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INFLUENCE OF GIANT ZEEMAN EFFECT ON THE DIELECTRIC CONSTANT OF AN ANTIFERROMAGNETIC CONDUCTOR

E.L. Nagaev and V.G. Polnikov
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It will be shown in this paper that an external magnetic field can exert an anomalously strong influence on the dielectric constant $\varepsilon(\vec{k}, \omega)$ of the conduction electrons in an antiferromagnetic semimetal or in a strongly doped semiconductor. It is assumed that the magnetic properties of the crystal are governed mainly by the localized moments of the magnetic atoms, and not by the conduction electrons (s-d model). Owing to the exchange interaction with the localized moments, the state of the conduction electrons depends strongly on the character and degree of magnetic ordering of the crystal, which can be varied with an external magnetic field.

In many magnetic conductors the width W of the conduction band greatly exceeds the product of the s-d exchange integral A by the spin S of the magnetic atom. According to [1], in the first-order approximation in AS/W, the appearance of an average moment $\bar{S}$ in the crystal causes a shift of the conduction electron with spin projection σ by an amount $AS\sigma$. The average moment $\bar{S}$ of the antiferromagnet in the field is determined from the condition that the total energy of the system be a minimum. At sufficiently low electron densities n, this energy consists of the exchange energy $\sim kT_N \cos 2\theta$ of the localized moments and the energy $\sim \mu H \cos\theta$ of these moments in the field. Here T_N is the Neel temperature, μ the Bohr magneton, and 2θ the angle between the moments of the antiferromagnet sublattices (the field is perpendicular to the antiferromagnetism vector, so that $\bar{S} = S \cos \theta$).

It follows from the foregoing that $\bar{S} \sim \mu HS/kT_N$, so that the spin splitting of the electronic levels turns out to be here $\sim(AS/kT_N)\mu H$, which is larger by several orders of magnitude than the usual Zeeman splitting μH. Indeed, according to [2], the energy AS of all magnetic materials without exception is of the order of the atomic energy, i.e., it amounts at least to several tenths of an electron volt, and in many cases also to several electron volts. At the same time kT_N is a quantity of second order of smallness relative to the overlap of the d-functions of the neighboring magnetic ions, and its typical value is therefore $\sim 10^{-4} - 10^{-2}$ eV [2].

At small n, if the Fermi energy of the conduction electrons is $E_F \lesssim AS$, the giant Zeeman effect (GZE) described above can lead to complete polarization of the electrons with respect to spin even in moderate fields

$$H \lesssim H_N = \frac{kT_N}{\mu} << \frac{E_F}{\mu}$$

In the case of ordinary antiferromagnets, the sublattice-collapse field is $H_N \lesssim 10^5$ Oe, but in the case of metamagnets it can be only $\sim 10^3$ Oe [1]. Since in such fields the number of electrons with $\sigma = 1/2$ is double the number of electrons having the same σ at $H = 0$, their kinetic energy $E_F(H)$ on the Fermi surface also increases appreciably, and this affects $\varepsilon(\vec{k}, \omega)$ strongly. The situation is particularly interesting in the case of energy bands with strong non-parabolicity or with several extrema that are close to one another. In this case the magnetic field can be used to decrease significantly the plasma frequence ω_p, and accordingly, to reverse the sign of $\varepsilon(\vec{k}, \omega)$ in a definite frequency interval $\omega \approx \omega_p$, making the conductor transparent to electromagnetic waves in this interval. The magnetic field also influences greatly the screening radius in the antiferromagnet.

It is assumed that the frequency of electron-defect collisions greatly exceeds the Larmor frequency, but is small compared with $E_F/\hbar$. This makes it possible to disregard the field-induced anisotropy of $\varepsilon(\vec{k}, \omega)$ and the quantization of the electron orbits.

The initial expression for $\varepsilon(\vec{k}, \omega)$ has the standard form

$$\epsilon(\mathbf{k}, \omega) = 1 - \frac{4\pi e^2}{\epsilon_0 k^2} \sum_{p,\sigma} \frac{n_{p+k,\sigma} - n_{p,\sigma}}{\omega_{p+k,p} - \omega - i\delta}. \tag{1}$$

Here $\omega_{p+k,p} = E_{p+k} - E_p$ and E_p is the electron energy and is given, assuming $p_F/m \gg a\omega$ and taking the non-parabolicity into account, by the expression

$$E_p = \frac{p^2}{2m}(1 + \beta p^2), \tag{2}$$

where m is the effective mass of the electron, $n_{p\sigma}$ is the Fermi distribution function of electrons with energies $E_{p\sigma} = E_p - AS_\sigma$ at $T = 0$, p_F is the Fermi momentum at $H = 0$, ε_0 is the dielectric constant of the crystal without allowance for the conduction electron, and $\underline{a}$ is the lattice constant. Terms proportional to $(AS)^2/W$ have been disregarded in (2).

Retaining only terms linear in βp_F^2, we get from (1) and (2)

$$\epsilon(k, \omega) = 1 - \frac{\omega_p\left[1 + \frac{3}{5}\beta\Pi_F^2\right]}{\omega^2}\left[1 + \frac{k^2\beta\left[1 + \frac{3}{10}\Pi_F^2/m^2\omega^2\beta\right]}{1 + \frac{3}{5}\beta\Pi_F^2}\right], \tag{3}$$

where $\omega_p = (4\pi e^2 n/\varepsilon_0 m)^{1/2}$ is the Langmuir frequency, $\Pi_F^2 = (p_{F_+}^5 + p_{F_-}^5)/p_F^3$, and p_{F_+} and p_{F_-} are the Fermi momenta for the electrons with $\sigma = 1/2$ and $\sigma = -1/2$. The dependence of $p_{F_\pm}$ on the magnetic field, with allowance for (2), is given by

$$p_{F_\pm} = \left(\frac{\sqrt{1 + 8m\beta(E_F' \pm AS/2)}}{2\beta} - 1\right)^{1/2} \tag{4}$$

Here E_F' is the Fermi level in the field H, reckoned from the bottom of the band at $H = 0$. The dependence of E_F' on H is determined from the condition for the conservation of the number of electrons:

$$p_{F_+}^3 + p_{F_-}^3 = 2p_F^3. \tag{5}$$

Thus, according to (3), the effective plasma frequency at k = 0, with allowance for the GZE, depends on the external field H like

$$\tilde{\omega}_p(H) = \omega_p \left[1 + \frac{3}{10}\beta\Pi_F^2\right] . \tag{6}$$

According to (5), in the critical field H_c at which all the electrons are fully polarized in spin, we have $p_{F_+} = 2^{1/3}p_F$. Taking this into account, it follows from (6) that the maximum shift of the plasma frequency in the field, at $\beta p_F^2 \ll 1$, amounts to $0.6(2^{2/3} - 1)\beta p_F^2$. It follows from the general properties of the electron spectrum that $\beta < 0$. Thus, the plasma frequency decreases with increasing field. If we choose for an estimate the value $\beta p_F^2 = 0.3$ at $n = 10^{20}cm^{-3}$, then this shift is approximately equal to 10%. Such an estimate of the possible amplitude of the effect can hardly be regarded as too high, since there are known semiconductors in which the degree of non-parabolicity is larger by several orders of magnitude (for example, InSb [3]).

If in addition to the principal minimum of the band there is another minimum close in energy to the principal one, then the magnetic field can be used to cause the electrons to go from the principal minimum to the auxiliary one. This also affects the value of $\tilde{\omega}_p$. The decrease of $\tilde{\omega}_p$ in the magnetic field, in accord with formula (3), causes an increase of $\varepsilon(0, \omega)$. In particular, in a definite interval of the frequencies ω, the magnetic field can change the sign of $\varepsilon(0, \omega)$ from negative to positive. By the same token, if the conductor is opaque to a field of frequency $\sim\tilde{\omega}_p(0)$ at H = 0, then it becomes transparent in the field. In other words, it is possible to control the transmission of electromagnetic waves through a conductor by means of a magnetic field.

The magnetic field alters also the dispersion law for plasmons. In the case of strong non-parabolicity, when $\beta p_F^2 \gg (E_F/\omega_p)^2$, we find from (3) that the GZE leads to a relatively strong shift of the Langmuir frequency $\tilde{\omega}_p$ and to a weak decrease of the plasmon effective mass. In the case of weak non-parabolicity, when $(E_F/\omega_p)^2 \gg \beta p_F^2$, there is no shift of the Langmuir frequency as a result of the GZE, but the effective mass of the plasmon in the field H_c decreases by a factor $2^{2/3}$ compared with the mass of H = 0.

As follows from (3), even when $\beta = 0$ the screening radius turns out to be significantly dependent on the magnetic field. Its value at H_c exceeds that at H = 0 by $2^{1/3}$ times. This can lead to a number of interesting physical effects. In particular, the increase of the screening radius in a strongly doped semiconductor can cause it to change from the conducting to the insulating state. To this end it is necessary that the carrier density in it not exceed greatly the critical value at which collectivization of the defects with formation of an impurity band occurs in the absence of the field. Turning on the field leads to an increase of the radius of the defect potential and can thus stabilize the state in which each electron remains on its own atom, and there is no impurity band. Of course, owing to the random distribution of the impurity, one can hardly expect the field-induced transition into the insulating state to be abrupt.

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CONTRIBUTION TO THE THEORY OF INSTABILITY OF CHARGED-PARTICLE BEAMS PASSING THROUGH AN INHOMOGENEOUS MEDIUM, AND INSTABILITY OF NON-UNIFORMLY MOVING BEAMS

V.A. Buts
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It was shown in [1] that when a homogeneous unbounded electron beam moves through a medium at a velocity exceeding the wave-propagation velocity in this medium, instability develops in the beam.

We shall show below that an exponential growth of the perturbations in the beam is produced also when the conditions for Cerenkov radiation are not satisfied, but the medium is inhomogeneous or the beam particles move with acceleration.

1. We consider a homogeneous unbounded beam passing with constant velocity ($\vec{v}_0 \parallel z$) through an inhomogeneous dielectric ($\varepsilon = \varepsilon(z)$). Let us trace the development of a small perturbation in such a beam. The linearized equations describing the interaction of the perturbing field with the electron beam take the form

$$\mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}; \quad \mathrm{rot}\,\mathbf{H} = \frac{\epsilon(z)}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}(\rho_o \mathbf{v} + \mathbf{v}_o \rho),$$

$$\frac{\partial \mathbf{v}}{\partial t} + v_o \frac{\partial \mathbf{v}}{\partial z} = \frac{e}{m}\mathbf{E} + \frac{e}{mc}[\mathbf{v}_o \times \mathbf{H}]. \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,(\rho_o \mathbf{v} + \mathbf{v}_o \rho) = 0.$$

The subscripts 0 denote here the unperturbed quantities. We choose the dependence on the time and on the transverse variable in the form $\exp[-i\omega t + ik_\perp x]$. We then have for the field of an E-wave (E_x, E_z, H_y) the following system of integro-differential equations:

$$E_x'' + \left(k^2\epsilon - \frac{\omega_b^2}{c^2}\right)E_x = ik_\perp\left[E_z' - \frac{\omega_b^2}{c^2}e^+\int_0^z E_z e^- dz_1\right],$$

$$E_z(k^2\epsilon - k_\perp^2) + k^2\frac{\omega_b^2}{v_o^2}\left(1 + \frac{k_\perp^2}{k^2}\frac{v_o^2}{c^2}\right)e^+\int_0^z dz_1\int_0^{z_1} E_z e^- dz_1 =$$

$$= ik_\perp\left[E_x' - \frac{\omega_b^2}{c^2}e^+\int_0^z E_x e^- dz_1\right], \tag{2}$$

where

$$E_x' \equiv \frac{\partial E_x}{\partial z}; \quad k \equiv \frac{\omega}{c}; \quad e^\pm \equiv e^{\pm i\frac{\omega}{v_o}z}; \quad \omega_b^2 \equiv \frac{4\pi e \rho_o}{m}.$$