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Subband mobilities and Dingle temperatures within a two-subband model in the presence of localized states

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We calculate the transport time and the single-particle relaxation times in a two subband system of a two-dimensional electron gas. We take into account screening and density of states effects under the assumption that disorder leads to localized states in both subbands. We find that the single-particle relaxation time of the second subband is always larger than for the first subband.The transport time of the second subband can be smaller or larger than the transport time of the first subband.

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Occupation of the second subband in a two dimensional electron gas (2DEG) has to lead to a change of the electron conductivity. This effect was under study for quite a long time. The main attention was paid to investigate the influence of the intersubband scattering processes on the transport in 2DEG [1 – 5]. Recently a new two subband systems were developed [6, 7] in which intersubband scattering is expected to be negligible [8] and very other effects, in the first line the change of the screening, define transport and quantum properties in small normal magnetic fields. In this particular case the two-subband system is very similar to the single subband one in the presence of a parallel magnetic field.

Some time ago we calculated the magnetoconductivity of a two-dimensional electron gas in a parallel magnetic field at zero temperature [9]. With a magnetic field one polarize the electron gas. When the Zeeman energy is smaller than the initial Fermi energy the 2DEG is partially spin-polarized. The system consist of two independent spin subbands if spin-flip processes are absent. The two spin-bands correspond to two electron gases with different Fermi energies. The two electron gases contribute to the screening while the density of states is modified. For zero magnetic field one has a spin-degeneracy of $g_s = 2$ and above a critical field one has a totally polarized system, which correspond to a single subband system with $g_s = 1$. We found a huge increase of the resistance between a unpolarized to a fully polarized system [9]. It was observed that the theory is in reasonable agreement with experimental results [9, 10].

In the present paper we apply an analogues approach to a very conventional two-subband 2DEG in which the intersubband scattering is negligible. We discuss the behavior of the single-particle relaxation time (defining the Dingle temperature) [5] and the transport scattering time as a function of the electron density. As an application of our calculations we consider Si(111)-MOSFET's [6, 7].

Transport time calculations for a low density electron system should include the transition of the electron system to strong localization (metal-insulator transition, MIT). The effects of a MIT on the magnetoresistivity in the two spin-band model has already been discussed [11]. In the region of localized states the conductivity should vanish when the temperature goes to zero. In the region of delocalized states the conductivity should be finite even at zero temperature. This argument only holds for the first subband. It is not clear what one would expect for the behavior of electrons in the second subband. We suppose that there exists a mobility edge in every subband. With this assumption we calculate the transport times [5, 12] of a two subband system as function of the electron density.

An important point in our approach is the absence of intersubband transitions. If different subbands are occupied by electrons of different valleys with a distance between energy minima in the k-space equal to k_0 the ratio between intersubband and intrasubband scattering time is about $(k_F/k_0)^3$. For typical electron density in Si-MOSETs this estimate gives $\leq 10^{-3}$.

Our model consists of a 2DEG with two subbands. For a total electron density $N < N_0$ only the first subband with $g_v^{(1)}$ is occupied. For $N > N_0$ two bands are occupied: the first subband with electron density $N^{(1)}$ and the second subband with $g_v^{(2)}$ and electron density $N^{(2)}$. For $N > N_0$ the density of electrons in the first subband is given by $N^{(1)} = (g_v^{(1)} N + 2g_v^{(1)} N_0)/(g_v^{(1)} +$

$+ g_v^{(2)}$) and in the second subband by $N^{(2)} = (g_v^{(2)}N - 2g_v^{(1)}N_0)/(g_v^{(1)} + g_v^{(2)})$. The two Fermi wave numbers are related by the relation $(4\pi N^{(1)}/g_s g_v^{(1)})^{1/2} = k_F^{(1)} \geq k_F^{(2)} = (4\pi N^{(2)}/g_s g_v^{(2)})^{1/2}$.

All electrons contribute to the screening. In the lowest order of the disorder the expression of the transport time for subband (l) is a q-integral $(0 \leq q \leq 2k_F^{(l)})$ over the random potential $< |U(q)|^2 >$ and the screening function $\varepsilon(q)$ with some weight factor. $\varepsilon_F^{(l)}$ is the Fermi energy $\varepsilon_F^{(l)} = \hbar^2(k_F^{(l)})^2/2m^*$ of band (l) with Fermi wave number $k_F^{(l)}$. Accordingly, the transport time $\tau_{t,0}^{(l)}$ of electrons in band (l) is given by [12]

$$\frac{\hbar}{\tau_{t,0}^{(l)}} = \frac{1}{2\pi\varepsilon_F^{(l)}} \int_0^{2k_F^{(l)}} dq \frac{< |U(q)|^2 >}{\varepsilon(q)^2} \frac{q^2}{(4(k_F^{(l)})^2 - q^2)^{1/2}}, \quad (1)$$

with the screening function $\varepsilon(q)=1+F_c(q)q_s(q)/q$, the q-dependent screening number

$$q_s(q)/q_s = g_v^{(1)} \left(1 - \mathrm{Re}\sqrt{1 - 4(k_F^{(1)})^2/q^2}\right) +$$

$$+ g_v^{(2)} \left(1 - \mathrm{Re}\sqrt{1 - 4(k_F^{(2)})^2/q^2}\right)$$

and $q_s = g_s/a_B^*$,where a_B^* is the effective Bohr radius; $F_c(q)$ is the form factor for the Coulomb interaction due to the finite extension of the electron gas [5]. We use charged impurity scattering $< |U(q)|^2 >= N_i(2\pi e^2 F_i(q)/\varepsilon_L q)^2$ with impurities at the interface with the density N_i and a form factor $F_i(q)$ for the finite extension of the electron gas [5]. The two bands give rise to a mobility $\mu_0^{(l)} = e\tau_{t,0}^{(l)}/m^*$. For transport characteristics calculated in the lowest order of the disorder we use the index (0).

In addition we introduce a metal-insulator transition in the mobility of both bands via [13]

$$\mu^{(l)}(N^{(l)} \geq N_c^{(l)}) = \mu_0^{(l)}(1 - N_c^{(l)}/N^{(l)}) \text{ and}$$
$$\mu^{(l)}(N^{(l)} < N_c^{(l)}) = 0. \quad (2)$$

with a mobility edge at $N_c^{(l)}$ in the two bands. For $N^{(l)} \gg N_c^{(l)}$ one has $\mu^{(l)} = \mu_0^{(l)}$. Both bands contribute to the conductivity $\sigma = N^{(1)}e\mu^{(1)} + N^{(2)}e\mu^{(2)}$ from which one can define an effective transport time $\tau_t = m^*\sigma/Ne^2 = \tau_t^{(1)}N^{(1)}/N + \tau_t^{(2)}N^{(2)}/N$.

The single-particle relaxation time $\tau_{s,0}^{(l)}$ of the two bands describes the Shubnikov-de Haas oscillations and the Dingles temperatures of the two bands $k_B T_D^{(l)} = \hbar/2\pi\tau_{s,0}^{(l)}$. The single-particle relaxation time is expressed by an expression similar to the one given in equation (1), but q^2 in the numerator must be replaced by $2k_F^2$ [12]. All scattering processes contribute to the single-particle relaxation time while backscattering processes prevail in the transport scattering time. The single-particle relaxation time is connected to density of states modifications and is not sensitive to a MIT. The density of states is finite at the electron density where the MIT occurs [12]. Therefore no critical density and no MIT is introduced for the calculation of the single-particle relaxation time.

Using Eq.(1), (2) below we calculate numerically the transport time and single-particle relaxation time. Having in mind the application to the 2DEG in Si-(111) MOSFET we use a two subband model [7] with $g_v^{(1)}$=2, $g_v^{(2)}$=4, and $N_0 = 1.8 \cdot 10^{11}\,\mathrm{cm}^{-2}$. The impurity density $N_i = 4 \cdot 10^{11}\,\mathrm{cm}^{-2}$ and $N_c^{(1)} = N_c^{(2)} = 0.6 \cdot 10^{11}\,\mathrm{cm}^{-2}$ have been chosen to achieve agreement with experimental data of Ref.[6] obtained for silicon(111).

Results of our calculations are shown in Fig.1. Accordingly, there is a MIT in the first subband at a

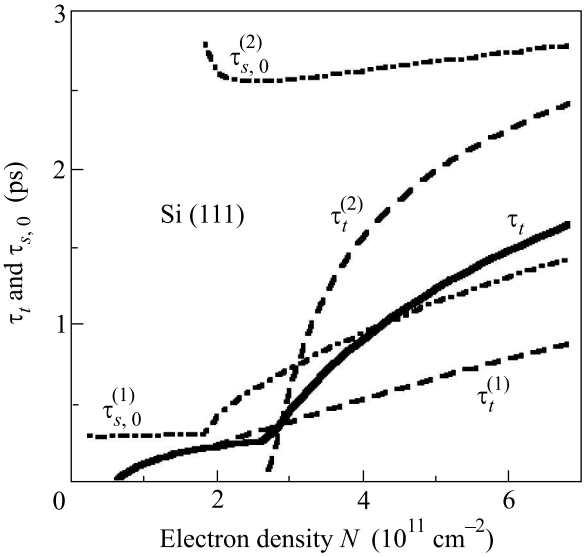

Fig.1. Transport times $\tau_t^{(l)}$ and single particle relaxation time $\tau_{s,0}^{(l)}$ of the two subbands $l = 1, 2$ versus total electron density as dashed and dash-dotted lines, respectively. The solid line is the effective transport time of the two bands. For the transport time two mobility edges have been assumed at $N_c^{(1)} = N_c^{(2)} = 0.6 \cdot 10^{11}\,\mathrm{cm}^{-2}$. The impurity density is $N_i = 4 \cdot 10^{11}\,\mathrm{cm}^{-2}$

total electron density $N_1 = N_c^{(1)} = 0.6 \cdot 10^{11}\,\mathrm{cm}^{-2}$ and for density $N < N_1$ we have $\tau_t = \tau_t^{(1)} = 0$. A MIT for the second subband occurs at a *total* electron density $N_2 = 2.7 \cdot 10^{11}\,\mathrm{cm}^{-2}$ where $\tau_t^{(2)} = 0$. There are localizes states in the second subband for $N_0 < N < N_2$ and this leads to a effective transport time which is lower than the transport time of the first subband: $\tau_t = \tau_t^{(1)}N^{(1)}/N <$

$< \tau_t^{(1)}$, see Fig.1. Due to increasing screening in the first subband, when the second subband becomes occupied for $N > N_0$, $\tau_t^{(1)}$ and $\tau_{s,0}^{(1)}$ increases with increasing density. We mention that within an one-subbad model $\tau_{t,0}^{(1)}$ and $\tau_{s,0}^{(1)}$ are only weakly dependent on the electron density. One can see explicitly that $\tau_{s,0}^{(2)}$ is nearly constant, the weak increase at small density $N \simeq N_0$ is due to weak confinement at low density. We note that $\tau_{s,0}^{(2)} \gg \tau_{s,0}^{(1)}$ due to better screening effects in the second subband.

In Fig.2 we show the ratio $\tau_t^{(2)}/\tau_t^{(1)}$ and $\tau_{s,0}^{(2)}/\tau_{s,0}^{(1)}$ versus total electron density N. Near N_2 there is a small

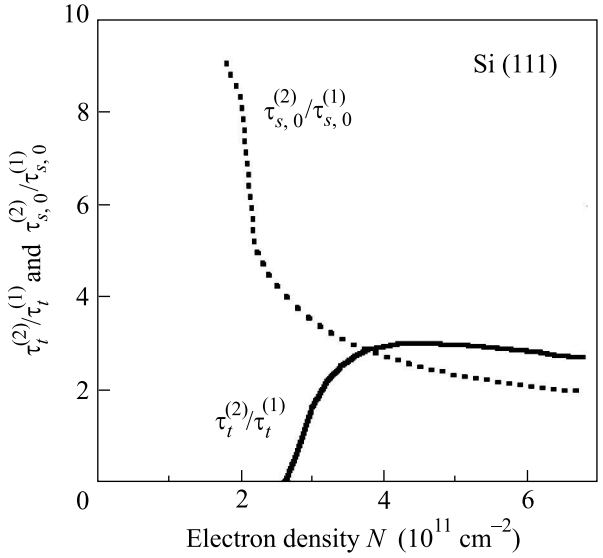

Fig.2. Ratio $\tau_t^{(2)}/\tau_t^{(1)}$ and $\tau_{s,0}^{(2)}/\tau_{s,0}^{(1)}$ versus total electron density N with $N_i = 4 \cdot 10^{11}$ cm^{-2}. Two mobility edges have been assumed at $N_c^{(1)} = N_c^{(2)} = 0.6 \cdot 10^{11}$ cm^{-2}. The ratio $\tau_{s,0}^{(2)}/\tau_{s,0}^{(1)}$ begins at a total electron density $N_0 = 1.8 \cdot 10^{11}$ cm^{-2}. The ratio $\tau_t^{(2)}/\tau_t^{(1)}$ begins at a total electron density $N_2 = 2.7 \cdot 10^{11}$ cm^{-2}

density range $2.7 \cdot 10^{11}\,\text{cm}^{-2} < N < 3 \cdot 10^{11}\,\text{cm}^{-2}$ where $\tau_t^{(2)}/\tau_t^{(1)} < 1$ while at higher density $\tau_t^{(2)}/\tau_t^{(1)} > 1$. For the single particle relaxation time we find $\tau_{s,0}^{(2)}/\tau_{s,0}^{(1)} > 1$ for all densities. With the occupation of the second subband we note a strong decrease of $\tau_{s,0}^{(2)}/\tau_{s,0}^{(1)}$ for $N > N_0$ due to the strong density dependence of $\tau_{s,0}^{(1)}$. The increase of $\tau_t^{(2)}/\tau_t^{(1)}$ with increasing density near N_2 has his origin in the existence of a mobility edge at N_2. Without localized states the ratio $\tau_{t,0}^{(2)}/\tau_{t,0}^{(1)}$ would be very similar to the ratio $\tau_{s,0}^{(2)}/\tau_{s,0}^{(1)}$.

Neglecting intersubband scattering is justified for silicon (111) [8], but might be questionable for other systems, for instance $GaAs/Al_xGa_{1-x}As$ heterostructures [5]. However, intersubband scattering begins at densities where a new subband becomes occupied, but there we assume localized states. Moreover, when discussed in terms of ratios, intersubband scattering might not be so important. Therefore, we apply now our results in a quantitative way to heterostructures, where, in addition, the disorder is of long-range order and theory predicts for high density $\tau_{t,0}^{(1)}/\tau_{s,0}^{(1)} \gg 1$ [12, 14].

SdH oscillations are characterized by the carrier mass and the Dingle temperature, which is proportional to the disorder present in the sample. In experiment on $GaAs/Al_xGa_{1-x}As$ heterostructures with double subband occupancy a double periodicity in Shubnikov-de Haas oscillations with two different Dingle temperatures was observed, see Fig.1 in Ref. [1]. From this Fig.1 it follows that the single-particle relaxation time of the first subband is smaller than the single particle relaxation time of the second subband.

A similar two-subband system in $GaAs/Al_xGa_{1-x}As$ heterostructures was studied in another experiment: it was found perplexing [3] that one can find in the same sample $\tau_t^{(2)} > \tau_t^{(1)}$ and $\tau_t^{(2)} < \tau_t^{(1)}$, depending on the density, whereas $\tau_{s,0}^{(2)} > \tau_{s,0}^{(1)}$ always holds. We explain the fact that $\tau_t^{(2)} < \tau_t^{(1)}$ by the existence of a mobility edge in the second subband, see Fig.2. It is generally believed [3] due to theoretical results [12, 14] that the scattering time is larger than the single-particle relaxation time. But this theoretical believe concerns systems without localized states: $\tau_{t,0}^{(l)} > \tau_{s,0}^{(l)}$. With localized states and near the mobility edge one finds $\tau_t^{(l)} < \tau_{s,0}^{(l)}$. Therefore, we claim that we have resolved a 20 year old puzzle [3] within our two-subband model by taking into account localized states in the second subband.

With very strong disorder one should be able to localize all the states in the lowest subband with $N = N_1 < N_0$ and $N_c^{(1)} \simeq N_0$. Due to the stronger screening in the presence of the second subband stronger disorder should have a very weak effect on $N_c^{(1)}$. Therefore we predict that $N_c^{(1)} \leq N_0$.

In this paper we have shown that a two-suband model with localized states give new insight into transport scattering times and Dingle temperatures in two-subband systems. Our predictions made with our model should be tested. For instance, we suggest that near N_2 one should find $\tau_t < \tau_t^{(1)}$. We claim that the strong density dependence of $\tau_t^{(1)}$ and $\tau_{s,0}^{(1)}$ is a strong indication for the presence of a second subband.

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