CHARACTERISTIC COSMOLOGICAL LENGTH

A. D. Chernin
A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences
Submitted 15 August 1968
ZhETF Pis. Red. 8, No. 11, 633-635 (5 December 1968)

If both nucleons and the isotropic electromagnetic radiation are taken into account in the Friedmann cosmological model, then the dependence of the radius of curvature a on the proper time τ takes the form [1]

$$a = A_m(\mathrm{ch}\eta - 1) + A_r \,\mathrm{sh}\eta,$$
$$c\tau = A_m(\mathrm{sh}\eta - \eta) + A_r(\mathrm{ch}\eta - 1). \quad (1)$$

Here η - parameter, A_m and A_r - arbitrary independent constants with dimension of length, expressed in terms of combinations of the densities of the matter ρ_m and of the radiation ρ_r, and the critical density ρ_c

$$A_m = \frac{1}{2}k^{-1/2}\rho_m(\rho_c - \rho)^{-3/2}, \quad (2)$$
$$A_r = k^{-1/4}\rho_r^{1/4}(\rho_c - \rho)^{-1},$$
$$k = \frac{8\pi G}{3c^2}, \quad \rho = \rho_m + \rho_r. \quad (3)$$

The solution (1) - (3) corresponds to an infinite volume and negative spatial curvature of a world that is isotropic in the mean ("open model"), which apparently represents correctly the main large-scale properties of real space-time.

To find the constants A_m and A_r, we shall use astronomical data on the densities ρ_r and ρ_m. The former has been measured by now with sufficient accuracy [2]:

$$\rho_r \simeq 6 \times 10^{-34} \ \mathrm{g/cm^3}.$$

On the other hand, the value of ρ_m is known with a great degree of uncertainty:

$$10^{-31} < \rho_m < 5 \times 10^{-28} \ \mathrm{g/cm^3}.$$

The upper limit is the upper bound for the contribution of all the forms of matter [3], the quantity 10^{-31} g/cm^3 corresponding to the minimum estimate of the average density of the visible matter of the galaxies [4]. The open model corresponds to a somewhat narrower interval

$$10^{-31} < \rho_m < \rho_c = \frac{3}{8\pi G}H^2 \simeq 2 \times 10^{-29} \ \mathrm{g/cm^3},$$
$$H = 100 \ \mathrm{km/sec\text{-}Mpsec}. \quad (4)$$

From formulas (2) - (3) and from the presented observational data it follows that the constants A_m and A_r are close in order of magnitude [1]. Moreover, the permissible interval

[1] We do not consider the degenerate case $\rho_c - \rho \to 0$ corresponding to a transition to a quasi-Euclidian model in which there is only one constant.

(4) of the values of the density of matter contains a value of ρ_m for which A_m and A_r are exactly equal:

$$\rho_m = 2 \frac{A_r}{A_m} \rho_r^{1/2} (\rho_c - \rho)^{1/2},$$
$$A_m = A_r \text{ when } \rho_m \simeq 2 \times 10^{-31} \text{ g/cm}^3. \tag{5}$$

We note one more remarkable numerical coincidence. At the values of ρ_m, ρ_r, and ρ_c given above, the constant length A_r turns out to be equal - accurate to a factor on the order of unity - to the numerical value of $h^2/Gm^3 \simeq 10^{26}$ cm, made up of the universal constants (h - Planck's constant, G - gravitational constant, m - nucleon mass).

These results allow us to conclude that in the cosmological theory there exists a characteristic length constant A such that

$$A_m \simeq A_r \simeq A \simeq \frac{h^2}{Gm^3} . \tag{6}$$

Relation (6) is an example wherein a numerical connection of surprising accuracy is established between the cosmological and microscopic constants. Searches for such coincidences, under the assumption that they are not accidental, are the subject of a large number of papers (see, e.g., [5-8]). The difficulties encountered in this matter are analyzed in a recent review [9] using the cosmological theory with a Λ term as an example. We call attention in this connection to the fact that the solution (1) has been obtained from standard gravitational equations [10] without a Λ term.

If we assume that A is an independent world constant, then relation (6) can be taken to mean a formula for one of the constants h, G, or m. For example, the proton mass is

$$m = (h^2/AG)^{1/3}.$$

With the aid of the cosmological length A and the microscopic constants h, G, c, and e (electron charge), we can set up three additional constants with dimension of mass:

$$M = Ac^2/G \simeq 10^{54} \text{ g},$$
$$\mu_1 = e^2/Ac^2 \simeq 10^{-66} \text{ g},$$
$$\mu_2 = h/Ac \simeq 10^{-63} \text{ g}.$$

The large mass M gives the total amount of matter inside a sphere of radius ~a at the instant when $a \sim A$. The small masses μ_1 and μ_2 (which are close in order of magnitude) satisfy, in particular, the experimental limitations on the rest mass of the photon [11] or neutrino [12].

We note in conclusion that the contemporary value of the radius of curvature a is larger than the characteristic length A (e.g., $a \simeq 10^{29}$ cm at $\rho_m \simeq 2 \times 10^{-31}$ g/cm^3). Since a increases with time (1), there existed in the past an instant τ_A ($\rho_m(\tau_A) \sim \rho_r(\tau_A) \sim 10^{-22}$ g/cm^3), at which the variable length a coincided with the characteristic length A. This instant is unique in a certain sense; if a qualitative change of the large-scale geometric structure of the metagalaxy took place in the past, it possibly occurred precisely in the epoch $\tau \sim \tau_A$. Such a change could be a transition (see [13]) from the pre-Friedmann stage (anisotropic, inhomogeneous) to the stage of large-scale isotropy described by solution (1).

The author thanks L. E. Gurevich for interesting discussions.

[1] A. D. Chernin, Astron. Zh. 42, 1124 (1965) [Sov. Astron.-AJ 9, 871 (1966)].
[2] A. A. Penzias and R. W. Wilson, Astr. J. 142, 419 (1965).
[3] Ya. B. Zel'dovich and Ya. A. Smorodinskii, Zh. Eksp. Teor. Fiz. 41, 907 (1961) [Sov. Phys.-JETP 14, 647 (1962)].
[4] J. Oort, La Structure et l'Evolution de l'Universe, Brussells, 1958.
[5] A. S. Eddington, Mathematical Theory of Relativity, Cambridge U. P.
[6] P. A. M. Dirac, Nature 139, 323 (1937).
[7] G. Gamow, Phys. Rev. Lett. 19, 759 (1967).
[8] Ya. B. Zel'dovich, ZhETF Pis. Red. 6, 922 (1967) [JETP Lett. 6, 345 (1967)].
[9] Ya. B. Zel'dovich, Usp. Fiz. Nauk 95, 209 (1968) [Sov. Phys.-Usp. 11 (1968)].
[10] L. D. Landau and E. M. Lifshitz, Teoriya polya (Field Theory), Fizmatgiz, 1962, p. 328 [Addison Wesley, 1962].
[11] I. Yu. Kobzarev and L. B. Okun', Usp. Fiz. Nauk 95, 131 (1968) [Sov. Phys.-Usp. 11 (1968)].
[12] S. S. Gershtein and Ya. B. Zel'dovich, ZhETF Pis. Red. 4, 174 (1966) [JETP Lett. 4, 120 (1966)].
[13] L. M. Ozernoi and A. D. Chernin, ibid. 7, 436 (1968) [7, 342 (1968).

NONLINEAR DEVIATIONS OF THE DIRECTION OF A REFLECTED WAVE IN VACUUM AT A PLASMA BOUNDARY

B. Chakraborti
Mathematics Division, Jadhavpur University, Calcutta, India
Submitted 5 August 1968
ZhETF Pis. Red. 8, No. 11, 636-638 (5 December 1968)

One of the nonlinear effects arising in the propagation of an electromagnetic wave in a semi-infinite plasma is the increase of the wave number in the third approximation of the solution of the boundary-value problem, when the electromagnetic wave is obliquely incident on the interface from the vacuum and excites only transverse waves in the plasma [1-4]. This makes it possible to obtain several nonlinear waves generated on the boundary and propagating in the vacuum along directions that differ little from the direction of propagation of the reflected wave; actually, these directions lie inside a cone whose axis coincides with the direction of propagation of the reflected wave. The vertical angle of the cone, i.e., the maximum deflection angle, is a second-order quantity and is proportional to the square of the amplitude of the incident wave. In a Vlasov plasma, which has only an intrinsic electromagnetic field, the first harmonic of the transverse wave generates (in the third approximation) first and third harmonic waves with amplitudes proportional to the cube of the amplitude of the linear wave [3,4]. The growth of the wave number follows also from the third-approximation solution and is proportional to the square of the amplitude of the linear wave.

Equality of the phases of the waves in the vacuum and in the plasma at the boundary yields

$$\omega \cos \alpha' = k'c \cos \beta',$$

where

$$\alpha' = \alpha + \delta\alpha, \quad k' = k + \delta k, \text{ and } \beta' = \beta + \delta\beta.$$