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BOUND STATES OF EXCITATIONS IN A BOSE SYSTEM WITH A CONDENSATE

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In this article we construct a theory of bound states of elementary excitations in liquid He^4 (or, in general, in a homogeneous Bose system with a condensate). The theory shows that the condensate plays a unique role which makes it necessary to review the usual correspondence between bound states and the poles of multiparticle Green's functions. This makes it possible, in particular, to explain why the branch observed in experiments with Raman scattering and characterizing the bound state of two rotons with nonzero angular momentum $\ell = 2$ [1] does not appear with any degree of clarity in experiments with neutron scattering [2]. The theory employs the Green's-function method and the Feynman diagram technique, but no reference is made to perturbation theory at all.

The starting point for the theoretical analysis of the bound states in He^4 is a fact, observed by Pitaevskii in his theory of the termination of a single-particle spectrum [3]. Namely, if we assume that the single-particle spectrum has a phonon-roton form (Fig. 1, ε_1), then in diagrams containing elements with two Green's functions  each element contains a logarithmic singularity connected with the roton minimum; the sum of the diagrams makes the contribution

$$\sim \frac{1}{1 + Q(p) \ln \frac{a}{2\Delta - \epsilon}} \qquad (1)$$

(Δ is the roton minumum).

Since the described diagrams enter in the self-energy part , a similar singularity arises in the denominator of the single-particle Green's function

$$G^{-1} = A^{-1}\left[\epsilon - \epsilon_p^0 + \frac{P(p) \ln \frac{a}{2\Delta - \epsilon}}{1 + Q(p) \ln \frac{a}{2\Delta - \epsilon}}\right] \qquad (2)$$

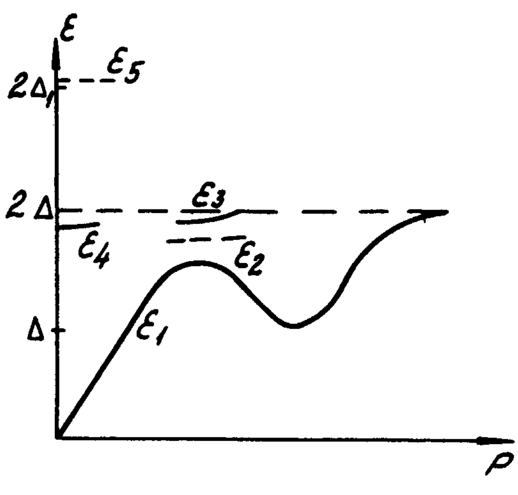


Fig. 1

$p > 0$; ε_p^0 is the "regular" part of the spectrum, i.e., the sum of the diagrams that do not contain the described singularity. It is seen from (1) and (2) that in the case of effective attraction between two rotons ($\theta(p) < 0$), a pole of the vertex (1) ε_2 appears regardless of the value of the interaction, and if $P/|Q| < \Delta$, an additional pole appears in G (ε_3, Fig. 1); this was pointed out in [4].

We now formulate two statements that serve as the basis for the proposed theory. The first concerns experiments with neutrons, i.e., the properties of the function of the linear response to the perturbations of the density $\chi(p, \varepsilon)$, namely, the set of poles of this function for a Bose system with condensate coincides identically with the set of poles of the single-particle Green's function of the supercondensate particles. It follows from this, in particular, that the branch ε_2 cannot be observed in experiments with neutrons. Let us describe the idea of the proof: By virtue of the nonconservation of the number of the supercondensate particles, the roton bound pair is capable of becoming transformed into a single-particle excitation , and vice versa. But if there are two Bose fields interacting with each other via mutual transformation (i.e., $H_{int} \sim a_1^+ a_2 + a_1 a_2^+$), then their nonrenormalized frequencies not only get shifted as a result of the interaction, but become doubled in each of the Green's functions $\langle T a_1 a_1^+ \rangle$ and $\langle T a_2 a_2^+ \rangle$. In other words, the Green's functions become transformed into the components of the matrix Green's function $\langle T a_i a_k^+ \rangle$ (with a single denominator). Thus, χ and

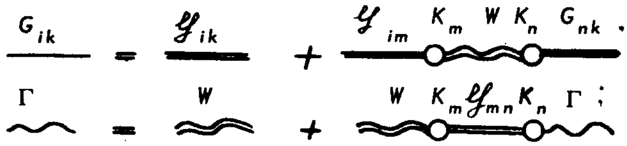
$$G = G_{ik} = \begin{pmatrix} G'(p) & \hat{G}(p) \\ \check{G}(p) & G'(-p) \end{pmatrix}$$

can be regarded as different components of a (3 × 3) matrix Green's function.

We now present the concrete scheme of the proof (a detailed exposition of the theory will be published in a more comprehensive article): χ is connected with the "interaction line" Γ

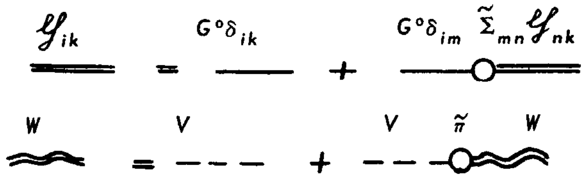
$$\delta n(p, \epsilon) =$$  $$= \chi(p, \epsilon)\,\delta U(p, \epsilon) = \frac{\delta U}{V_p}\Gamma(p, \epsilon)\tilde{\pi}(p, \epsilon) \qquad (3)$$

(n is the density and V_p the Fourier component of the paired potential). The poles of Γ and G_{ik} coincide, as follows from the equations



(4)

The latter can be easily obtained by using the auxiliary functions

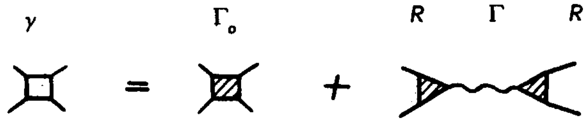


(5)

and the irreducible "self-energy" parts k, $\tilde{\Sigma}$, and $\tilde{\pi}$, at the inputs and outputs of which there are lines of either particles or the potential. It remains to note that the poles of χ and Γ coincide, since $\tilde{\pi}\Gamma = 0$ at the pole.

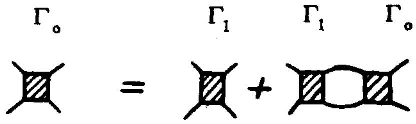
It is seen that ε_2 (pole of W) is only the nonrenormalized frequency of the roton pair, i.e., it does not correspond to any excitations in the system, similar to ε_p^0 (pole of $\mathcal{G}$), which is the nonrenormalized frequency of the supercondensate particles. To the contrary, ε_1 and ε_2 are the particle and pair frequencies "corrected" with allowance for the interaction.

The second statement is as follows: there exist excitations of the bound-pair type, which do not enter in χ and G, and consequently have no bearing on the neutron experiments. To prove this, we represent the total vertex for the scattering of two particles in the form of a sum:



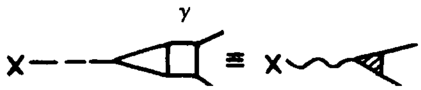
(6)

where



$$\sim \frac{\Gamma_1}{1 + Q_1 \ln \frac{a}{2\Delta - \epsilon}} \tag{7}$$

(The vertices Γ_0 and R are irreducible with respect to one line of particles and the potential, and Γ_1 is also irreducible with respect to two particle lines). It can be shown (the details will be given in the more comprehensive article) that the pole of $\Gamma_0(\varepsilon = \varepsilon^{\Gamma_0})$ is simultaneously a pole of γ, i.e., it corresponds to real excitations $|\varepsilon_4|$, and is not contained in χ: at $\varepsilon = \varepsilon^{\Gamma_0}$ we have



$$= \chi\, \delta U(R/\pi) = 0, \tag{8}$$

i.e., $\chi(\varepsilon = \varepsilon^{\Gamma_0}) = 0$ (---=---+---○══○--- ; R/π is finite). The character of the corresponding bound state is determined by the fact that it does not contain density oscillations; its contribution to δn is equal to zero even at $\delta U \neq 0$ (cf. (8) and (3)). This means that the internal wave function of the pair vanishes when the arguments coincide, i.e., it contains harmonics with $\ell \neq 0$. Further, the wave function of the pair should in general not contain single-particle states, since they, too, are connected with the density oscillations, i.e., the matrix element of the transition of a pair into a single-particle state should be equal to zero. This corresponds to a wave function with helicity $(\vec{l}\cdot\vec{p}/|p|) \equiv m \neq 0$ (m is conserved and classifies the possible bound states with given p by virtue of cylindrical symmetry; $Q_{m=0} \equiv Q$, $Q_{m\neq 0} \equiv Q_1$, see (1) and (7)).

Thus, if the transition of a pair into a single-particle state is allowed (m = 0), the pole of the irreducible vertex $\Gamma^{(m=0)} \equiv W$ is a bare one for a two-particle pole of G or χ; if the transition is forbidden ($m \neq 0$), $\Gamma^{(m\neq 0)} \equiv \Gamma_0$ characterizes independent two-particle excitations and is a pole of γ but not of G or χ. Although branches with $m \neq 0$ are not observable in neutron experiments, where the scattering is single, they should become manifest in effects of second order in the external perturbation, particularly in Raman scattering [1] (Fig. 2; in the experiment $p \to 0$ and $\ell = 2$).

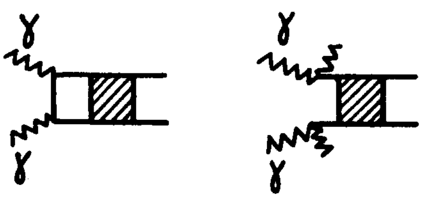


Fig. 2

Let us consider the damping. If the branch $\varepsilon_1 \leq 2\Delta$ is stable against decay into single-particle excitations, then p, Q, and α in (1) and (2) are real, but, taking into account the damping explicitly, it is to make in (1) and (2) the replacement ln → ln + $iF(\varepsilon)\theta(\varepsilon - \varepsilon_1)$ (since ε_1 is the lowest of all the branches with m = = 0). We see that ε_3 attenuates everywhere. The question of the existence of ε_3 for real He^4 is apparently left open by experiments with neutron scattering [2]. Branches with $m \neq 0$ can in principal turn out to be lower than ε_1, i.e., to be strictly non-damped.

In conclusion we note that it is possible in principle to realize other bound states (in addition to two-roton states). Under certain conditions a weakly damped branch $\varepsilon_5 > 2\Delta_1$ is produced. In analogy with the preceding, using the logarithmic singularities, we can predict (and in simple models also calculate) the bound levels of three and more particles; for example if ε_4 has a minimum, then a bound state of ε_4 excitations with rotons can occur. Similar states with m = 0 would enter as new branches in χ and G, and those with $m \neq 0$ would enter in the Green's functions of the pairs.

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HIGH-ENERGY ASYMPTOTIC FORM OF SCATTERING AMPLITUDES AND VERTEX FUNCTIONS

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1. In many recent papers (see, for example, [1 - 4]), use is made of the connection between the high-energy asymptotic form of the scattering amplitudes (and vertex functions) and the space-time structure of the matrix elements of the commutators of the corresponding currents on the light cone. The indicated papers are based on the universally accepted a priori conviction that the high-energy asymptotic form is wholly governed by the indiciated space-time structure only on the light cone. This conviction is based in turn on the fact that the asymptotic form of the Fourier integrals is governed only by the discontinuity points of the integrand or of its derivatives. In a recent paper [5], attention was called to the fact that the last circumstance is rigorous only for one-dimensional Fourier integrals, and generally speaking does not hold for multidimensional Fourier integrals. On the basis of this remark, an example was constructed in [5], wherein the high-energy asymptotic form is determined by the space-time structure not on the light cone.

2. On the basis of the representation of the high-energy asymptotic form from [6], we explain below the following: a) why it is possible to determine this asymptotic form from the space-type structure not on the light cone, b) the conditions under which this takes place, and c) experimental tests for verifying these conditions. In this brief communication all is demonstrated using as an example an investigation of the asymptotic form of the imaginary part of the amplitude for forward scattering of a virtual γ quantum with "mass" $|q^2|$ and energy q_0 by a nucleon [2]: