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CONTRIBUTION TO THE THEORY OF INSTABILITY OF CHARGED-PARTICLE BEAMS PASSING THROUGH AN INHOMOGENEOUS MEDIUM, AND INSTABILITY OF NON-UNIFORMLY MOVING BEAMS

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It was shown in [1] that when a homogeneous unbounded electron beam moves through a medium at a velocity exceeding the wave-propagation velocity in this medium, instability develops in the beam.

We shall show below that an exponential growth of the perturbations in the beam is produced also when the conditions for Cerenkov radiation are not satisfied, but the medium is inhomogeneous or the beam particles move with acceleration.

1. We consider a homogeneous unbounded beam passing with constant velocity ($\vec{v}_0 \parallel z$) through an inhomogeneous dielectric ($\epsilon = \epsilon(z)$). Let us trace the development of a small perturbation in such a beam. The linearized equations describing the interaction of the perturbing field with the electron beam take the form

$$\begin{aligned} \text{rot } \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t}; \quad \text{rot } \mathbf{H} = \frac{\epsilon(z)}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} (\rho_0 \mathbf{v} + \mathbf{v}_0 \rho), \\ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}_0 \frac{\partial \mathbf{v}}{\partial z} &= \frac{e}{m} \mathbf{E} + \frac{e}{mc} [\mathbf{v}_0 \times \mathbf{H}], \\ \frac{\partial \rho}{\partial t} + \text{div} (\rho_0 \mathbf{v} + \mathbf{v}_0 \rho) &= 0. \end{aligned} \quad (1)$$

The subscripts 0 denote here the unperturbed quantities. We choose the dependence on the time and on the transverse variable in the form $\exp[-i\omega t + ik_\perp x]$. We then have for the field of an E-wave (E_x, E_z, H_y) the following system of integro-differential equations:

$$\begin{aligned} E_x'' + \left(k^2 \epsilon - \frac{\omega_b^2}{c^2}\right) E_x &= ik_\perp \left[E_z' - \frac{\omega_b^2}{c^2} e^+ \int_0^z E_z e^- dz_1 \right], \\ E_z (k^2 \epsilon - k_\perp^2) + k^2 \frac{\omega_b^2}{v_0^2} \left(1 + \frac{k_\perp^2}{k^2} \frac{v_0^2}{c^2} \right) e^+ \int_0^z dz_1 \int_0^{z_1} E_z e^- dz_1 &= \\ = ik_\perp \left[E_x' - \frac{\omega_b^2}{c^2} e^+ \int_0^z E_x e^- dz_1 \right], \end{aligned} \quad (2)$$

where

$$E_x' \equiv \frac{\partial E_x}{\partial z}; \quad k \equiv \frac{\omega}{c}; \quad e^\pm \equiv e^{\pm i \frac{\omega}{v_0} z}; \quad \omega_b^2 \equiv \frac{4\pi e \rho_0}{m}.$$

If $k_{\perp} = 0$, then the system (2) breaks up into two independent equations, the first of which describes transverse waves and the second longitudinal oscillations. We seek the solution of the system (2) in the form

$$E_x = \mathcal{E}(z) \exp[i \int_0^z k_{\parallel}(z_1) dz_1]; \quad E_z = A \exp[i \int_0^z k_{\parallel}(z_1) dz_1], \quad (3)$$

where $\mathcal{E}(z)$ and $k_{\parallel}(z)$ are slowly varying functions. Substituting these expressions in (2) and confining ourselves to the terms proportional to the first power of the parameter k'_{\parallel}/k^2 , we obtain the following dispersion equation ($k_{\perp} = 0$):

$$\epsilon^* \Delta - k_{\perp}^2 \frac{\omega_b^2}{\omega^{*2}} \epsilon \frac{v_0^2}{c^2} = -if, \quad (4)$$

where

$$\begin{aligned} \epsilon^* &= \epsilon - \frac{\omega_b^2}{\omega^{*2}}; \quad \omega^* = \omega - k_{\parallel} v_0; \quad \Delta = k^2 \epsilon - k_{\parallel}^2 - k_{\perp}^2 - \frac{\omega_b^2}{c^2}; \\ f &= \frac{k_{\perp}^2}{k^2} \left[2k_{\parallel} \alpha \left(\frac{\alpha}{\Lambda_{\parallel}} \right)' + \frac{k_{\parallel}' \alpha^2}{\Lambda_{\parallel}} \right] + 3\Lambda_{\parallel} \frac{\omega_b^2}{\omega^{*2}} \frac{k_{\parallel}' v_0^2}{\omega^{*2}} \left(1 + \frac{k_{\perp}^2}{k^2} \frac{v_0^2}{c^2} \right) + \\ &+ k_{\perp}^2 \Lambda_{\parallel} \left[\frac{\alpha}{\Lambda_{\parallel}} \left(1 - \frac{\omega_b^2}{\omega^{*2}} \frac{v_0^2}{c^2} \right) \right]' - \alpha \frac{k_{\perp}^2}{k^2} \frac{\omega_b^2}{\omega^{*2}} \frac{v_0^2}{c^2} - \frac{k_{\parallel}' v_0^2}{\omega^{*2}}. \end{aligned}$$

If $\epsilon(v_0/c^2) \ll 1$, instability is possible only when account is taken of the inhomogeneity of the medium. Solving Eq. (4) for this case, we obtain

$$k_{\parallel} = k_{\parallel 0} (1 \pm i\gamma) \quad (5)$$

$$\begin{aligned} k_{\parallel 0} &= \frac{\omega}{v_0} = \frac{\omega_b}{v_0 \sqrt{\epsilon}}; \quad \gamma = \frac{1}{2} \frac{\epsilon'}{(\epsilon)^{3/2}} \frac{\omega_b \omega}{v_0^2} - \frac{k_{\perp}^2}{k_{\parallel 0}^2 (k_{\perp}^2 + k_{\parallel 0}^2)} = \\ &= \frac{3}{4} \frac{\epsilon'}{\epsilon} \frac{1}{k_{\parallel 0}} \end{aligned}$$

for longitudinal waves and

$$\begin{aligned} k_{\parallel} &= \pm k_{\parallel 0} + \frac{i}{4} \frac{\epsilon'}{\epsilon^*} \frac{k_{\perp}^2 + 3k_{\parallel 0}^2}{k_{\parallel 0}^2} \\ k_{\parallel 0} &= \sqrt{k^2 \epsilon - k_{\perp}^2 - \frac{\omega_b^2}{c^2}}; \quad \frac{\omega_b^2}{\omega^2} \ll 1; \quad \frac{v_0}{c} \ll 1 \end{aligned} \quad (6)$$

for transverse waves.

It follows from (5) and (6) that the longitudinal oscillations grow exponentially. The amplitude of the transverse waves is altered only by the pre-exponential factor.

2. We consider now a beam in which the particle velocity varies with time. The initial system of equations in (1), in which we must put $\epsilon = 1$. Here

$E_0(t)$ is the external electric field, which controls the beam motion. From the system (1) for the E-field wave, the coordinate dependence of which is of the form $\exp[ik_{\parallel}z + ik_{\perp}x]$, we obtain

$$\begin{aligned} H_y'' + (k_{\parallel}^2 c^2 + \omega_b^2) H_y &= ik_{\perp} c [E_z' + \omega_b^2 e^{-\int_{-\infty}^t E_z e^+ dt_1}]; \\ E_z' + \omega_b^2 e^{-\int_{-\infty}^t E_z e^+ dt_1} - ik_{\parallel} v_0 \omega_b^2 \left(1 + \frac{k_{\perp}^2}{k_{\parallel}^2}\right) e^{-\int_{-\infty}^t dt_1 \int_{-\infty}^{t_1} E_z e^+ dt_2} &= \\ = ik_{\perp} c \left[H_y + i \frac{v_0}{c} \frac{\omega_b^2}{k_{\parallel} c} e^{-\int_{-\infty}^t H_y e^+ dt_1} \right], \end{aligned}$$

where

$$E_z' = \frac{\partial E_z}{\partial t}; \quad e^{\pm} = \exp\left[\pm i \int_{-\infty}^t k_{\parallel} v_0(t_1) dt_1\right].$$

We seek the solution of the system (7) in the form

$$E_z = E \exp\left(-i \int_{-\infty}^t \omega(t_1) dt_1\right); \quad H_y = H(t) \exp\left(-i \int_{-\infty}^t \omega(t_1) dt_1\right). \quad (8)$$

Substituting these expressions in (7) and retaining the terms proportional to the first powers of the parameters $\omega'/k^2 c^2$ and v_0'/kc^2 , we obtain the following dispersion equation

$$\epsilon^* \Delta = if, \quad (9)$$

where

$$\begin{aligned} f &= \frac{\omega_b^2}{\omega^{*2}} \frac{\omega^{*'}}{\omega^{*2}} \Delta_{\parallel} \left[\frac{\omega^*}{\omega} + \frac{3k_{\parallel} v_0}{\omega} \left(1 + \frac{k_{\perp}^2}{k_{\parallel}^2}\right) \right] + k_{\perp}^2 \left(1 - \frac{\omega_b^2 v_0}{\omega^* k_{\parallel} c^2}\right) \times \\ &\times \left\{ \frac{\omega^{*'}}{\omega^{*2}} \left(\frac{\omega_b^2}{\omega \omega^*} - \frac{2\omega_b^2}{c^2 \Delta_{\parallel}} \right) - \frac{\omega'}{c^2 \Delta_{\parallel}} \left[2 + \frac{3\omega^2 + k_{\parallel}^2 c^2 + \omega_b^2}{c^2 \Delta_{\parallel}} \left(1 - \frac{\omega_b^2}{\omega \omega^*}\right) \right] \right. \\ &\quad \left. - k_{\perp}^2 \frac{\omega_b^2 v_0}{\omega^{*2} c} \frac{1}{k_{\parallel} c} \left[\frac{\omega'}{\omega} - \frac{\omega^{*'}}{\omega^*} \left(1 - 2 \frac{\omega_b^2}{\omega \omega^*}\right) \right] \right\}. \end{aligned}$$

Solving this equation with respect to $\omega(t)$, we obtain

$$\omega = k_{\parallel} v_0 \pm \omega_b (1 + i\gamma); \quad \gamma = v_0' / k_{\parallel} c^2 \quad (10)$$

for longitudinal waves and

$$\begin{aligned} \omega &= \pm \omega_s (1 + i\gamma_1), \\ \gamma_1 &= \frac{k_{\parallel} v_0'}{\omega_s^2} \frac{\omega_b^2}{\omega^{*2}} \frac{1}{c}; \quad \omega_s = [(k_{\parallel}^2 + k_{\perp}^2) c^2 + \omega_b^2]^{1/2} \end{aligned} \quad (11)$$

for transverse waves.

Formulas (10) and (11) show that the instability develops both when the beam is decelerated and when it is accelerated. In addition, the frequency of the longitudinal oscillations varies in proportion to the change of the beam velocity.

If follows from the foregoing results that, just as Cerenkov radiation of an individual beam particle causes oscillations to build up, the radiation of individual particles passing through an inhomogeneous medium or having non-uniform motion leads to the development of collective instabilities of the beam. The growth increment of the waves depends in this case both on the emissivities of the individual particles ($\partial\epsilon/\partial z$ or $\partial v_0/\partial t$) and on the collective characteristics of the beam (ω_b).

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DRAGGING OF A LIQUID BY A LIQUID THROUGH A STATIONARY SOLID WALL

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The influence of acoustic fluctuations (phonons) on the hydrodynamic properties of a liquid was considered in [1]. An account of these fluctuations is of particular importance in those cases when one deals with phenomena that might exist in principle, but are absent in ordinary hydrodynamics. For example, the thermomechanical effect, which arises only when phonons are taken into account [1], does not exist in ordinary hydrodynamics.

The purpose of the present article is to call attention to the following effect, which is possibly the most pronounced manifestation of phonons. We consider two liquid layers separated by a stationary solid partition (Fig. 1). Assume that Poiseuille flow takes place in region I. According to the equations of ordinary hydrodynamics, the liquid in region II remains stationary. The situation is altered if account is taken of the possibility of momentum transfer from region I into region II by phonons passing through the solid partition. This should be accompanied by dragging of the liquid in region II, so that the velocity distribution should have the form shown in Fig. 1.

The drag velocity can be calculated on the basis of the equations of motion derived in [1]. We write first the kinetic equation for the phonons in the liquid

$$\frac{\partial \chi}{\partial z} + \frac{\chi}{c n_z \tau} = \frac{\epsilon n_x}{c} \frac{dv}{dz}, \quad (1)$$

where the function χ determines the deviation of the phonon distribution function from the equilibrium value n_0 in accord with the formula

$$n = n_0 + \chi \frac{\partial n_0}{\partial \epsilon},$$

ϵ is the phonon energy, c the speed of sound, τ the phonon free-path time, v the velocity of the liquid, and \mathbf{n} a unit vector along the phonon momentum.

We assume that the coefficient W of penetration of the phonons through the solid wall is

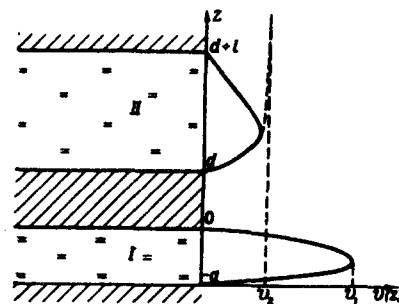


Fig. 1