to one of the velocities of transverse sound in tin, which was determined in [2] for the [010] direction. It can thus be assumed that a shear sound wave is excited in the tin.

Electromagnetic excitation of sound in a metal was first observed in bismuth. The hsarp increase of the amplitude of the quantum oscillations, apparently likewise connected with excitation of standing sound waves, was observed in aluminum [4]. Unlike in aluminum, however, the helicoidal sound-generation mechanism proposed in [4] is impossible in tin. We propose that the sound wave in tin is excited by the skin current, which produced in a magnetic field a periodic shear stress along the sample surface. The additional contribution made to the surface impedance by such an excitation mechanism was considered in [5, 6]. It is possible that the increase of the amplitude of the observed quantum oscillations is also connected with this excitation mechanism.

It should be noted in conclusion that the additional contribution made to the surface impedance upon production of standing sound waves in tin is so large, that a resonance curve is readily observed on an S-1-1 oscilloscope (we used an NMR procedure with deep frequency modulation). The signal is detected directly from the resonant circuit of the autodyne.

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T-MODELS OF "SPHERE" IN GENERAL RELATIVITY THEORY

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It is assumed that Tolman's solution [1-3] represents all the possible general relativity theory (GRT) nonstatic models of a sphere made of dust without pressure. However, in the integration of Einstein's equations, it is implicitly assumed that the co-moving condition $G_0^1 = 0$ does not reduce to a trivial identity. Therefore, Tolman's solution implies an additional requirement with respect to the initial metric, namely $(\partial R/\partial \chi)_T \neq 0$, and accordingly with respect to the permissible distributions of the relativistic per-unit energy of the "dust," $W = 1 + f > 0$, which determines the ratio of the increment s of the current active and proper masses of the sphere, $m' = W\mathcal{M}'$, and also the geometry of the space-like cross sections V_3 [3].

Actually these limitations do not follow from spherical symmetry or from the field equa equations, and thus sight is lost of a special type of paradoxical configuration of the general-relativity sphere, having a constant active mass $M = r_0/2$ at an arbitrary total rest mass $\mathcal{M}(\chi)$ of the constituent "dust" with $W \equiv 0$, namely T-models, for which the co-moving reference frame is also the synchronous T-system [2, 4]:

$$ds^2 = d\tau^2 - e^{\omega(\chi,\tau)} d\chi^2 - r^2(\tau)\{d\nu^2 + \sin^2\nu\, d\phi^2\}. \quad (1)$$

The purpose of this note is to point out this additional class of exact internal solutions for the metric (1):

$$r = r_0/2(1-\cos\eta),\ \tau = r_0/2(\eta - \sin\eta)\ ,$$

$$e^{\omega/2} = \epsilon\,\mathrm{ctg}\,\eta/2 + \mathcal{M}'(1 - \eta/2\,\mathrm{ctg}\,\eta/2),\quad \rho = \frac{\mathcal{M}'}{4\pi r^2 e^{\omega/2}} \geqslant 0 \tag{2}$$

$$\epsilon = 0,\ \pm 1;\quad \mathcal{M}'(\chi) \geqslant 0,\quad 0 < r_0 < \infty$$

and to discuss briefly its important physical consequences.

The T-models of the "sphere" are simple inhomogeneous generalizations of the anisotropic cosmological model of the "quasi-open" type [5,6,2] with hypercylindrical space-like sections $V_3 = (S_2 \times R_1)$, which have no symmetry center, are open in the "radial" direction $(-\infty < \chi < \infty)$, and have an infinite proper volume.

The transverse pulsations of the hypercylinder V_3 recall the Friedmann open model [1,2]: the general expansion phase begins from a singularity ($\rho = \infty$), and is then also simultaneously replaced by unlimited contraction back to a line or a point. The peculiarity of the T-models lies in the aperiodic dynamics of their inhomogeneous longitudinal deformations along the generatrices of V_3, which can lead to additional singularities $e^{\omega(\chi,\tau)} = 0$ - collapse of individual parts or of the entire hypercylinder V_3 to a sphere S_2.

2. In contrast to the Newton-like Tolman-Friedmann models, the T-models have no classical analog, and their existence and unusual properties are due to nonlinearities of the Einstein equations. They give the simplest example of the general-relativity models and nonstatic fields of anomalous "longitudinal" type, which are nonexistent in the classical and linearized gravitational theories, and thus, demonstrate qualitatively a new aspect of the relation between the latter and GRT from the point of view of the correspondence principle.

In view of the impossibility of gravitational radiation for the case of spherical symmetry, a far reaching similarity remains between the local properties of the relativistic and Newtonian models, and their differences are due essentially to the nonlinearity of the GRT, which is manifest primarily in the fact that the active mass $m(\chi)$ of the sphere coincides with its total energy and includes, besides the rest mass and the kinetic energy of the "dust," also the gravitational potential binding energy. The latter is negative, just as in the Newtonian theory, and leads to a mass defect if $W(\chi) < 1$. It is just this nonlinear contribution which exactly cancels the rest mass of each layer of "dust" ($W \equiv 0$) and ensures constancy of the active mass of the T-models as they increase without limit.

The T-models yield in principle a method, different from that of the closed Friedmann model [2], of realizing the maximum total gravitational mass effect that is possible in the GRT, equal to the total rest mass of the matter; this becomes uniquely manifest via the hypercylindrical geometry of the co-moving space V_3.

In view of the complete gravitational binding of matter, the active mass $M > 0$ should be of "bare" character. It can be interpreted as the geometro-dynamic "massless mass" of the T-regions of the Schwarzschild vacuum field [2,4], into which the T-models go over in the limit as $\rho = 0$, and to which they are closely analogous.

3. As is well known [1,2,7], the external Schwarzschild metric in vacuum is not limited to a pseudosingularity on the gravitational radius R = 2M (which determines only the boundary of the static R-regions, in which a rigid polar system is physically admissible), and can be analytically continued into principally non-static T-regions with $R \leq 2M$, where in a manner of speaking the selected temporal and radial coordinates interchange roles, and a canonical T-system is realized in the form [4]

$$ds^2 = \left(\frac{2M}{T} - 1\right)^{-1} dT^2 - \left(\frac{2M}{T} - 1\right) d\chi^2 - T^2(d\nu^2 + \sin^2\nu\, d\phi^2) \qquad (3)$$

$$(0 < T < 2M, \quad -\infty < \chi < \infty).$$

An entire series of rather strange properties of the Schwarzschild field appears in the T-region (inherent also in T-models), namely: non-static character, homogeneity, temporal character of the geometrical singularity in the "center" T = 0, finite time extent, non-Euclidean hypercylindrical structure of the invariant space-like sections T = const, while others, particularly global anomalies of space-time V_4, exclude an interpretation of the vacuum expansion of the Schwarzschild-Kruskal manifold as a field produced by a localized pointlike mass.

The Schwarzschild sphere is a result of the general-relativity effects of nonlinear growth of the attracting field, and corresponds to the radius of the gravitational capture of all the light rays, so that the T-regions cannot have Euclidean analogs. The correspond to a limiting strong field of an anomalous "longitudinal" type without material sources, a field which cannot be identified with gravitational waves. The T-spheres are constructed on the basis of these vacuum T-regions, and appear as a modification of the latter in the sense that the material is bound gravitationally and is retained by their extremely strong fields whitin the Schwarzschild sphere. Because of total neutralization of the rest mass, the "dust" has little influence on the local properties and replaces, as it were, the trial reference liquid of the synchronous T-system (3).

However, when the T-regions of the initial Schwarzschild-Kruskal field are filled with such a passive matter with $W \equiv 0$, a radical change takes place in the topological structure of V_4, particularly the orientation and the properties of their isotropic boundaries, inasmuch as for the internal solution (2) there is no pseudosingularity on the gravitational radius: $e^{\omega/2} = 2\mathcal{M}(\chi) \neq 0$ when $r(\tau) = 2M$. The Schwarzschild **null-hypersphere** made up of two different semipermeable causal membranes is transformed into a single space-like impenetrable barrier, as a result of which the pair of globally nonequivalent T-regions forms a geodetically complete and "closed in itself" T-model of a sphere, in which the vacuum R-regions vanish.

4. In view of the compactness of the infinite ($-\infty < \chi < \infty$) T-models of the "sphere," the concept of the total energy for them has no physical meaning, just as for the topologically closed Friedmann model [1]. However, in analogy with the "semiclosed" worlds [2], one can consider bounded T-spheres containing empty T- and R-regions of the Schwarzschild field, and by joining the internal ($-\infty < \chi \leq \chi_0$) and external vacuum metrics on the boundary sphere

$\chi = \chi_0$ it is possible to determine for them correctly the Schwartzschild integral of the total mass-energy $M = 1/2r_0 > 0$.

The T-spheres are a new type of relativistic hypothetical objects which contain, within the Schwarzschild sphere, an unlimited amount of matter, and yet behave in vacuum as an ordinary sphere with finite gravitational mass [1)], the latter containing in general no material contribution and having the pure field nature of Wheeler's geometro-dynamic "massless mass" [8].

The boundary of the T-sphere is made up of particles that move along the radial geodesics of the external Schwarzschild field, similar to the trial reference "dust" of the T-system [2]. It is therefore easy to discern even from the Kruskal diagram [7,2] (on which $\chi = \chi_0 = 0$ is represented by a segment of the time axis $u = 0$) that it is possible in principle to have unilateral exchange of information, energy, and matter between the T-spheres and the R-regions.

From the point of view of an observer in the R-region, the evolution of the T-sphere does not differ from the case of the "semi-closed" world [2] with an "equatorial" sphere as a boundary, and is characterized by an asymmetry between the unobservable stage of collapse T_- and the phase of anticollapse T_+, in which the expansion of the boundary sphere begins from a point-like dimension and terminates with asymptotic "cooling" on the Schwarzschild sphere with the typical gravitational self-closing picture.

5. The existence of T-spheres demonstrates the feasibility in principle of realizing an ideal "gravitational machine," which transforms the entire rest mass of matter into radiation energy, and leads to an interesting astrophysical consequence: it is possible, at least in principle, to bind together all the rest mass of any arbitrary amount of matter inside the Schwarzschild sphere produced by collapse, and release completely all its energy equivalent.

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