wave relative to the normal. We shall call this direction the second direction in the plasma (see the figure). From the two optical boundary conditions (namely, continuity of the tangential components of the electric and magnetic fields) we can determine the amplitudes of these nonlinear transverse waves, which propagate along the two following directions for each first or third harmonic: 1) the radiation direction in vacuum and 2) the propagation direction along the second direction in the plasma. It can be easily shown that the assumption that waves propagating along the second direction in the plasma exist does not violate the fact that the nonlinear waves pass through the lower boundary of the plasma layer.

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MECHANISM OF GENERATION OF ULTRAHIGH-ENERGY MUONS WITH NEARLY ISOTROPIC AZIMUTHAL DISTRIBUTION BY COSMIC RAYS

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The results of a recent experiment [1] on the observation of muons from cosmic rays with energy $E > 10^3$ GeV can be simply explained if there exists a mechanism whereby muons can be produced in the upper layer of the atmosphere.

In this paper we call attention to the fact that such a mechanism is contained in natural fashion in the theory of renormalizable weak interactions, developed earlier by the author [2] on the basis of the Kummer-Segre model [3]. According to [2], the weak interaction is described by a Lagrangian

$$\begin{aligned} L = g_1(\bar{\mu}^*(1+3\gamma_5)\mu^+ + \bar{n}^*(1+3\gamma_5)p + \bar{e}(1-3\gamma_5)e^*)B^+ + \\ + g_2(\bar{\mu}^*(1+\gamma_5)\nu_\mu + \bar{n}^*(1+\gamma_5)n + \bar{\nu}_e(1-\gamma_5)e^*)B^+ + \\ + g_3(\bar{n}^*(1+\gamma_5)\Lambda)B^0 + h.c., \end{aligned} \qquad (1)$$

where $B^{\pm,0}$ - bosons with zero spin and mass $M \leq 30$ GeV, μ^* and e^* - neutral leptons, with $m_{\mu^*} \geq m_\mu$, and n^* - neutral baryon. The fields B, μ^*, e^*, and n^* carry a new quantum number, which is rigorously conserved. On the basis of the methods analogous to the current-algebra methods, it was shown in [2] that (1) leads in the low-energy limit to four-fermion interaction with equality of the vector constants of the muon and neutron decay, and with absence of experimentally forbidden interactions with neutral lepton currents.

An important property of the model (1) is that the bosons B should be sufficiently heavy: $M \sim 15 - 30$ GeV, and consequently can be created in noticeably amounts only by primary radiation with energy $E \gtrsim 10^3$ GeV.

The constants $g_{1,2}$ are connected with the weak-interaction constant G by the relation $G/\sqrt{2} = g_1^2 g_2^2/4\pi^2 M^2$, so that when $M \approx 30$ GeV and $g_1^2 > g_2^2$ the constant g_1 turns out to be on the order of unity. More accurately, the effective constant of the transition

$$p \to n^* + B^+ \tag{2}$$

is, in accordance with (1) (see also [2])

$$g_{eff}^2 \sim 10 g_1^2, \quad g_1^2 \sim 2. \tag{3}$$

We recall that in the theory with intermediate vector meson at $M_W = 30$ GeV the constant of the transition

$$p \to n + W^+ \tag{4}$$

is

$$f^2 = \frac{GM_W^2}{\sqrt{2}} \sim 10^{-2}. \tag{5}$$

We note also that, unlike the theory with the vector W-meson, the dependence of the matrix element $\langle n^*|p\rangle$ on the momentum transfer $(k_p - k_{n^*})^2$ is determined by a scalar form factor. And whereas the form factor of the vector transition $\langle n|p\rangle$ in the theory with the W-meson is quite small in the production of a meson with a large mass, the form factor of the transition $\langle n^*|p\rangle$ should more readily be a slowly varying function of the momentum transfer if the theory is to be self-consistent.

We can conclude from the foregoing that the particles n^* and B should be created in nucleon-nucleon collisions, with a cross section characteristic of ordinary strong interactions. If we recognize that the boson B decays into a $\mu\mu^*$ pair within a time $\sim 10^{-24}$ sec and n^* decays at a mass $2m_p$ into a proton and a $\mu\mu^*$ pair within $\sim 10^{-16}$ sec (the latter under the assumption that the form factor of the vertex $\bar{n}^*pB$ changes little when $(k_p - k_{n^*})^2$ changes from M^2 to m_N^2), then it can be concluded that the particles B and n^* can serve as a source of isotropic muons of ultrahigh energy.

If we assume that the cross section for the production of the pions that carry away a considerable fraction of the primary-proton energy equals the cross section for the creation of the system Bn^*, and if we recognize that the fraction of the pions with energy $E > 10^3$ GeV, which decay into muons, is less than 10^{-1} (see [1,4]), we get a weakening of the dependence of the intensity $I_\mu(\cos\theta)$ at $\theta = \pi/4$ by less than a factor of 4 [1)]. Such a weakening does not contradict the experimentally acceptable [1] dependence $I_\mu(\cos\theta) - \cos\theta^{-1/4}$.

We emphasize that the mechanism considered above generates equally well isotropic electrons of ultrahigh energy.

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ELECTROMAGNETIC RADIUS OF THE PION

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1. In earlier papers [1,2] we indicated the method of estimating the electromagnetic radius of the pion, using essentially the analytic properties of the form factor $G(t)$ [1]. An advantage of this method is that it employs experimental data on $G(t)$ in the finite interval $[t_2, t_1]$, $t_2 < t_1 < 0$ [3], and data on $G(t)$ at $t \geq t_0 = 4m_\pi^2$, obtained from e^-e^+ collision experiments [4]. It is important to emphasize that this yields a new method of determining $G(t)$ in the vicinity of $t \simeq 0$, in which direct extrapolation of $G(t)$ from $[t_2, t_1]$ to the point $t = 0$ is excluded. This is particularly important in connection with the fact that at small t the value of $G(t)$ is known experimentally very poorly, and also in connection with the possible "halo" effects [5], which change radically the behavior of $G(t)$ at $t \simeq 0$.

2. In [2] we analyzed the sum rules for the form factor of the π meson, i.e., we verified the consistency of the experimental data of [3] with those of [4]. It is noted in [2] that for $|G(t)|$ in the vicinity of $t \simeq t_0$ it is necessary to have a more accurate formula than simply the resonance formula [4]. Consequently, an attempt to determine the pion radius on the basis of the dispersion relation (8) of [2] does not lead to reasonable results. In principle, the threshold behavior of $|G(t)|$ could be obtained if the $\pi\pi$ scattering phases were to be known in detail. However, the latter have not yet been sufficiently well determined. We therefore used a modification of the dispersion relations (8) of [2] [2]:

$$G(t) = \frac{1}{\pi}\sqrt{\frac{(t-t_1)(t-t_2)}{(t_0-t)}}\left\{\int_{t_0}^{\infty}\sqrt{\frac{t'-t_0}{(t'-t_1)(t'-t_2)}}\,\frac{\ln|G(t')|}{t'-t}\,dt' + \int_{t_2}^{t_1}\sqrt{\frac{t_0-t'}{(t_1-t')(t'-t_2)}}\,\frac{\ln G(t')}{t'-t}\,dt'\right\}, \quad (1)$$

in which the contribution from the near-threshold region in the first integral of formula (1) is suppressed as a result of the factor $(t' - t_0)$, and the region of the ρ resonance becomes most important for the calculation of this integral.

3. On the basis of (1) we determined numerically r_π, the electromagnetic radius of the pion. We used here the experimental data described by the following expressions: when

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1) We use here the notation of [2].

2) Formula (1) is valid for $t \in [t_1, t_0]$ provided $G(t)$ has no complex zeros.