nu - i\omega| << \Omega$, it is necessary to retain in the sum over n only the term with $n = 0$. Assuming $\kappa_\ell D >> 1$, this leads to the result

$$u_z(x) = u_z(0)\left[e^{-\kappa_\ell x}\left(1 - \frac{G_0}{2\pi}\frac{x}{D}\right) - e^{-\kappa_\ell|x-D|}G_0\frac{\kappa_\ell|x-D| + 2}{4\pi\kappa_\ell D} + \dots\right], \quad k_y = 0. \qquad (6)$$

The amplitude of the r-th burst is $(G_0/\kappa_\ell D)^r \sim (\kappa_\ell D)^{-r}$, and the field attenuates rapidly in the interior of the metal. The formation of small bursts is in this case a size effect. When $k_y \neq 0$ sets in, the amplitude of the field bursts decreases exponentially like $\exp[-\kappa_\ell D\mu^2/2]$.

The effect predicted above can be observed experimentally under conditions of acoustic cyclotron resonance with surface hypersonic waves ($\omega \sim 10^{10} - 10^{11}$ sec^{-1}) in pure single crystals of metals at low temperatures.

The authors thank E.A. Kaner for a stimulating criticism and valuable discussions.

[1] L.D. Landau and E.M. Lifshitz, Teoriya uprugosti (Theory of Elasticity), Nauka, 1965, Sec. 24 [Addison Wesley].

[2] V.G. Skobov and E.A. Kaner, Zh. Eksp. Teor. Fiz. 46, 273 (1964) [Sov. Phys.-JETP 19, 189 (1964)].
[3] M.Ya. Azbel', ibid. 39, 400 (1960) [12, 283 (1961)].
[4] E.A. Kaner, ibid. 44, 1036 (1963) [17, 700 (1963)].

STATIC DISTRIBUTIONS OF MAGNETIZATION OF AN ELECTRON LIQUID IN A QUANTIZING MAGNETIC FIELD

V.I. Okulov and V.N. Bagaev
Institute of Metal Physics, USSR Academy of Sciences
Submitted 12 October 1970
ZhETF Pis. Red. 12, No. 10, 493 - 495 (20 November 1970)

The sharp increase of the magnetic susceptibility of an electron gas in the peaks of quantum oscillations leads to the possible occurrence of magnetization jumps as functions of the magnetic field [1] and of phase transitions into states with "diamagnetic" domains [2] and periodic distributions of the magnetization [3]. We wish to call attention in this article to peculiarities exhibited by these phenomena in an electron liquid and caused by the presence of an exchange Fermi-liquid interaction. These peculiarities reduce essentially to the existence of spin ordering of the electrons under certain conditions. To study the effects of interest to us, it is necessary to consider the tensor $\hat{\chi}(\vec{k})$ describing the reaction of the electrons to a small inhomogeneous addition to the magnetic induction (we denote the Fourier component of this addition by $\vec{b}_k$). This tensor is defined by the equation $m_k = \hat{\chi} b_k$, where m_k is the Fourier component of the addition to the electron magnetization. We confine ourselves henceforth to a situation in which the field inhomogeneity exists only in a plane perpendicular to the quantizing magnetic field $\vec{H} = (0, 0, H)$, and direct $\vec{k}$ along the y axis. Then, in the isotopic case, only the diagonal elements of the tensor $\hat{\chi}$ are different from zero. Only one of these components, $\chi_{\parallel}$, describes the reaction in the case when $\vec{b}_k \parallel \vec{H}$, and the other, $\chi_{\perp}$, in the case $\vec{b}_k \perp \vec{H}$. For an isotropic electron fluid with a contact exchange Fermi-liquid interaction described by a constant ψ, we can easily obtain, by using the general results of [4], the values of $\chi_{\parallel}$ and $\chi_{\perp}$ in the following form:

$$\chi_{\parallel} = \kappa + \frac{\mu_0^2 X + 2\mu_0 P - \psi P^2}{1 + \psi X_{\parallel}} = \chi_{\parallel}^{\circ} - \psi \frac{(\mu_0 X_{\parallel} + P)^2}{1 + \psi X_{\parallel}}, \quad (1)$$

$$\chi_{\perp} = \kappa_{\perp} + \frac{\mu_0^2 X_{\perp}}{1 + \psi X_{\perp}} = \chi_{\perp}^{\circ} - \psi \frac{(\mu_0 X_{\perp})^2}{1 + \psi X_{\perp}}, \quad (2)$$

$$\begin{pmatrix} \kappa_{\parallel} \\ \kappa_{\perp} \end{pmatrix} = - \frac{\omega_0^2}{4\pi c^2 k^2} \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{4\pi e^2}{\omega_0^2} \sum_{aa'} \frac{\rho_a - \rho_{a'}}{\epsilon_a - \epsilon_{a'}} \begin{pmatrix} V^x_{aa'}(-k) V^x_{a'a}(k) \\ V^z_{aa'}(-k) V^z_{a'a}(k) \end{pmatrix} \right\}, \quad (3)$$

$$X_{\parallel} = - \sum_{aa'} \frac{\rho_a - \rho_{a'}}{\epsilon_a - \epsilon_{a'}} | I_{a'a}(k)|^2, \quad P = \frac{ie}{ck} \sum_{aa'} \frac{\sigma_a - \sigma_{a'}}{\epsilon_a - \epsilon_{a'}} I_{aa'}(-k) V^x_{a'a}(k), \quad (4)$$

$$X_{\perp} = - \sum_{aa'} \frac{\rho_{a'} - \rho_a + \sigma_a + \sigma_{a'}}{\epsilon_{a'} - \epsilon_a + \hbar\Omega_0} | I_{a'a}(k)|^2 . \quad (5)$$