$$p_2(z) \approx \frac{\pi}{2}\, \rho\, \kappa\left(\frac{\kappa}{2\pi u}\ \ln\frac{|\kappa|}{ua}\right)^2.$$

The trajectories intersect at the point

$$k_0 \approx \frac{|\kappa|}{2\pi u\beta}\ln\frac{|\kappa|}{ua}\ .$$

From this we get (see (4)):

$$\operatorname{Im} S \approx \frac{\rho\kappa^4}{24\pi^2 u^3\beta}\left(\ln\frac{|\kappa|}{ua}\right)^3. \tag{6}$$

Thus, the inhomogeneity shape most favorable for vortex formation is that of half a flat disk ($\beta \to \infty$).

In conclusion, the author thanks S.V. Iordanskii for suggesting the problem and directing the work.

[1] S.V. Iordanskii, Zh. Eksp. Teor. Fiz. 48, 708 (1965) [Sov. Phys.-JETP 21, 467 (1965)].
[2] W.F. Vinen, Progr. Low Temp. Phys. 3, Amsterdam (1961).
[3] I.M. Lifshitz and Yu. Kagan, Zh. Eksp. Teor. Fiz. 62, 385 (1972) [Sov. Phys.-JETP 35, No. 1 (1972)].
[4] S.V. Iordanskii and A.M. Finkel'shtein, ibid. 62, 403 (1972) [35, No. 1 (1972)].
[5] A. Walraven, Phys. Rev. AI, 145 (1970).
[6] H. Lamb, Hydrodynamics, Dover, 1932.
[7] L.D. Landau and E.M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), M., 1963 [Addison-Wesley, 1965].

DESTRUCTIVE INTERFERENCE IN THE SCATTERING OF FAST ELECTRONS IN A SINGLE CRYSTAL

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1. As is well known, elastic scattering in a single crystal exhibits interference properties even for relativistic electrons of small wavelength, inasmuch as the wavelength transmitted in the longitudinal direction (along the particle motion) ($\hbar = c = 1$, $\kappa = me^2Z^{1/3}$)

$$(p_{1x} - p_{2x})^{-1} \sim (p\theta^2)^{-1} \sim p\kappa^{-2}$$

can greatly exceed the interatomic distances. Ter-Mikaelyan [1] investigated this effect in the Born approximation for scattering by a single crystal as a whole. The condition for the applicability of perturbation theory

$$\frac{1}{v}\int dx\, U \approx \frac{1}{v} UL \ll 1 \tag{1}$$

($v = p/E$ is the velocity of the relativistic electron) imposes a rigid upper limit on the thickness L of the single crystal in the electron motion direction. The purpose of the present communication is to call attention to the fact that for crystal thicknesses exceeding the Born limit (1) the interference effect in scattering differs qualitatively from that considered in [1]. Let

$$L < p\kappa^{-2} \tag{2}$$

and assume that one can use the well-known high-energy approximation [2] for the the amplitude of scattering by the single crystal as a whole

$$f(q) = ip\int_0^\infty r\,dr\,J_0(qr)\left\{1 - \exp\left[-\frac{i}{v}\int_{-\infty}^{\infty} dx\,U(r_\perp, x)\right]\right\}, \tag{3}$$

where J_0 is a Bessel function and q the momentum transfer.

2. When (3) is applied to scattering by a single crystal, it must be recognized that the region of applicability of (3) is limited to rather small angles $\theta < \sqrt{\kappa/p}$ and is restricted by the condition (2). In the limiting case $p\kappa^{-2} \gg L \gg (Ze^2)^{-1}a$ we obtain from (3), if the electrons are incident along the crystallographic axis, the following expression for the total cross section for scattering in the single crystal:

$$\sigma = N_\perp \pi \kappa^{-2} \ln^2\left(LZe^2\gamma\frac{\sqrt{\pi}}{av}\right), \tag{4}$$

where $N_\perp$ is the number of atoms in the transverse plane, and $\gamma \simeq 1.781$ is Euler's constant. The differential cross section in the same case is equal to

$$\frac{d\sigma}{d\Omega} = N_\perp \kappa^{-2}\ln^2\left(LZe^2\frac{\gamma\sqrt{\pi}}{av}\right)\frac{J_1\left(2p\sin\frac{\theta}{2}\kappa^{-1}\ln\left(LZe^2\frac{\gamma\sqrt{\pi}}{av}\right)\right)}{4\sin^2\frac{\theta}{2}}. \tag{5}$$

Formulas (4) and (5) depend little on the crystal thickness and have a diffraction character. All that remains in (4) and (5) from the potential of the individual atom is the action radius κ^{-1} and the coupling constant Ze^2. (We emphasize that formulas (4) and (5) do not admit of transition to the limit of small L, owing to the condition $L > (Ze^2)^{-1}a$!). The obtained relations (4)-(5) are valid if scattering by thermal oscillations can be neglected, i.e., when

$$L \ll \frac{a\ln^2\left(LZe^2\frac{\gamma\sqrt{\pi}}{av}\right)}{(Ze^2)^2\,\kappa^2\overline{u^2}}, \tag{6}$$

where $\overline{u^2}$ is the mean-squared thermal displacement of the atom.

3. The meaning of the obtained solution can be explained by considering scattering by two centers located at the points $\vec{r} = 0$ and $\vec{r} = \vec{a}$. When the conditions $\sin(\vec{p}\cdot\vec{a}) \ll 1$ and $a \ll p\kappa^{-2}$ are satisfied, it follows from (3) that the amplitude $f_2(\vec{q})$ for scattering by two centers is connected with the one-center amplitude $f_1(\vec{q})$ by the relation

$$f_2(q) = f_1(q) + f_1(q)e^{iq_\perp a_\perp} + \frac{i}{2\pi p}\int f_1(s_\perp)f_1(-s_\perp)e^{is_\perp a_\perp}d^2s_\perp. \tag{7}$$

If the centers are of the Coulomb type, we obtain for the total cross section for scattering by two centers

$$\sigma_2 = 2\sigma_1 + 2\sigma_1 \{\theta a\kappa K_1(\theta a\kappa)\} - \sigma_1 \frac{(Ze^2)^2}{4} \exp\left\{ -\frac{1}{4} \theta^2 a^2 \kappa^2 \right\}, \quad (8)$$

where K_1 is the Macdonald function. At $\theta a\kappa \ll 1$ we have

$$\sigma_2 \cong 4\sigma_1 - \sigma_1 \frac{(Ze^2)^2}{4} .$$

The first term coincides here with the result of the Ter-Mikaelyan approximation [1], so that interference results in addition of the amplitudes for scattering from each center. However, scattering by the first atom changes the particle flux incident on the second atom, and this circumstance upsets the interference conditions. In other words, the presence of the shadow produced by the first atom leads to destructive interference.

The foregoing interpretation of the effect is confirmed by an analysis of diffraction by two inpenetrable spheres. In this case the deep shadow of the first sphere can suppress the contribution of the second sphere to the scattering cross section. In the limiting case when the velocity direction coincides with the line joining the centers, the two-center cross section simply coincides with the one-center one. In the Coulomb case, shadowing is likewise present, but it is slight because the interaction constant Ze^2 is small, and a deep shadow is produced by superposition of shadows from a chain of atoms on the crystallographic axis. For a long chain, the deep atoms are completely in the shadow and have no effect on the scattering; hence the weak dependence of formulas (4) and (5) on L.

4. It follows from the foregoing that the interference effect in the scattering of fast electrons in a single crystal takes place only in a sufficiently thin crystal, so long as its thickness is $L < (a/Ze^2)$. Even in the favorable case of small Z, this thickness does not exceed 10^2 monatomic layers. It should also be noted that the interference effect depends strongly on Z: for single crystals in the end of the periodic system, deep shadowing sets in already when shadows from several atoms are superimposed, and the interference effects becomes of no importance.

[1] M.L. Ter-Mikaelyan, Zh. Eksp. Teor. Fiz. 25, 289 (1953).
[2] L.I. Schiff, Phys. Rev. 103, 443 (1956).
[3] R.J. Glauber, Phys. Rev. 100, 242 (1955).

INTERACTION OF VIRTUAL PHOTONS WITH PRODUCTION OF A PAIR OF PIONS

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The interaction of electrons and positrons in colliding beams proceeds, in a definite kinematics, via interaction of two virtual photons [1, 2]. We know the connection between the cross sections of the reactions $ee \rightarrow ee$ + hadrons and $\gamma + \gamma \rightarrow$ hadrons, so that in principle we can obtain information concerning processes in which light interacts with light [3]. This explains why physicists are greatly interested in processes $\gamma + \gamma \rightarrow$ hadrons, and particularly in the process $\gamma + \gamma \rightarrow \pi + \pi$ [4, 5].