$\ln \to \ln + iF(\varepsilon)\theta(\varepsilon - \varepsilon_1)$ (since ε_1 is the lowest of all the branches with $m = 0$). We see that ε_3 attenuates everywhere. The question of the existence of ε_3 for real He^4 is apparently left open by experiments with neutron scattering [2]. Branches with $m \neq 0$ can in principal turn out to be lower than ε_1, i.e., to be strictly non-damped.

In conclusion we note that it is possible in principle to realize other bound states (in addition to two-roton states). Under certain conditions a weakly damped branch $\varepsilon_5 > 2\Delta_1$ is produced. In analogy with the preceding, using the logarithmic singularities, we can predict (and in simple models also calculate) the bound levels of three and more particles; for example if ε_4 has a minimum, then a bound state of ε_4 excitations with rotons can occur. Similar states with $m = 0$ would enter as new branches in χ and G, and those with $m \neq 0$ would enter in the Green's functions of the pairs.

I am deeply grateful to L.P. Pitaevskii for a detailed discussion of the results and valuable remarks, and also to O.K. Kalashnikov, L.V. Keldysh, D.A. Kirzhnits, N.M. Plakida, A.A. Sobyanin, and E.S. Fradkin for fruitful discussions.

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HIGH-ENERGY ASYMPTOTIC FORM OF SCATTERING AMPLITUDES AND VERTEX FUNCTIONS

L.A. Khalfin
Submitted 10 January 1972
ZhETF Pis. Red. 15, No. 4, 215 - 218 (20 February 1972)

1. In many recent papers (see, for example, [1 - 4]), use is made of the connection between the high-energy asymptotic form of the scattering amplitudes (and vertex functions) and the space-time structure of the matrix elements of the commutators of the corresponding currents on the light cone. The indicated papers are based on the universally accepted a priori conviction that the high-energy asymptotic form is wholly governed by the indiciated space-time structure only on the light cone. This conviction is based in turn on the fact that the asymptotic form of the Fourier integrals is governed only by the discontinuity points of the integrand or of its derivatives. In a recent paper [5], attention was called to the fact that the last circumstance is rigorous only for one-dimensional Fourier integrals, and generally speaking does not hold for multidimensional Fourier integrals. On the basis of this remark, an example was constructed in [5], wherein the high-energy asymptotic form is determined by the space-time structure not on the light cone.

2. On the basis of the representation of the high-energy asymptotic form from [6], we explain below the following: a) why it is possible to determine this asymptotic form from the space-type structure not on the light cone, b) the conditions under which this takes place, and c) experimental tests for verifying these conditions. In this brief communication all is demonstrated using as an example an investigation of the asymptotic form of the imaginary part of the amplitude for forward scattering of a virtual γ quantum with "mass" $|q^2|$ and energy q_0 by a nucleon [2]:

$$\operatorname{Im} M(q_0, q^2) = \iint\limits_{-\infty}^{\infty} e^{iq_0 t - i\sqrt{q_0^2 - q^2}\, z} f(z, t)\,dz, dt, \tag{1}$$

where f(z, t) is connected with the matrix element of the commutator of the corresponding currents, and the z axis is directed along the vector $\vec{q}$.[1)]

We consider below two asymptotic regimes: 1) $q_0 \to \infty$, $|q^2| \to \infty$, $\omega = q^2 - q_0$ = const (Bjorken A-regime) and 2) $q_0 \to \infty$, q^2 = const (Regge R-regime). On the basis of the causality condition we obtain [6]

$$\operatorname{Im} M(q_0, q^2) = 2\operatorname{Re}\left\{ \int\limits_0^\infty \exp\left[- iq_0 z + i\frac{\omega}{r} z\right] dz \int\limits_{-\infty}^{-z} \exp(iq_0 t) f(z, t)\,dt + \right. \\ \left. + \int\limits_0^\infty \exp - \left[iq_0 z + i\frac{\omega}{2} z\right] dz \int\limits_z^\infty \exp[iq_0 t] f(z, t)\,dt \right\} \tag{2}$$

for the A-regime. The formula for the R-regime is obtained from (2) by making the substitution $\omega \to q^2/q_0$. It follows from these formulas that an investigation of the asymptotic form in the A-regime reduces to an investigation of the asymptotics of one-dimensional Fourier integrals at infinitely large values of the argument ($q_0 \to \infty$), and the investigation of the R-regime reduces to an investigation of the asymptotic forms of one-dimensional Fourier integrals for both infinitely large values of the argument ($q_0 \to \infty$) and infinitesimally small values of the argument ($q^2/2q_0 \to 0$). Assuming f(z, t) to be regular inside the light cone and the singularities to be homogeneous on the cone[2)], we get from (2) [6]:

$$\operatorname{Im} M(q_0, q^2) \underset{q_0 \to \infty,\ \omega = \text{const}}{\approx} 2\operatorname{Re}\left\{ \Phi_1(q_0) \int\limits_0^\infty \exp[-2iq_0 z]\, \phi(z, -z)\,dz + \right. \\ \left. + \Phi_2(q_0) \int\limits_0^\infty \exp\left[i\frac{\omega}{2} z\right] \phi(z, z)\,dz \right\} \tag{3}$$

and an analogous formula for the R-regime by making the substitution $\omega \to q^2/q_0$. In (3) we have

$$\int\limits_{-z}^{\infty} e^{iq_0 t} f(z, t)\,dt \underset{q_0 \to \infty}{\to} \Phi_1(q_0)\, e^{-iq_0 z} \phi(z, -z), \\ \int\limits_z^\infty e^{iq_0 t} f(z, t)\,dt \underset{q_0 \to \infty}{\to} \Phi_2(q_0)\, e^{iq_0 z} \phi(z, z)\,. \tag{4}$$

It follows from (3) and (4) that the asymptotic form of Im $M(q_0, q^2)$ is determined by the space-time structure on the light cone, since this structure determines the asymptotic forms of the corresponding (see (2)) one-dimensional Fourier integrals, but this takes place only if these principal terms of the asymptotic forms do not cancel each other in (3). If this calcellation does take place, then the asymptotic form of Im $M(q_0, q^2)$ is already determined by the next terms of the asymptotic expansions of the one-dimensional Fourier integrals, which generally speaking are no longer determined by the space-time

1)The integration with respect to x and y has already been performed in (1); the notation is that of [2, 6].

2)It is easy to obtain analogous results also for inhomogeneous singularities.

structure on the light cone. The conditions under which this takes place follows directly from (3). We present, for concreteness, an example of these conditions. For (1), subject to the additional assumption that f(z, t) is an absolutely integrable function, $f(z, -z) \neq 0$, $f(z, z) \neq 0$:

$$\mathrm{Im}\int_0^\infty e^{i\frac{\omega}{2}z} f(z, z)dz = 0, \quad -\frac{1}{2}\mathrm{Re}\, f(0, 0) = \mathrm{Re}\int_0^\infty e^{i\frac{\omega}{2}z} \left.\frac{\partial f(z, t)}{\partial t}\right|_{t=z} dz. \qquad (5)$$

Satisfaction of conditions of the type (5) makes it possible to use the analytic properties in ω, and consequently the dispersion relations and the corresponding sum rules. Thus, knowing the space-time structure only on the light cone we can uniquely determine whether or not the high-energy asymptotic form is determined by the space-time structure on the light cone. It is important that this question can be answered also by investigating the experimental data, namely, by studying the dependence of the asymptotic form on the invariants q^2 in the R-regime and ω in the A-regime. Indeed, it can be seen from (3) that if the asymptotic form of Im $M(q_0, q^2)$ is a non-decreasing function of q_0 and depends on q^2 in the R-regime or on ω in the A-regime, then this asymptotic form is determined by the space-time structure only on the light cone (this fact has been noted in [6]). On the other hand, if the asymptotic form does not depend on these invariants, then it can be determined by the space-time structure also not on the light cone. In this connection, an experimental investigation of the asymptotic as a function of the invariants is particularly urgent.

If we assume the experimental data of [7] to be really asymptotic, then Im $M(q_0, q^2)$ does not decrease as $q_0 \to \infty$, and depends explicitly on q^2 and ω in the asymptotic region, and consequently the asymptotic form of Im $M(q_0, q^2)$ is determined completely in the A- and R-regimes by the space-time structure on the light cone.

Analogous results can be obtained by investigating other scattering amplitudes and vertex functions, since the mathematical problems reduce to an investigation of asymptotic forms of integrals of the type (1).

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INELASTIC CONTRIBUTIONS TO ELECTROMAGNETIC FORM FACTOR OF THE PION

V.N. Baier and V.S. Fadin

Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences

Submitted 11 January 1972

ZhETF Pis. Red. 15, No. 4, 219 - 224 (20 February 1972)

In recent experiments on the production of $\pi^+\pi^-$ pairs in colliding electron-positron beams in Novosibirsk [1] and in Frascati [2], in which the electromagnetic form factor $F_\pi(s)$ of the pion was measured, there was a considerable rise of $|F_\pi(s)|$ above the Breit-Wigner curve, which approximates $|F_\pi(s)|$ in the region of the δ resonance [3, 4], was observed at energies $\sqrt{s} = 2\varepsilon > 1$ GeV. In experiments [5, 6], in the same energy region, it was also established