

per unit mass, we obtain from (3)

$$\frac{v_2}{v_1} = \zeta \left(\frac{3}{2} \right) \frac{T}{4\eta} \left(\frac{\rho^3 c}{\pi^5 \gamma^3 a} \right)^{1/2} \frac{\rho c}{D c_1} \Phi \left(\frac{c}{c_1}, \frac{c \ell}{c_1} \right), \quad (4)$$

where

$$\Phi(x, y) = x^4 \left\{ \int_0^{1/y^2} \frac{(1-y^2 t)^{1/2} t (1-x^2 t)^{1/4} dt}{y(1-2t)^2 + 4t(1-t)^{1/2}(1-y^2 t)^{1/2}} + \right. \\ \left. + \int_{1/y^2}^1 \frac{4(y^2 t - 1)t^2(1-t)^{1/2}(1-x^2 t)^{1/4} dt}{y^2(1-2t)^4 + 16t^2(1-t)(y^2 t - 1)} \right\},$$

c_ℓ is the velocity of the longitudinal sound in the solid, and ζ is the Riemann function. A plot of $\Phi(x, y)$ obtained by numerical calculation is shown in Fig. 2.

In the derivation of the foregoing formula we used for the equilibrium distribution function n_0 its classical value T/ϵ , since most liquids solidify upon cooling long before quantum effects come into play. We note also that in order for (4) to be valid it is necessary that there be no dispersion of sound in the liquid up to frequencies corresponding to a mean free path equal to \underline{a} . In other words, the thickness \underline{a} must be larger than $\rho c^3/\gamma \omega_0^2$, where ω_0 is the frequency above which appreciable dispersion sets in.

For the water-polystyrene (or Plexiglas) pair we obtain from (4) that v_2/v_1 is approximately equal to $10^{-7}/\sqrt{\underline{a}}$ (\underline{a} is in cm) at $T = 353^\circ\text{K}$ and $10^{-8}/\sqrt{\underline{a}}$ at $T = 293^\circ\text{K}$; for the mercury-silver pair we obtain approximately $10^{-8}/\sqrt{\underline{a}}$ at $T = 293^\circ\text{K}$.

The ratio of the total liquid fluxes through regions II and I differs from v_2/v_1 , as can be readily seen, by the factor $3\ell/4a$. On the other hand, if the liquid in region II is at rest, then a pressure gradient is produced in this region, and its ratio to the pressure gradient in region I is equal to $(3a^2/4\ell^2)(v_2/v_1)$.

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OPTICAL BREAKDOWN OF MOLECULAR GASES

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Submitted 11 November 1971
ZhETF Pis. Red. 15, No. 1, 60 - 63 (5 January 1972)

1. In connection with the progress in the development of high-power lasers (e.g., lasers using molecular gases such as CO_2 , CO, etc.) the problem of optical breakdown of molecular gases has become quite vital. The solution of this problem has a direct bearing on the determination of the limiting parameters of lasers for the infrared band.

The breakdown of atomic gases in the giant-pulse regime has been thoroughly investigated (see the review [1]). At sufficiently high pressures ($p \gtrsim 1$ atm) the optical-breakdown mechanism is the cascade ionization process. Within the framework of the simple classical model, the process evolves in the following manner: The priming electrons acquire energy from the wave field via elastic collisions with the neutral atoms; the rate of electron energy acquisition is $d\epsilon/dt = \alpha = \epsilon_0 v_{\text{eff}}$, where $\epsilon_0 = e^2 E_0^2 / 2m\omega^2$ is the energy of the electron oscillations in the field E_0 of frequency ω , and v_{eff} is the frequency of the elastic collisions. Moving in energy space, the electron attains an energy $\epsilon \approx I$, where I is the characteristic value of the threshold energy of the inelastic processes, within a time $\tau \approx I/\alpha$, and consequently the cascade-development constant γ is given by the relation $\gamma = k/\tau$, where k is the probability of electron passage through the excitation zone. With decreasing radiation frequency, γ increases in proportion to ω^{-2} , leading, in the breakdown of atomic gases, to a corresponding decrease of the threshold radiation density q .

When the radiation interacts with a molecular gas, however, this effect can be compensated to a considerable extent by the deceleration of the electrons on the vibrational levels of the molecules. As will be shown below, the vibrational levels of a molecule form a unique type of potential barrier, and the motion of the electron in energy space acquires a tunneling character. This circumstance changes qualitatively the field dependence of the cascade-development constant, and the breakdown of the gas becomes much more sensitive to the light flux, namely, an exponentially-attenuating factor appears in the dependence of γ on the field.

2. The kinetic equation for the electron distribution function $f(\epsilon, t) = n_0 F(\epsilon) \exp[\gamma t]$ in the light field is

$$\gamma F = - \frac{\partial J_q}{\partial \epsilon} + \left(\frac{\partial F}{\partial t} \right)_{in} + \left(\frac{\partial F}{\partial t} \right)_v; \quad \int_0^\infty F(\epsilon) d\epsilon = 1, \quad (1)$$

where n_0 is the priming electron density,

$$J_q = \frac{a}{3} \left(F - 2\epsilon \frac{\partial F}{\partial \epsilon} \right) \quad (2)$$

the electron flux in energy space, determined by the radiation field,

$$\left(\frac{\partial F}{\partial t} \right)_{in} = \begin{cases} 0 & \text{for } \epsilon < I \\ \infty & \text{for } \epsilon > I \end{cases} \quad (3)$$

is the change of $F(\epsilon)$ due to ionization and excitation of the electronic terms of the molecule [2],

$$\left(\frac{\partial F}{\partial t} \right)_v = N_0 \sum_{m=0}^{\infty} \{ \sigma_{0m}(\epsilon + \hbar\omega_{0m}) F(\epsilon + \hbar\omega_{0m}) v(\epsilon + \hbar\omega_{0m}) - \sigma_{0m}(\epsilon) F(\epsilon) v(\epsilon) \} \quad (4)$$

is the collision term connected with the excitation of the vibrational levels of the molecule, N_0 is the density of the molecules, σ_{0m} is the cross section for the excitation of the n -th vibrational level with quantum energy $\hbar\omega_{0m}$, and v is the electron velocity.

It is known that only the first few (approximately four) vibrational levels are effectively excited in molecular gases [3]. Since the typical energy of the vibrational quantum is approximately 0.1 eV, the average energy lost by the electron upon excitation of the molecule is ~0.2 eV. At the same time, the energy at which the vibrational structure of the molecule is excited most effectively lies in the range 1 - 3 eV. Taking this into account, we can expand (4) in powers of $\hbar\omega_{Om}$. As a result (1) takes the form

$$\gamma F = - \frac{\partial}{\partial \epsilon} \{ J_q + J_v \}, \quad J_v = \alpha^*(\epsilon) F(\epsilon), \quad (4')$$

where $\alpha^*(\epsilon) = N_0 v(\epsilon) \int \sigma_{Om}(\epsilon) \hbar\omega_{Om}$ is the rate of electron energy loss upon excitation of the vibrational levels. In practice, the function $\alpha^*(\epsilon)$ differs from zero only in a small vicinity Δ (compared with 1) of the point $\epsilon = i$ corresponding to the maximum of $\alpha^*(\epsilon)$. We can therefore put in the calculation $\alpha^*(\epsilon) = \alpha^*(i) \Delta \delta(\epsilon - i)$. Introducing now the effective probability k of electron penetration through the band of excitation of the electronic terms of the molecule, we write down the boundary conditions for Eq. (4'):

$$J_q(0) = J_q(1)(1 + k), \quad F(1) = 0. \quad (5)$$

It is physically clear that at the point $\epsilon = i$, under the condition $\alpha \ll \alpha^*(i)$, the function $F(\epsilon)$ has a discontinuity, whereas the flux J_q is continuous at this point. This leads to the conditions

$$F(i-0) = F(i+0) \exp \left\{ - \frac{\alpha^*(i) \Delta}{\alpha} \frac{1}{i} \right\}, \quad (6)$$

$$\left(F - 2\alpha \frac{\partial F}{\partial \epsilon} \right)_{i-0} = \left(F - 2\alpha \frac{\partial F}{\partial \epsilon} \right)_{i+0}.$$

Equation (1) with the conditions (5 - 6) defines our problem completely. In particular, the expression for the determination of the cascade-development constant γ takes the form

$$\text{sh } x \left[1 + \frac{x^3}{3} \frac{i}{i} \left[\exp \left(\frac{\Delta}{i} \frac{\alpha^*(i)}{\alpha} \right) - 1 \right] \right] = x(1 + k), \quad x = \sqrt{\frac{2\gamma i}{\alpha}}; \quad (7)$$

Upon satisfaction of the condition $\Delta\alpha^*/i\alpha \ll 1$, which means low losses at the vibrational levels, we have from (7)

$$\gamma = k \frac{\alpha}{i}, \quad (8)$$

which coincides with the expression obtained in [2] for the breakdown of atomic gases. Under the condition $\Delta\alpha^*/i\alpha > 1$ we obtain the "tunnel" effect:

$$\gamma = k \frac{\alpha}{i} \exp \left\{ - \frac{\Delta\alpha^*(i)}{i\alpha} \right\}. \quad (9)$$

It follows from (9) that in the case of breakdown of a molecular gas the threshold radiation flux density q^* depends logarithmically on the pressure and on the light-pulse duration and is determined in practice by the molecular characteristics of the gas and by the radiation frequency. Indeed, in the pressure range 1 - 10 atm and at $\tau \geq 10^{-6}$ sec we obtain with the aid of (9) the following expression for q^* (at the breakdown criterion $\gamma\tau \approx 40$)

$$q^* (\text{W/cm}^2) = 1.6 \cdot 10^{-18} \omega^2 \frac{\Delta}{i} \frac{\langle \hbar \omega_{0m} \rangle \text{eV} \sum_m \sigma_{0m}}{\sigma_{tr} \ln [k \cdot 10^{-3} \nu_{\text{eff}} / i (\text{eV})]} \quad (10)$$

We present numerical estimates for molecular nitrogen. In this case, according to [4], $\sigma_{0m} \approx 3 \times 10^{-16} \text{ cm}^2$, $\sigma_{tr} \approx 1.2 \times 10^{-15} \text{ cm}^2$, $\langle \hbar \omega_{0m} \rangle = 0.4 \text{ eV}$, $i \approx \Delta \approx 2 \text{ eV}$, and at $\tau \approx 10^{-6} \text{ sec}$, $\omega = 2 \times 10^{14} \text{ sec}^{-1}$ (CO_2 laser), $k = 0.1$, $p \approx 1 \text{ atm}$, and $\nu_{\text{eff}} \approx 3 \times 10^{11} \text{ sec}^{-1}$ we obtain $q^* \approx 10^{19} \text{ W/cm}^2$.

We note that a corresponding estimate, without allowance for the deceleration of the electrons on the vibrational levels in accord to (8), leads under the same conditions to $q \leq 10^7 \text{ W/cm}^2$. Thus, the effect under consideration leads to high power characteristics of molecular-gas lasers.

The authors are grateful to N.G. Basov and O.N. Krokhin for interest in the work and for a discussion of the results.

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TUNNELING OF DISLOCATIONS

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Submitted 15 November 1971

ZhETF Pis. Red. 15, No. 1, 63 - 66 (5 January 1972)

As is well known, the motion of dislocations in the glide plane through the Peierls barriers occurs via production and further expansion of double kink. At sufficiently low temperatures, such a phenomenon has the character of sub-barrier quantum penetration (tunneling). The purpose of the present paper is to calculate the time of tunnel formation of the double kink.

We assume the simplest model wherein the dislocation is represented by a string in a periodic potential field $U_0(y)$ (the Peierls relief). Assume that the string is initially at rest at the bottom of one of the valleys (the zero-point oscillations are assumed small). In the field of a constant external stress F , such a position becomes unstable, and the string will move after a finite average time to the neighboring valley. We shall henceforth consider a potential relief $U(y) - U_0(y) - Fy$ only for two neighboring valleys. The Hamiltonian of the string is

$$H = \int \frac{\rho}{2} \left(\frac{\partial y}{\partial t} \right)^2 dx + V\{y\}; \quad V\{y\} = \int \left[\frac{\kappa}{2} \left(\frac{\partial y}{\partial x} \right)^2 + U(y) \right] dx. \quad (1)$$

We consider motion only in the glide plane, the coordinate x is directed along the valley, y is the transverse coordinate, ρ is the string density, and κ is its stiffness.

A quantum string is described by a wave function $\Psi\{y\}$, which is a functional of $y(x)$. It is impossible to solve the quantum-mechanical problem in