instant is unique in a certain sense; if a qualitative change of the large-scale geometric structure of the metagalaxy took place in the past, it possibly occurred precisely in the epoch $\tau \sim \tau_A$. Such a change could be a transition (see [13]) from the pre-Friedmann stage (anisotropic, inhomogeneous) to the stage of large-scale isotropy described by solution (1).

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NONLINEAR DEVIATIONS OF THE DIRECTION OF A REFLECTED WAVE IN VACUUM AT A PLASMA BOUNDARY

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One of the nonlinear effects arising in the propagation of an electromagnetic wave in a semi-infinite plasma is the increase of the wave number in the third approximation of the solution of the boundary-value problem, when the electromagnetic wave is obliquely incident on the interface from the vacuum and excites only transverse waves in the plasma [1-4]. This makes it possible to obtain several nonlinear waves generated on the boundary and propagating in the vacuum along directions that differ little from the direction of propagation of the reflected wave; actually, these directions lie inside a cone whose axis coincides with the direction of propagation of the reflected wave. The vertical angle of the cone, i.e., the maximum deflection angle, is a second-order quantity and is proportional to the square of the amplitude of the incident wave. In a Vlasov plasma, which has only an intrinsic electromagnetic field, the first harmonic of the transverse wave generates (in the third approximation) first and third harmonic waves with amplitudes proportional to the cube of the amplitude of the linear wave [3,4]. The growth of the wave number follows also from the third-approximation solution and is proportional to the square of the amplitude of the linear wave.

Equality of the phases of the waves in the vacuum and in the plasma at the boundary yields

$$\omega \cos \alpha' = k'c \cos \beta',$$

where

$$\alpha' = \alpha + \delta\alpha, \quad k' = k + \delta k, \text{ and } \beta' = \beta + \delta\beta.$$

Consequently,

$$\omega \sin \alpha \delta\alpha = c(k \sin \beta \delta\beta - \delta k \cos \beta)$$

and $\delta\alpha$ is the angle between the direction of propagation of the nonlinear wave generated by the plasma in vacuum and the direction of propagation of the reflected wave. The nonlinear waves considered in [3] and [4] depend on the condition

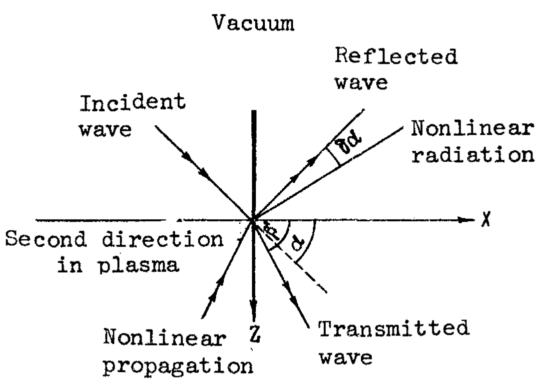
$$\delta\alpha = 0, \quad \delta\beta = (\delta k/k) \cot \beta .$$

If $\delta\beta \neq (\delta k/k) \cot \beta$, then $\delta\alpha \neq 0$ and $\delta\alpha$ can be regarded as a deviation from the laws of geometrical optics, brought about by the nonlinearity of the exact theory. From the results of [3], the maximum value of $\delta\alpha$, obtained when $\delta\beta = 0$, is given by

$$(\delta \alpha)_{max} = \frac{-\delta k}{k} \frac{kc}{\omega} \frac{\cos \beta}{\sin \alpha} = E_t^2 \left(\frac{e}{mc}\right)^2 \frac{\cos \beta}{\sin \alpha} \frac{\omega^2 - \omega_0^2}{16 \omega H} ,$$

which is obtained in the cold-plasma approximation (E is the amplitude of the transmitted nonlinear wave). It should be noted that the nonlinear wave generated in the vacuum is not the reflected wave as in the case of the linear approximation, since it is not due to reflection in the classical sense, but is generated by the plasma radiation as a result of interaction between the incident wave and the plasma.

Waves generated in vacuum and satisfying the dondition $\delta \neq 0$ on the boundary have phases that differ from the phase of the incident wave, and therefore these waves should be regarded independently, and not in parallel with the first harmonic of the linear approximation. The source of the linear waves is a wave propagating in vacuum, whereas there exists no such source for the transmitted nonlinear waves, since they are produced as a result of interaction between the incident wave and the nonlinear plasma. From the equality of the phases of the boundary it follows that there are only two wave propagation directions on each side of the boundary, and wave propagation in the opposite direction is absurd even theoretically. The frequency, amplitude, and direction of the incident wave are determined by the properties of the wave source, and not by the properties of the nonlinear plasma, so that the possibility of obtaining waves in the propagation direction of the incident wave in vacuum are excluded. Consequently, the only theoretically possible direction is one making an angle $\alpha + \delta\alpha$ with ox, where $\delta\alpha$ is the angle of deviation from the propagation of the linear reflected wave. On that side of the boundary where the plasma is located, equality of the phases determines the direction of the passage of the wave and the radiation direction towards the boundary: this direction lies in the plane of incidence of the wave and is symmetrical to the direction of the transmitted



Schematic representation of the nonlinear waves together with the linear waves at the plasma-vacuum boundary.

wave relative to the normal. We shall call this direction the second direction in the plasma (see the figure). From the two optical boundary conditions (namely, continuity of the tangential components of the electric and magnetic fields) we can determine the amplitudes of these nonlinear transverse waves, which propagate along the two following directions for each first or third harmonic: 1) the radiation direction in vacuum and 2) the propagation direction along the second direction in the plasma. It can be easily shown that the assumption that waves propagating along the second direction in the plasma exist does not violate the fact that the nonlinear waves pass through the lower boundary of the plasma layer.

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MECHANISM OF GENERATION OF ULTRAHIGH-ENERGY MUONS WITH NEARLY ISOTROPIC AZIMUTHAL DISTRIBUTION BY COSMIC RAYS

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The results of a recent experiment [1] on the observation of muons from cosmic rays with energy $E > 10^3$ GeV can be simply explained if there exists a mechanism whereby muons can be produced in the upper layer of the atmosphere.

In this paper we call attention to the fact that such a mechanism is contained in natural fashion in the theory of renormalizable weak interactions, developed earlier by the author [2] on the basis of the Kummer-Segre model [3]. According to [2], the weak interaction is described by a Lagrangian

$$L = g_1(\bar{\mu}^*(1 + 3\gamma_5)\mu^+ + \bar{n}^*(1 + 3\gamma_5)p + \bar{e}(1 - 3\gamma_5)e^*)B^+ + \\ + g_2(\bar{\mu}^*(1 + \gamma_5)\nu_\mu + \bar{n}^*(1 + \gamma_5)n + \bar{\nu}_e(1 - \gamma_5)e^*)B^+ + \\ + g_3(\bar{n}^*(1 + \gamma_5)\Lambda)B^0 + h.c., \tag{1}$$

where $B^{\pm,0}$ - bosons with zero spin and mass $M \leq 30$ GeV, μ^* and e^* - neutral leptons, with $m_{\mu^*} \geq m_\mu$, and n^* - neutral baryon. The fields B, μ^*, e^*, and n^* carry a new quantum number, which is rigorously conserved. On the basis of the methods analogous to the current-algebra methods, it was shown in [2] that (1) leads in the low-energy limit to four-fermion interaction with equality of the vector constants of the muon and neutron decay, and with absence of experimentally forbidden interactions with neutral lepton currents.

An important property of the model (1) is that the bosons B should be sufficiently heavy: $M \sim 15 - 30$ GeV, and consequently can be created in noticeably amounts only by primary radiation with energy $E \gtrsim 10^3$ GeV.

The constants $g_{1,2}$ are connected with the weak-interaction constant G by the rela-