$$\chi_\perp^r = -\sum_\alpha \partial f(\epsilon_\alpha - \hbar\Omega_0/2)/\partial\epsilon_\alpha |I_{n-1\,n}(k)|^2 ,$$

which experiences quantum oscillations of large amplitude (if the appropriate conditions are satisfied). At the peaks of these oscillations, the denominator in (2) may become small when $\psi < 0$. For small k such a situation occurs if the ratio of the quantum density of states with a Fermi energy to the classical one is of the order of $\psi(kR)^2$, where R is the radius of the electron cyclotron orbit. Since the equation $1 - 4\pi\chi_\perp(k) = 0$ reduces essentially to $1 + \psi\chi_\perp(k) \simeq 0$, a periodic distribution of the spin density is produced under the foregoing contribution; the periods are determined by the equation $1 + \psi\chi_\perp^r(k) = 0$.

To observe the aforementioned spin-ordering effects it is necessary that the state density of the electrons with the Fermi energy exceed its classical value by at least several times in the peaks of the quantum oscillations. This condition is more stringent than the conditions for the observation of diamagnetic domains.

The foregoing conclusions are based essentially on the assumption that the exchange Fermi-liquid interaction can be described by the constant ψ. The question of how the results are altered in a more thorough analysis calls for a detailed evaluation.

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BOUND STATES OF AN ELECTRON AND A PHONON IN A STRONG MAGNETIC FIELD

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In the present paper we point out the existence of bound states of an electron situated in a quantizing magnetic field at the lower Landau level $\ell = 0$, and of an optical phonon. The following is assumed here:

1) The electrons interact only with the optical phonons; this interaction is weak (coupling constant $\alpha << 1$) and is realized by the deformation potential (the matrix element is independent of the phonon momentum $\vec{q}$); the phonons have no dispersion, $\omega(\vec{q}) = \omega_0$.

2) The magnetic field is strong, i.e., $\omega_c = 4H/mc >> \omega_0$, but the effective coupling constant in the magnetic field is $\bar{\alpha} = \alpha\omega_c/\omega_0 << 1$.

3) The temperature $T = 0$ and the electron density $N = 0$; this means physically that the temperature is so low that the phonon absorption, which is

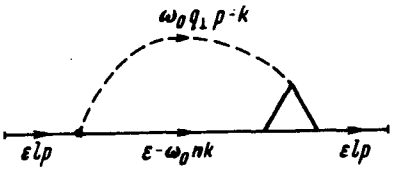

Fig. 1a

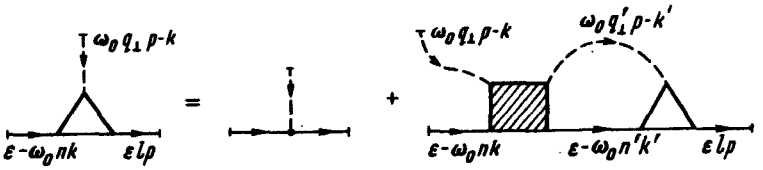

Fig. 1b

proportional to $\exp(-\omega_0/T)$, can be neglected, and the electron density is low enough to regard the electron gas as degenerate and to ignore the change of the phonon spectrum.

4) The investigated elementary excitation have a component $p \ll p_c = (2m\omega_c)^{1/2}$ along H.

The bound states are determined as the branches of the electron spectrum from the equation

$$\epsilon - \frac{p^2}{2m} - M(\epsilon p) = 0, \qquad (1)$$

where M is the electron mass operator. It is clear from physical considerations that the branches, which can be interpreted as bound states, should lie below the threshold, $\varepsilon < \varepsilon_0$, at least at $p = 0$, and $\varepsilon \simeq \omega_0$ near the threshold (the energy is reckoned from the bottom of the Landau band $l = 0$). It is known [1] that in this region perturbation theory for the mass operator diverges, this being connected with the singularity of the state density in a magnetic field. The spectrum near the threshold can be investigated by the method developed by us jointly with Matulis and Shcherbakov [2]. We start from the equations for the mass operator M (Fig. 1a) and the vertex part Γ (Fig. 1b).

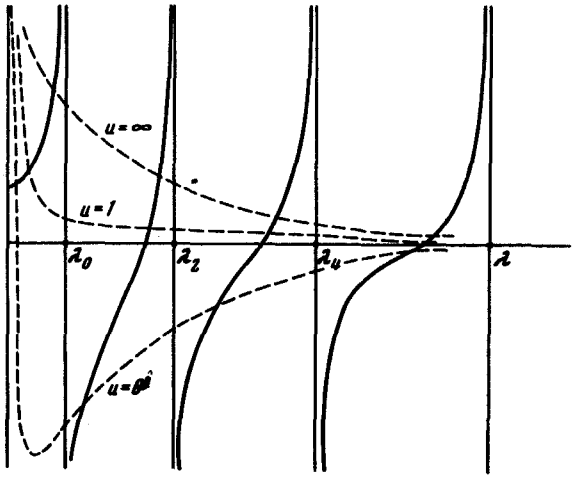

Fig. 2

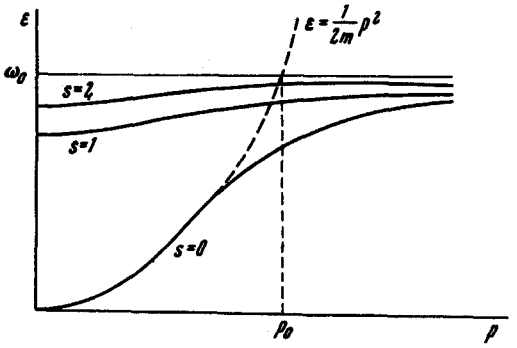

Fig. 3

The dashed lines representing the phonon Green's function D are marked with the transverse and longitudinal phonon momenta. The solid line represents the electron Green's function in the Landau representation. The shaded rectangle is the irreducible vertex Δ.

In the case $\ell = 0$ and $\varepsilon \simeq \omega_0$ of interest to us, in the lowest order in α, we can introduce the following simplifications into the equation for Γ: G and Δ can be expanded in powers of α, and are therefore replaced by the simplest terms. Upon integration with respect to k', only the term with n' = 0 in the sum over n' has a pole with a large residue, proportional to $(\varepsilon - \omega_0)^{-1/2}$; the terms with $n' \neq 0$ can therefore be omitted. The factors D, Δ, and Γ can be taken outside of the integral with respect to k' at the point k' = 0. Assuming also that n = 0 and $k_{\vec{}} = 0$, we obtain for Γ with $n = \ell = 0$ and k = 0 an integral equation in terms of $\vec{q}$, with ε and p as parameters. If we make similar simplifications in the equation for M, then it turns out that this is precisely the vertex needed for the calculation of M. After changing over to the dimensionless variables

$$\bar{\Gamma}(\lambda) = -\frac{1-v^2}{1+v^2}\frac{1}{\lambda} + \frac{\bar{a}^2}{(1+v^2)^3}\frac{1}{\lambda^3} \equiv f(\lambda) \quad . \tag{2}$$

M takes the form

$$M = \tilde{M}\bar{\Gamma}, \tag{3}$$

where

$$\tilde{M} = -i\omega_0 \bar{a} w^{-1/2}, \quad \bar{\Gamma} = \int_0^\infty dt e^{-\frac{1}{2}t}\Gamma(t) \quad , \tag{4}$$

The equation for Γ is

$$\Gamma(t) = e^{-\frac{1}{2}t} + \lambda \int_0^\infty dt' e^{-\frac{1}{2}(t+t')} J_0(2\sqrt{tt'})\Gamma(t'), \tag{5}$$

where J_0 is a Bessel function and

$$\lambda = -i\bar{a}(1+v^2)^{-1}w^{-\frac{1}{2}}. \tag{6}$$

$w^{-1/2}$ in (4) and (6) is defined in the w plane with a cut Re w > 0.

The Fredholm equation for I is solved first in the form of a Neumann series in powers of λ, and is then continued analytically over the entire λ plane. As a result we get

$$\bar{\Gamma}(\lambda) = \frac{3\rho - 1}{\rho^2} \sum_{k=0}^{\infty} \frac{-\lambda_k^{-1}}{\lambda - \lambda_k}, \quad \lambda_k = (-1)^k \rho^{-(2k+1)}, \tag{7}$$

$$\rho = \frac{1}{2}(\sqrt{5} - 1).$$

After using (3) and changing over to the variables (2), Eq. (1) takes the form

$$\bar{\Gamma}(\lambda) = -\frac{1-v^2}{1+v^2}\frac{1}{\lambda} + \frac{\bar{a}^2}{(1+v^2)^3}\frac{1}{\lambda^3} \equiv f(\lambda) \quad . \tag{8}$$

If we are interested in the true spectrum without damping, which lies below the threshold, $w < 0$, then $\lambda > 0$. A graphic solution of (8) with $\lambda > 0$ is shown in Fig. 2, which shows $\bar{\Gamma}(\lambda)$ (solid curve) and $f(\lambda)$ (dashed) at $u = 0$, 1, and ∞. Each root of (8) $\lambda_s^*(u)$, $s = 0, 1, 2\ldots$, determines the spectrum branch (Fig. 3)

$$w_s(u) = -\bar{\alpha}^2 \lambda_s^*(u)^{-2} (1 + u^2)^{-2}. \tag{9}$$

Equation (8) has two types of roots. The roots with $s = 1, 2, \ldots$, resulting from the intersection of the curve of f with the "tangensoid" branches of $\bar{\Gamma}$ depends little on u and are bounded from above and from below by two poles of the same order of magnitude as u varies from 0 to ∞. When $s \simeq 1$ we have $\lambda_s^* \simeq 1$ and the roots λ_s^* increase exponentially with increasing s. These roots determine the spectrum branches lying above the threshold and near it in the interval $\Delta w \simeq \bar{\alpha}^2$; they describe, at least at $p = 0$, the bound states of the electron and the phonon. The binding energies decrease like p^{-4} with increasing momentum, and exponentially with increasing number of the branch. The singular root λ_0^* of (8) arises when the f curve intersects the first branch of $\bar{\Gamma}$. The value of λ_0^* depends strongly on u. When $u < 1$ we have $\lambda_0^* \simeq \bar{\alpha}$. The branch $w_0(u)$ then corresponds to the unperturbed spectrum of the free electron. When $u = 1$ we have $\lambda_0^* \simeq \bar{\alpha}^{2/3}$; this yields $|w_0| \simeq \bar{\alpha}^{2/3}$, i.e., the branch $w_0(u)$ is still quite far from the branches describing the bound states. When $u > 1$ we have $\lambda_0^* \simeq 1$, i.e., the branch $w_0(u)$ already falls in the same energy region as the other branches. It is obvious that in this region of momenta the distinction between the proper electronic spectrum and the spectrum of bound states of an electron with a phonon has no clear-cut meaning.

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