[2] V.G. Skobov and E.A. Kaner, Zh. Eksp. Teor. Fiz. 46, 273 (1964) [Sov. Phys.-JETP 19, 189 (1964)].
[3] M.Ya. Azbel', ibid. 39, 400 (1960) [12, 283 (1961)].
[4] E.A. Kaner, ibid. 44, 1036 (1963) [17, 700 (1963)].

STATIC DISTRIBUTIONS OF MAGNETIZATION OF AN ELECTRON LIQUID IN A QUANTIZING MAGNETIC FIELD

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The sharp increase of the magnetic susceptibility of an electron gas in the peaks of quantum oscillations leads to the possible occurrence of magnetization jumps as functions of the magnetic field [1] and of phase transitions into states with "diamagnetic" domains [2] and periodic distributions of the magnetization [3]. We wish to call attention in this article to peculiarities exhibited by these phenomena in an electron liquid and caused by the presence of an exchange Fermi-liquid interaction. These peculiarities reduce essentially to the existence of spin ordering of the electrons under certain conditions. To study the effects of interest to us, it is necessary to consider the tensor $\hat{\chi}(\vec{k})$ describing the reaction of the electrons to a small inhomogeneous addition to the magnetic induction (we denote the Fourier component of this addition by $\vec{b}_k$). This tensor is defined by the equation $m_k = \hat{\chi} b_k$, where m_k is the Fourier component of the addition to the electron magnetization. We confine ourselves henceforth to a situation in which the field inhomogeneity exists only in a plane perpendicular to the quantizing magnetic field $\vec{H} = (0, 0, H)$, and direct $\vec{k}$ along the y axis. Then, in the isotopic case, only the diagonal elements of the tensor $\hat{\chi}$ are different from zero. Only one of these components, $\chi_{\parallel}$, describes the reaction in the case when $\vec{b}_k \parallel \vec{H}$, and the other, $\chi_{\perp}$, in the case $\vec{b}_k \perp \vec{H}$. For an isotropic electron fluid with a contact exchange Fermi-liquid interaction described by a constant ψ, we can easily obtain, by using the general results of [4], the values of $\chi_{\parallel}$ and $\chi_{\perp}$ in the following form:

$$\chi_{\parallel} = \kappa + \frac{\mu_0^2 X + 2\mu_0 P - \psi P^2}{1 + \psi X_{\parallel}} = \chi_{\parallel}^{\circ} - \psi \frac{(\mu_0 X_{\parallel} + P)^2}{1 + \psi X_{\parallel}}, \qquad (1)$$

$$\chi_{\perp} = \kappa_{\perp} + \frac{\mu_0^2 X_{\perp}}{1 + \psi X_{\perp}} = \chi_{\perp}^{\circ} - \psi \frac{(\mu_0 X_{\perp})^2}{1 + \psi X_{\perp}}, \qquad (2)$$

$$\begin{pmatrix} \kappa_{\parallel} \\ \kappa_{\perp} \end{pmatrix} = - \frac{\omega_0^2}{4\pi c^2 k^2} \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{4\pi e^2}{\omega_0^2} \sum_{aa'} \frac{\rho_a - \rho_{a'}}{\epsilon_a - \epsilon_{a'}} \begin{pmatrix} V^x_{aa'}(-k) V^x_{a'a}(k) \\ V^z_{aa'}(-k) V^z_{a'a}(k) \end{pmatrix} \right\}, \qquad (3)$$

$$X_{\parallel} = - \sum_{aa'} \frac{\rho_a - \rho_{a'}}{\epsilon_a - \epsilon_{a'}} | I_{a'a}(k) |^2, \quad P = \frac{ie}{ck} \sum_{aa'} \frac{\sigma_a - \sigma_{a'}}{\epsilon_a - \epsilon_{a'}} I_{aa'}(-k) V^x_{a'a}(k), \qquad (4)$$

$$X_{\perp} = - \sum_{aa'} \frac{\rho_{a'} - \rho_a + \sigma_a + \sigma_{a'}}{\epsilon_{a'} - \epsilon_a + \hbar \Omega_0} | I_{a'a}(k) |^2 . \qquad (5)$$

Here μ_0 is the spin magnetic moment of the free electron, $\chi_{\parallel}^0$ and $\chi_{\perp}^0$ are the values of $\chi_{\parallel}$ and $\chi_{\perp}$ for an electron gas, ω_0 is the plasma frequency, $\alpha = npx_0$ are the orbital quantum numbers of the electron in the magnetic field, $\varepsilon_{\alpha} = \varepsilon_{np}$ are the eigenvalues of the orbital energy,

$$\rho_{np} = f(\epsilon_{np} + \hbar\Omega_o/2) + f\left(\epsilon_{np} - \frac{\hbar\Omega_o}{2}\right);$$

$$\sigma_{np} = f\left(\epsilon_{np} + \frac{\hbar\Omega_o}{2}\right) - f\left(\epsilon_{np} - \frac{\hbar\Omega_o}{2}\right);$$

$f(E)$ is the Fermi distribution function, $\hbar\Omega_0$ the magnitude of the spin splitting of the Landau levels, and $I_{\alpha'\alpha}(k)$ and $V_{\alpha'\alpha}(k)$ are the matrix elements of the Fourier components of the particle-number and electron-flux operators. To investigate the stability of the homogeneous state of an electron liquid against small changes of the magnetic induction, it is necessary to compare the values of $\chi_{\parallel}(k)$ and $\chi_{\perp}(k)$ with $1/4\pi$. If at some values of the external magnetic field and of the temperature we have $4\pi\chi_{\parallel}(0) \geq 1$, then domains are produced in the system, or else the system goes over jumpwise (when the equality $4\pi\chi_{\parallel}(0) = 1$ is reached) into a state with a different value of the magnetization; on the other hand, if the conditions $4\pi\chi_{\parallel}(k) = 1$ and $4\pi\chi_{\parallel}(0) < 1$ are satisfied, then a periodic distribution of the magnetization appears [5]. When analogous criteria are satisfied for $\chi_{\perp}(k)$, the conditions for the stability of the system against perturbations $\vec{b}_k$ perpendicular to the magnetic field are violated. Let us consider first the case $\vec{b}_k \parallel \vec{H}$, which was investigated for $\psi = 0$ in detail in [1 - 3, 5]. Using the fact that the functions $\kappa_{\parallel}(k)$, $\chi_{\parallel}(k)$, and $P(k)$ reach their maxima at $k = 0$, it is easy to verify that $\chi_{\parallel}(0) \geq \chi_{\parallel}(k)$. Therefore, just as in the gas model, periodic distributions of magnetization are possible only when there are several cyclotron periods. We shall not consider such a situation. As to the possible occurrence of domains and magnetization jumps, an essentially new circumstance, arising when account is taken of the exchange Fermi-liquid interaction, is connected with the presence of the denominator $1 + \psi\chi_{\parallel}(0)$ in the formula for $\chi_{\parallel}(0)$. At negative vales of ψ, this denominator can be small. Indeed, the quantity $\chi_{\parallel}(0) = -\sum_{\alpha}\partial\rho_{\alpha}/\partial\varepsilon_{\alpha}$ is the density of states having the Fermi energy, and exceeds appreciably its classical value at the peaks of the quantum oscillations. Owing to the smallness of the spin contribution to $\chi_{\parallel}^0(0)$, the equality $4\pi\chi_{\parallel}(0)$ can be satisfied, generally speaking, either when $1 - 4\pi\kappa_{\parallel}(0) \simeq 0$ or when $1 + \psi\chi_{\parallel}(0) \simeq 0$. The latter equality is the condition for the existence of an ordered distribution of the electron spin density. Therefore in an electron liquid at $\psi < 0$ there can occur, in general, both ordinary "diamagnetic" domains and jumps of the orbital magnetization, and domains connected mainly with the spin ordering and jumps of the spin magnetization.

We now proceed to consider the stability against perturbing fields perpendicular to the field $\vec{H}$. No such question has apparently been considered in the literature for an electron gas. It follows from the expressions for $\kappa_{\perp}(0)$ and $\chi_{\perp}(0)$ that $\chi_{\perp}(0)$ oscillates relatively little when the magnetic field is varied, and in order of magnitude we have $\chi_{\perp}(0) \simeq M/H$, where M is the magnetization of the electron gas. Therefore $\chi_{\perp}(0) \ll 1$ under ordinary conditions. An appreciable increase of $\chi_{\perp}(k)$ at nonzero k can take place in the case when the spin splitting of the Landau levels $\hbar\Omega_0$ is approximately equal to the cyclotron quantum $h\Omega$ (i.e., $|\Omega - \Omega_0| \lesssim T/\hbar$, $1/\tau$, where T is the temperature and τ is the relaxation time. In this case $\chi_{\perp}(k)$ contains a term

$$\chi_\perp^r = -\sum_\alpha \partial f(\epsilon_\alpha - \hbar\Omega_0/2)/\partial\epsilon_\alpha |I_{n-1\,n}(k)|^2 ,$$

which experiences quantum oscillations of large amplitude (if the appropriate conditions are satisfied). At the peaks of these oscillations, the denominator in (2) may become small when $\psi < 0$. For small k such a situation occurs if the ratio of the quantum density of states with a Fermi energy to the classical one is of the order of $\psi(kR)^2$, where R is the radius of the electron cyclotron orbit. Since the equation $1 - 4\pi\chi_\perp(k) = 0$ reduces essentially to $1 + \psi\chi_\perp(k) \approx 0$, a periodic distribution of the spin density is produced under the foregoing contribution; the periods are determined by the equation $1 + \psi\chi_\perp^r(k) = 0$.

To observe the aforementioned spin-ordering effects it is necessary that the state density of the electrons with the Fermi energy exceed its classical value by at least several times in the peaks of the quantum oscillations. This condition is more stringent than the conditions for the observation of diamagnetic domains.

The foregoing conclusions are based essentially on the assumption that the exchange Fermi-liquid interaction can be described by the constant ψ. The question of how the results are altered in a more thorough analysis calls for a detailed evaluation.

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[1] A.B. Pippard, Proc. Roy. Soc. A272, 192 (1963).
[2] J.H. Condon, Phys. Rev. 145, 526 (1966); I.A. Privorotskii and M.A. Azbel', Zh. Eksp. Teor. Fiz. 56, 388 (1969) [Sov. Phys.-JETP 29, 214 (1969)].
[3] M.Ya. Azbel', Zh. Eksp. Teor. Fiz. 53, 1751 (1967) [Sov. Phys.-JETP 26, 1003 (1968)]; S.C. Ying and J.J. Quinn, Phys. Rev. Lett. 22, 231 (1969).
[4] P.S. Zyryanov, V.I. Okulov, and V.P. Silin, Zh. Eksp. Teor. Fiz. 58, 1295 (1970) [Sov. Phys.-JETP 31, 696 (1970)].
[5] M.Ya. Azbel', Usp. Fiz. Nauk 98, 601 (1969) [Sov. Phys.-Usp. 12, 507 (1970)].

BOUND STATES OF AN ELECTRON AND A PHONON IN A STRONG MAGNETIC FIELD

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In the present paper we point out the existence of bound states of an electron situated in a quantizing magnetic field at the lower Landau level $\ell = 0$, and of an optical phonon. The following is assumed here:

1) The electrons interact only with the optical phonons; this interaction is weak (coupling constant $\alpha \ll 1$) and is realized by the deformation potential (the matrix element is independent of the phonon momentum $\vec{q}$); the phonons have no dispersion, $\omega(\vec{q}) = \omega_0$.

2) The magnetic field is strong, i.e., $\omega_c = 4H/mc \gg \omega_0$, but the effective coupling constant in the magnetic field is $\bar{\alpha} = \alpha\omega_c/\omega_0 \ll 1$.

3) The temperature T = 0 and the electron density N = 0; this means physically that the temperature is so low that the phonon absorption, which is