It is obvious that the foregoing analysis is valid for closed equal-energy surfaces of any shape. The long period should also change the topology of the open equal-energy surfaces. For example, for an initial equal-energy surface of the corrugated cylinder type, the additional period in the direction perpendicular to the opening leads to a transformation of such a surface into a system of corrugated planes.

3. The appearance of open equal-energy surfaces should lead to the following new effects: a) oscillations of the conductivity in the sound propagation direction when the wavelength is changed. The period $\Delta(\lambda)$ of these oscillations determines the extremal dimension p_z^F of the Fermi surface in this direction: $p_z^F = \pi\hbar/\Delta(\lambda)$. b) A qualitative change in the galvanomagnetic properties of the crystal, namely, a transition from saturation of the magnetoresistance to a quadratic growth in the field at $\vec{H} \perp \vec{q}$. c) Magnetic breakdown at $H = H_c$, accompanied by a sharp decrease of the magnetoresistance. The value of H_c is determined by the value of $\Delta\varepsilon_g$ (i.e., by the sound intensity). Magnetic breakdown should be observed only at values $\lambda > \pi\hbar/p_z^F$. d) A qualitative change in the oscillatory effects at different orientations of the magnetic field relative to the sound propagation direction, namely, the appearance of new and the vanishing of old periods as a result of the change in the topology (at $H < H_c$) and as a result of magnetic breakdown (at $H > H_c$). e) A possibility of observing non-extremal (at different p_z) sections S of the Fermi surface and a dependence of S on p_z, and also, under certain conditions, a dependence of ε on p_z. f) A possibility of determining the dependence of $\Delta\varepsilon_g$ on the value of the effective potential.

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NONLINEAR MODULATION OF A QUASIMONOCHROMATIC PACKET OF WHISTLERS IN THE MAGNETOSPHERE

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Many researches have been recently reported on the propagation of monochromatic whistlers along the geomagnetic field in the upper ionosphere and magnetosphere. Included among the very interesting experiments of this type are studies in which the waves were emitted by a loud-based transmitter and registered by a receiver located at the magnetically-conjugate point (see, for example, [1] and a large number of analogous investigations).

In the interpretation of the experimental results it is necessary to recognize that the state of the plasma in the equatorial region of the magnetosphere is very frequently characterized by an anisotropic distribution function

($T_\perp > T_\parallel$). The whistlers passing through it can therefore be amplified to such an extent that nonlinear effects become significant. Among the latter are the effects discussed in [2], namely generation of satellites and broadening of the spectrum by an amount $\Delta f \sim (\tau)^{-1}$, where

$$\tau = (k\,\omega_c\, v_{T_\perp} h/H)^{-1/2} \tag{1}$$

is the characteristic period of the longitudinal-velocity oscillations of the particles that interact resonantly with the wave (h/H is the ratio of the amplitude of the alternating field in the wave to the dc field. ω_c is the electron cyclotron frequency, and k is the wave number). In [2], however, monochromatic waves of constant amplitude were considered, whereas the transmitter emits usually quasimonochromatic wave packets with sharp boundaries. In this article we discuss some nonlinear effects due to the bounded nature of the packet.

Let us consider a rectangular wave packet entering into the active region of the magnetosphere, where the lienar increment is $\gamma_L > 0$. If the distribution function ahead of the packet is stationary, then the expression for the nonlinear increment is

$$\gamma(z) = \frac{8\omega_p^2\omega}{k^2c^2}\left(1 - \frac{\omega}{\omega_c}\right)\int_0^\infty dw\,w^3 f_0'(w) \iint_S \frac{\sin[2\,\mathrm{am}(F,\kappa)]}{\kappa^3} \times \\ \times\, \mathrm{dn}\left[F - \frac{z}{\kappa\tau V},\ \kappa\right] dF\,d\kappa, \tag{2}$$

where z is the coordinate measured into the interior of the packet from its leading front in the reference frame where the latter is at rest, V is the average velocity of the resonant particles relative to the packet

$$V = v_g - \frac{\omega - \omega_c}{k} = v_g\left(1 + \frac{\omega_c}{2\omega}\right);\quad v_g = \frac{\partial\omega}{\partial k};$$

$$f_0'(W) = \left(\frac{\partial f_0}{\partial v_z} + \frac{\omega_c}{kw}\frac{\partial f_0}{\partial w}\right)_{v_z = \frac{\omega - \omega_c}{k}}$$

$f_0(v_z, w)$ is the distribution function ahead of the packet, v_z and w are the longitudinal and transverse velocity components, $\mathrm{am}(F, \kappa)$ and $\mathrm{dn}(F, \kappa)$ are elliptic functions with modulus κ, and the integration region S is shown in Fig. 1. The main contribution to the integral (2) is made by the values $\kappa \lesssim z/\tau V$. At small z, i.e., near the leading front of the packet, expression (2) can be easily integrated and we obtain for $z \ll \tau V$

$$\gamma(z) = \gamma_L = \frac{\pi^2\omega_p^2\omega}{k^2c^2}\left(1 - \frac{\omega}{\omega_c}\right)\int_0^\infty f_0'(w)\,w^3 dw,$$

where γ_L is the increment in the linear theory. At $z \gtrsim \tau V$ the increment oscillates (with period $\sim\tau V$) and tends to zero (Fig. 2). The amplitude of the packet in the initial state has a time

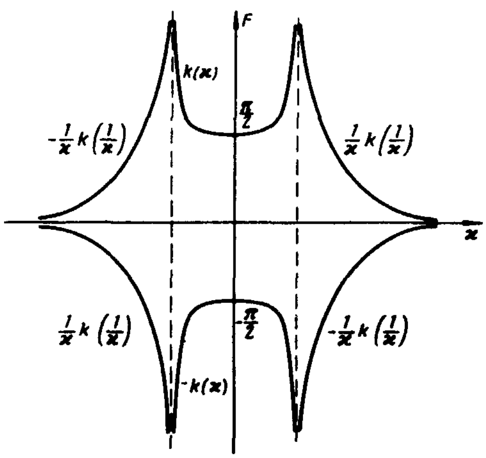

Fig. 1. The integration region S.

dependence given by

$$h(z, t) = h_0 \exp[\gamma(z) t] , \qquad (3)$$

from which we see that in the course of time there should appear amplitude modulations with a characteristic length $V\tau$. At large t ($\gamma_L t \gtrsim 1$) the packet becomes strongly modulated and expression (3) is no longer valid. It is clear, however, that since the vanishing of the increment of a rectangular packet at $z \gg \tau V$ is a consequence of ergodicity [3], it follows that as the modulation develops in the leading part of the packet, the modulation should propagate with time also into the interior of the packet. It is also clear that if the initial amplitude of the packet increases along z sufficiently smoothly, then the modulation can proceed so slowly that it has no time to become manifest during the time of motion of the packet in the magnetosphere. An investigation of all the processes at times $\gamma_L t > 1$ requires, however, the use of a numerical model.

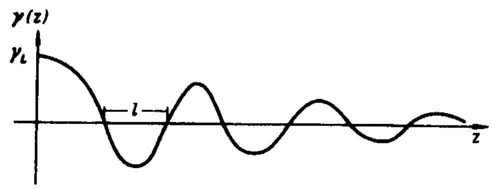

Fig. 2. Dependence of the increment γ on the distance z for a rectangular packet, $\ell \sim \tau V$.

For a strongly modulated packet, the modulation period is determined by the time τ corresponding to a certain effective value of the amplitude of the packet envelope, which generally speaking is several times smaller than the maximum amplitude. Indeed the deviation from the average resonant velocity, for particles interacting with a wave, is of the order of magnitude $(k\tau)^{-1}$, so that the particles for which τ is calculated from the maximum amplitude are relatively rarely at resonance with the wave.

In addition to amplitude modulation, naturally, there should take place also frequency modulation characterized by the quantity $\Delta f \sim (\tau)^{-1}$, where τ corresponds to the local amplitude. This modulation, as already noted above, is due to the broadening of the spectrum as a result of the oscillations of the resonant frequencies with period τ. Thus, the period of the amplitude-frequency modulation T, observed when the signal is received on the earth, should be of the order of

$$T \sim \frac{V}{v_g (\Delta f)_{eff}} , \qquad (4)$$

where $(\Delta f)_{eff}$ is a certain effective broadening of the spectrum in a modulated packet and is smaller by several times than $(\Delta f)_{max}$.

The foregoing considerations apparently explain the nature of the modulation effects reported recently in [4]. The period of the amplitude-frequency modulation of the complete packet passing along a force line of the magnetic field, measured in that reference, is T = 0.1 - 0.2 sec, and $(\Delta f)_{max} \sim 100$ Hz.

Recognizing that $\omega_c/2 \sim 2$ in this experiment (in the equatorial region, where the phenomena in question mainly take place), we obtain a perfectly satisfactory agreement between our estimates and experiment.

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BOUND STATES OF EXCITATIONS IN A BOSE SYSTEM WITH A CONDENSATE

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In this article we construct a theory of bound states of elementary excitations in liquid He^4 (or, in general, in a homogeneous Bose system with a condensate). The theory shows that the condensate plays a unique role which makes it necessary to review the usual correspondence between bound states and the poles of multiparticle Green's functions. This makes it possible, in particular, to explain why the branch observed in experiments with Raman scattering and characterizing the bound state of two rotons with nonzero angular momentum $\ell = 2$ [1] does not appear with any degree of clarity in experiments with neutron scattering [2]. The theory employs the Green's-function method and the Feynman diagram technique, but no reference is made to perturbation theory at all.

The starting point for the theoretical analysis of the bound states in He^4 is a fact, observed by Pitaevskii in his theory of the termination of a single-particle spectrum [3]. Namely, if we assume that the single-particle spectrum has a phonon-roton form (Fig. 1, ε_1), then in diagrams containing elements with two Green's functions  each element contains a logarithmic singularity connected with the roton minimum; the sum of the diagrams makes the contribution

$$\sim \frac{1}{1 + Q(p) \ln \frac{a}{2\Delta - \epsilon}} \qquad (1)$$

(Δ is the roton minumum).

Since the described diagrams enter in the self-energy part , a similar singularity arises in the denominator of the single-particle Green's function

$$G^{-1} = A^{-1}\left[\epsilon - \epsilon_p^0 + \frac{P(p) \ln \frac{a}{2\Delta - \epsilon}}{1 + Q(p) \ln \frac{a}{2\Delta - \epsilon}}\right] \qquad (2)$$

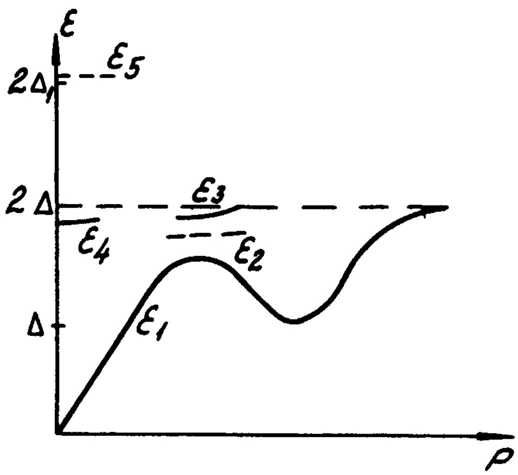

Fig. 1

$p > 0$; ε_p^0 is the "regular" part of the spectrum, i.e., the sum of the diagrams that do not contain the described singularity. It is seen from (1) and (2) that in the case of effective attraction between two rotons ($\theta(p) < 0$), a pole of the vertex (1) ε_2 appears regardless of the value of the interaction, and if $P/|Q| < \Delta$, an additional pole appears in G (ε_3, Fig. 1); this was pointed out in [4].