# CHANGE OF TOPOLOGY OF FERMI SURFACE IN CRYSTALS WITH AN ADDITIONAL LONG PERIOD, AND SOME ASSOCIATED EFFECTS

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1. Interest in single crystals with superlattices (systems of thin planar layers at a distance of several hundred Å) has greatly increased recently. This is due to the possibility of observing in such systems negative differential conductivity [1, 2], generation and amplification of electromagnetic waves [3], and specific optical effects [4]. An additional long period $\lambda$ of this type can be produced also with the aid of hypersound. Of these two methods, the latter is more convenient, since it makes it possible to change, in the same crystal, not only the magnitude of the period $\lambda$ but also the effectiveness of the periodic potential produced by it.

It is shown in [5] that when a longitudinal hypersonic wave with frequency $\nu \simeq 10^{10}$ Hz propagates in a crystal, the energy spectrum $\varepsilon = \varepsilon(\vec{p})$ of the electrons moving in the direction of the wave vector $\vec{q}$ breaks up into a number of alternating allowed and forbidden subbands. The width $\Delta\varepsilon$ of the allowed band is determined by the wavelength $\lambda$ of the hypersound, and the width $\Delta\varepsilon_g$ of the forbidden band is determined by the intensity of the hypersound:

$$\Delta\epsilon_n \approx \frac{\pi^2\hbar^2 n^2}{2m_z^*\lambda^2} = \frac{\pi^2\hbar^2\nu^2 n^2}{2m_z^* c_{ac}}, \quad n = 1, 2 \ldots; \ q \parallel z \tag{1}$$

$$\Delta\epsilon_g \approx 2V_{eff} = \sqrt{\frac{w}{c_z}}\, V_g \tag{2}$$

($n$ is the number of the allowed bands, $m_z^*$ is the effective mass of the electron and the direction of the z axis, $c_{ac}$ is the velocity of the longitudinal hypersonic wave in the given direction, $V_{eff}$ is the effective potential produced by the hypersound in the crystal, w is the energy density of the hypersound, $c_z$ is the diagonal component of the elasticity tensor, and $V_g$ is the deformation potential of the given substance).

2. We wish to call attention to the fact that the appearance of an additional long period in crystals should lead also to a change in the topology of the Fermi surface, and as a consequence also to new interesting phenomena.

Let us consider for simplicity a closed electron equal-energy Fermi surface. Its intersections with the plane $p_x = 0$ is shown in Fig. 1. The

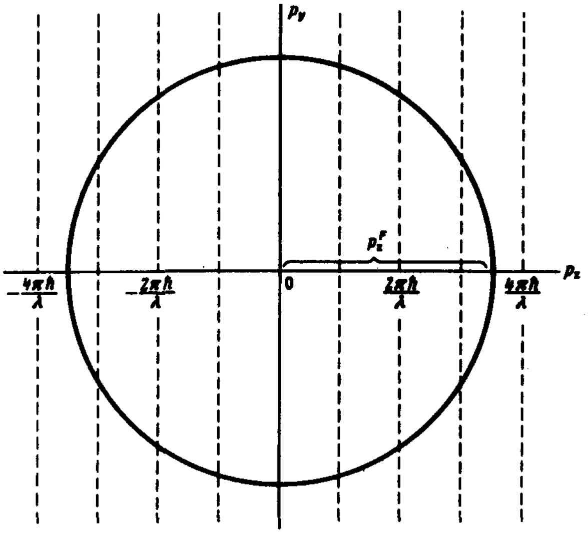

Fig. 1. Intersection of the closed electron Fermi surface with the plane $p_x = 0$. The dashed lines show the family of additional energy-discontinuity planes perpendicular to the hypersound propagation direction.

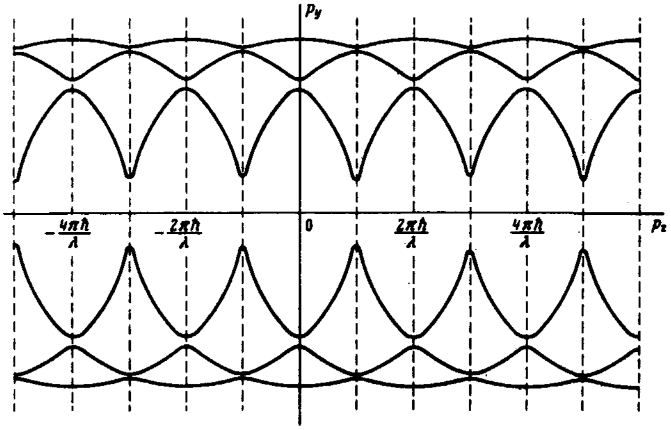

Fig. 2

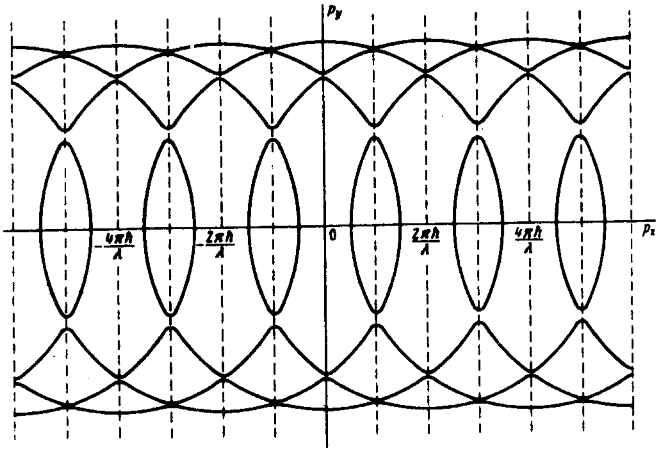

Fig. 3

Fig. 2. Electron equal-energy surface of a crystal with an additional long period, consisting of a system of coaxial corrugated cylinders in subbands 1, 2, and 3 and a closed lens-like surface in the fourth subband. (Particular case when $3\pi\hbar/\lambda < p_z^F < 4\pi\hbar/\lambda$, where $\lambda$ is the additional long period.)

Fig. 3. Fermi surface of a crystal with an additional long period, consisting only of a system of coaxial corrugated cylinders ($p_z^F \simeq 3\pi\hbar/\lambda$).

additional long-period potential in the direction of the z axis in the crystal leads to the appearance of a family of additional energy discontinuity planes perpendicular to $p_z$ in momentum space. Since the energy becomes a periodic function of $p_z$,

$$\epsilon\,(p_z) = \epsilon\left(p_z \pm \frac{2\pi\hbar n}{\lambda}\right), \quad n = 1, 2, \ldots \tag{3}$$

it follows that we can use the following procedure to construct the equal-energy surfaces in the subbands. We construct analogous Fermi surfaces with centers at the equivalent points $\pm 2\pi\hbar n/\lambda$, $n = 0, 1, 2, \ldots$ on the $p_z$ axis. Allowance for the finite values of $\Delta\varepsilon_g$ of the energy discontinuities on the planes $p_z = \pm\pi\hbar n/\lambda$ leads to a listing of the degeneracy on the intersection lines with the surfaces constructed in the indicated manner. As a result we get a system of open equal-energy surfaces of the coaxial corrugated cylinder type in subbands 1, 2, and 3 and a closed lens-like surface in the fourth subband (Fig. 2).

With decreasing period $\lambda$, the distances between the planes bounding the Brillouin subzones increases. The depths of the corrugations of the open surfaces is also increased and the volume of the closed surface decreases. If

$$\frac{\pi\hbar n}{\lambda} \approx p_z^F, \quad n = 0, 1, 2\ldots \tag{4}$$

($p_z^F$ is the component of the Fermi momentum), the closed surfaces contract to a point (Fig. 3).

It is obvious that the foregoing analysis is valid for closed equal-energy surfaces of any shape. The long period should also change the topology of the open equal-energy surfaces. For example, for an initial equal-energy surface of the corrugated cylinder type, the additional period in the direction perpendicular to the opening leads to a transformation of such a surface into a system of corrugated planes.

3. The appearance of open equal-energy surfaces should lead to the following new effects: a) oscillations of the conductivity in the sound propagation direction when the wavelength is changed. The period $\Delta(\lambda)$ of these oscillations determines the extremal dimension $p_z^F$ of the Fermi surface in this direction: $p_z^F = \pi\hbar/\Delta(\lambda)$. b) A qualitative change in the galvanomagnetic properties of the crystal, namely, a transition from saturation of the magnetoresistance to a quadratic growth in the field at $\vec{H}\perp\vec{q}$. c) Magnetic breakdown at $H = H_c$, accompanied by a sharp decrease of the magnetoresistance. The value of $H_c$ is determined by the value of $\Delta\varepsilon_g$ (i.e., by the sound intensity). Magnetic breakdown should be observed only at values $\lambda > \pi\hbar/p_z^F$. d) A qualitative change in the oscillatory effects at different orientations of the magnetic field relative to the sound propagation direction, namely, the appearance of new and the vanishing of old periods as a result of the change in the topology (at $H < H_c$) and as a result of magnetic breakdown (at $H > H_c$). e) A possibility of observing non-extremal (at different $p_z$) sections S of the Fermi surface and a dependence of S on $p_z$, and also, under certain conditions, a dependence of $\varepsilon$ on $p_z$. f) A possibility of determining the dependence of $\Delta\varepsilon_g$ on the value of the effective potential.

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## NONLINEAR MODULATION OF A QUASIMONOCHROMATIC PACKET OF WHISTLERS IN THE MAGNETOSPHERE

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Many researches have been recently reported on the propagation of monochromatic whistlers along the geomagnetic field in the upper ionosphere and magnetosphere. Included among the very interesting experiments of this type are studies in which the waves were emitted by a loud-based transmitter and registered by a receiver located at the magnetically-conjugate point (see, for example, [1] and a large number of analogous investigations).

In the interpretation of the experimental results it is necessary to recognize that the state of the plasma in the equatorial region of the magnetosphere is very frequently characterized by an anisotropic distribution function