$$\sigma_2 = 2\sigma_1 + 2\sigma_1 \{\theta a\kappa K_1(\theta a\kappa)\} - \sigma_1 \frac{(Ze^2)^2}{4} \exp\left\{ -\frac{1}{4}\theta^2 a^2 \kappa^2 \right\}, \quad (8)$$

where K_1 is the Macdonald function. At $\theta a\kappa \ll 1$ we have

$$\sigma_2 \simeq 4\sigma_1 - \sigma_1 \frac{(Ze^2)^2}{4} .$$

The first term coincides here with the result of the Ter-Mikaelyan approximation [1], so that interference results in addition of the amplitudes for scattering from each center. However, scattering by the first atom changes the particle flux incident on the second atom, and this circumstance upsets the interference conditions. In other words, the presence of the shadow produced by the first atom leads to destructive interference.

The foregoing interpretation of the effect is confirmed by an analysis of diffraction by two inpenetrable spheres. In this case the deep shadow of the first sphere can suppress the contribution of the second sphere to the scattering cross section. In the limiting case when the velocity direction coincides with the line joining the centers, the two-center cross section simply coincides with the one-center one. In the Coulomb case, shadowing is likewise present, but it is slight because the interaction constant Ze^2 is small, and a deep shadow is produced by superposition of shadows from a chain of atoms on the crystallographic axis. For a long chain, the deep atoms are completely in the shadow and have no effect on the scattering; hence the weak dependence of formulas (4) and (5) on L.

4. It follows from the foregoing that the interference effect in the scattering of fast electrons in a single crystal takes place only in a sufficiently thin crystal, so long as its thickness is $L < (a/Ze^2)$. Even in the favorable case of small Z, this thickness does not exceed 10^2 monatomic layers. It should also be noted that the interference effect depends strongly on Z: for single crystals in the end of the periodic system, deep shadowing sets in already when shadows from several atoms are superimposed, and the interference effects becomes of no importance.

[1] M.L. Ter-Mikaelyan, Zh. Eksp. Teor. Fiz. 25, 289 (1953).
[2] L.I. Schiff, Phys. Rev. 103, 443 (1956).
[3] R.J. Glauber, Phys. Rev. 100, 242 (1955).

INTERACTION OF VIRTUAL PHOTONS WITH PRODUCTION OF A PAIR OF PIONS

V.I. Khleskov
Joint Institute for Nuclear Research
Submitted 15 December 1971
ZhETF Pis. Red. 15, No. 2, 123 - 126 (20 January 1972)

The interaction of electrons and positrons in colliding beams proceeds, in a definite kinematics, via interaction of two virtual photons [1, 2]. We know the connection between the cross sections of the reactions $ee \to ee$ + hadrons and $\gamma + \gamma \to$ hadrons, so that in principle we can obtain information concerning processes in which light interacts with light [3]. This explains why physicists are greatly interested in processes $\gamma + \gamma \to$ hadrons, and particularly in the process $\gamma + \gamma \to \pi + \pi$ [4, 5].

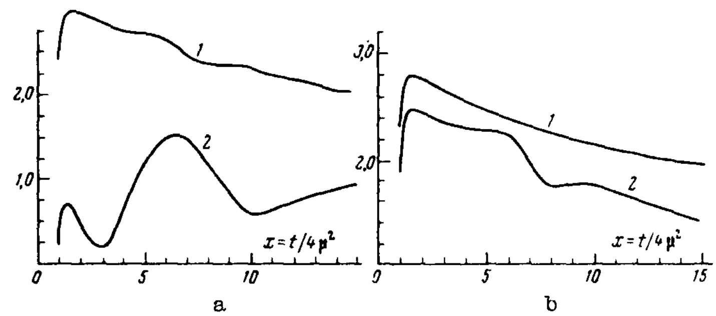

Fig. 1. Interaction of transverse photons: a - log $(10^{30}\sigma_{\gamma\gamma\to\pi\pi^0}$ cm^2) - curve 1 log $(10^{31}\sigma^{\perp\perp}_{\gamma\gamma\to\pi^+\pi^-}$ cm^2) - curve 2; b - log $(10^{30}\sigma^{\perp\perp}_{\gamma\gamma\to\pi\pi(T=2)}$ cm^2) -curve 1, log$(10^{30}\sigma^{\perp\perp}_{\gamma\gamma\to\pi\pi(T=0)}$ cm^2) - curve 2.

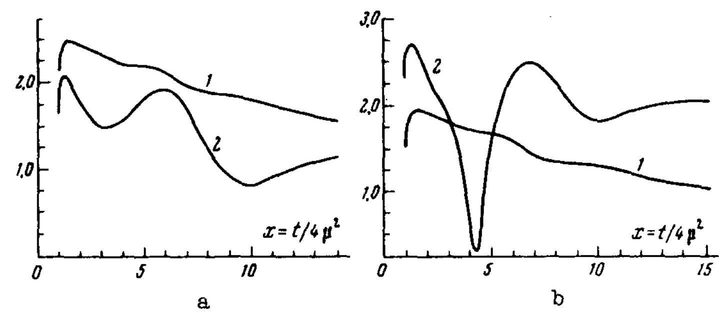

Fig. 2. a - Interaction between longitudinal and transverse photons: log $(10^{30}\sigma^{\parallel\perp}_{\gamma\gamma\to\pi\pi^0}$ cm^2) - curve 1 and log$(10^{32}\sigma^{\parallel\perp}_{\gamma\gamma\to\pi^+\pi^-}$ cm^2)- curve 2; b - interaction of two longitudinal photons: log $(10^{30}\sigma^{\parallel\parallel}_{\gamma\gamma\to\pi^0\pi^0}$ cm^2) - curve 1, log $(10^{33}\sigma^{\parallel\parallel}_{\gamma\gamma\to\pi^+\pi^-}$ cm^2) - curve 2.

We analyze in this paper the reaction $\gamma+\gamma \to \pi + \pi$ by the method of dispersion relations for virtual space-like photons with specified squared 4-momenta. The dispersion relations for the two-photon scattering amplitudes $T_{\alpha\beta}(t, \cos\phi_t)$ are obtained in the c.m.s. with respect to the energy variable $t = (k + k')^2$ (k and k' are the photon momenta) at a fixed scattering angle ϕ_t. In the direct channel, the cut in the complex t-plane is connected with the two-meson state $t \geq 4\mu^2$. The crossing channels were represented by the Born terms and by the nearest ρ and ω resonances. We used a two-particle unitarity condition, i.e., the analysis was limited to low energies (up to E $\sim$ 1 GeV). In this case we can confine ourselves to the contribution of the s and d partial waves: $T^{(T)}_{\alpha\beta} \simeq T^{(T)}_{\alpha\beta}(\ell=0) + 5P_2(\cos\phi_t)T^{(T)}_{\alpha\beta}(\ell=2)$ (ℓ is even, owing to C-invariance). Assuming $\cos\phi_t = 1/\sqrt{3}$, we separate in the amplitude the s-wave, $T^{(T)}_{\alpha\beta}(t, \cos\phi_t = 1/\sqrt{3}) \simeq T^{(T)}_{\alpha\beta}(t)_s$.

The dispersion equation for the amplitude of the process under consideration is given by

$$T^{(T)}_{\alpha\beta}(t)_s = \frac{1}{\pi}\int_{4\mu^2}^{\infty} \frac{e^{-i\delta^T_s(t')}\sin\delta^T_s(t')\,T^{(T)}_{\alpha\beta}(t')_s}{t'-t-i\epsilon}\,dt' + $$

$$+ B_{\alpha\beta}\left(t, \cos\phi_t = \frac{1}{\sqrt{3}}\right) + P_{\alpha\beta}\left(t, \cos\phi_t = \frac{1}{\sqrt{3}}\right), \qquad (1)$$

where $B_{\alpha\beta}$ and $P_{\alpha\beta}$ are the contributions of the Born terms and of the resonances, and δ^T_s is the phase shift of the s-wave of $\pi\pi$ scattering with isospin T.

The solution of the singular equation (1) reduces to a solution of the Riemann boundary problem [6]. The cross section of the process $\gamma + \gamma \to \pi + \pi$ for virtual photons in colliding beams (the electron and positron are scattered forward) is written in the form [7]:

$$\sigma_{\gamma\gamma\to\pi\pi}(t) = \sigma^{\perp\perp}_{\gamma\gamma\to\pi\pi}(t) + 4K(t)\sigma^{\perp\parallel}_{\gamma\gamma\to\pi\pi}(t) + 4K^2(t)\sigma^{\parallel\parallel}_{\gamma\gamma\to\pi\pi}(t)$$

where

$$K(t) = 16\frac{m_\gamma^4}{t^2}\left[\frac{(2E-\sqrt{t})E}{m_\gamma^2} + \frac{1}{2}\right]; \quad k^2 = k'^2 = -m_\gamma^2;$$

E is the particle energy in the colliding beams.

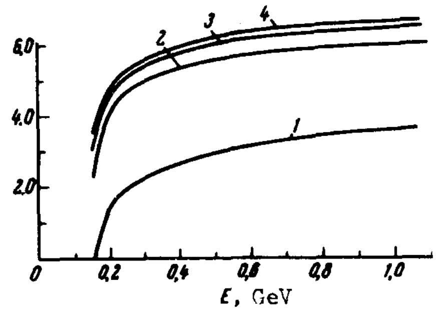

Fig. 3. Curves 1, 2, 3, and 4 represent log $(10^{36}\sigma_{ee\to ee\pi\pi}\ \mathrm{cm}^2)$ for $\pi^+\pi^-$, $\pi\pi(T=0)$, $\pi\pi(T=2)$, and $\pi^0\pi^0$ final two-meson states.

We note that for real light ($m_\gamma^2 = 0$) the cross section of the processes with participation of longitudinal photons make no contribution. The solutions of Eq. (1) by means of the exact formulas were calculated with a computer using for the $\pi\pi$ scattering phase shifts $\delta_s^T(t)$ expressions that agree well with the experimental points. Resonant parametrization was chosen for $\delta_s^0(t)$, with resonance at the point $M_\sigma = 730$ MeV.

Plots of the cross sections of the processes $\gamma + \gamma \to \pi + \pi$ with transverse photons are shown in Figs. 1a and 1b. The cross sections of the same process with participation of one or two longitudinal photons are shown in Figs. 2a and 2b. It is of interest to compare these results with those of [8], where this reaction is considered for real photons.

In our case the cross section for the production of a $\pi\pi$ pair (T = 0) has an anomalous behavior in the region $M_\sigma = 730$ MeV, due to the σ meson. The cross sections for the production of the $\pi^+\pi^-$ pair reveal a broad peak. No such peak is observed in [8]. This is apparently connected with the parametrization of the $\pi\pi$ phase shifts.

Figure 3 shows the differential cross sections of the processes ee → ee + $\pi\pi$, obtained by the equivalent-photon method. The cross section of the process ee → ee + $\pi^0\pi^0$ turns out to be larger by one order of magnitude than the cross section of the process ee → ee + $\pi^+\pi^-$. The situation is reversed in [8], owing to the fact that no account was taken in [8] of the contribution of the ω and ρ resonances (the ω meson contributes to the reaction $\gamma + \gamma \to \pi^0 + \pi^0$ and the ρ meson to both reactions), the constants of which are connected by the relation

$$g^2_{\omega\pi\gamma} = 9g^2_{\rho\pi\gamma} > g^2_{\rho\pi\gamma}.$$

In conclusion, I am grateful to P.S. Isaev for directing the work and to I.F. Ginzburg for fruitful discussions.

[1] N. Arteago-Romero, A. Jaccarini, J. Parisi, and P. Kessler, Lett. Nuovo Cim. 4, 933 (1970); 1, 935 (1971).
[2] S.J. Brodsky, T. Kihoshita, and H. Terazawa, Phys. Rev. Lett. 25, 972 (1970).

[3] V.M. Budnev and I.F. Ginzburg, Yad. Fiz. 13, 519 (1971) [Sov. J. Nuc. Phys. 13, 370 (1971)].
[4] A. Bramon and M. Greco, Preprint, LNF-71/34, Roma (1971).
[5] H. Cheng and T.T. Wu, Preprint, DESY 71/23, Hamburg (1971).
[6] D. Gakhov, Kraevye zadachi (Boundary Value Problems), Gostekhizdat, 1960.
[7] P.S. Isaev and V.I. Khleskov, JINR Preprint R2-5505, Dubna, 1971.
[8] D.H. Luth, Preprint, University of Lancaster, 1971.

E R R A T A

The title of the article by N. K. Zhabitenko et al., Vol. 14, No. 8, p. 312 should read: "Absorption and Amplification of Ultrasound in a Two-layer System Consisting of a Nonpiezoelectric Ceramic with Large Dielectric Constant and a Semiconductor" instead of "... Consisting of a Piezoelectric Ceramic..."

In the article by E. M. Lipmanov, Vol. 14, No. 9, p. 365, formula (3) should read

$$1 + \sum_{k=1}^{N} \epsilon_i^k \epsilon_j^k = y_i \delta_{ij} .$$

In the same article, p. 366, line 9 from the top should begin with "between $G_{diag}^{(ev)}$ and G_F" instead of "between G_{diag} and G_F." On the same page, in the first line after Eq. (9), read "...$G_{diag}^{0} = G_{diag}^{(ev)}/2$" instead of "...$G_{diag} = G_F/2$."