

structure on the light cone. The conditions under which this takes place follows directly from (3). We present, for concreteness, an example of these conditions. For (1), subject to the additional assumption that $f(z, t)$ is an absolutely integrable function, $f(z, -z) \neq 0$, $f(z, z) \neq 0$:

$$\operatorname{Im} \int_0^\infty e^{i\frac{\omega}{2}z} f(z, z) dz = 0, \quad -\frac{1}{2} \operatorname{Re} f(0, 0) = \operatorname{Re} \int_0^\infty e^{i\frac{\omega}{2}z} \left. \frac{\partial f(z, t)}{\partial t} \right|_{t=z} dz. \quad (5)$$

Satisfaction of conditions of the type (5) makes it possible to use the analytic properties in ω , and consequently the dispersion relations and the corresponding sum rules. Thus, knowing the space-time structure only on the light cone we can uniquely determine whether or not the high-energy asymptotic form is determined by the space-time structure on the light cone. It is important that this question can be answered also by investigating the experimental data, namely, by studying the dependence of the asymptotic form on the invariants q^2 in the R-regime and ω in the A-regime. Indeed, it can be seen from (3) that if the asymptotic form of $\operatorname{Im} M(q_0, q^2)$ is a non-decreasing function of q_0 and depends on q^2 in the R-regime or on ω in the A-regime, then this asymptotic form is determined by the space-time structure only on the light cone (this fact has been noted in [6]). On the other hand, if the asymptotic form does not depend on these invariants, then it can be determined by the space-time structure also not on the light cone. In this connection, an experimental investigation of the asymptotic as a function of the invariants is particularly urgent.

If we assume the experimental data of [7] to be really asymptotic, then $\operatorname{Im} M(q_0, q^2)$ does not decrease as $q_0 \rightarrow \infty$, and depends explicitly on q^2 and ω in the asymptotic region, and consequently the asymptotic form of $\operatorname{Im} M(q_0, q^2)$ is determined completely in the A- and R-regimes by the space-time structure on the light cone.

Analogous results can be obtained by investigating other scattering amplitudes and vertex functions, since the mathematical problems reduce to an investigation of asymptotic forms of integrals of the type (1).

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INELASTIC CONTRIBUTIONS TO ELECTROMAGNETIC FORM FACTOR OF THE PION

V.N. Baier and V.S. Fadin

Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences

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In recent experiments on the production of $\pi^+\pi^-$ pairs in colliding electron-positron beams in Novosibirsk [1] and in Frascati [2], in which the electromagnetic form factor $F_\pi(s)$ of the pion was measured, there was a considerable rise of $|F_\pi(s)|$ above the Breit-Wigner curve, which approximates $|F_\pi(s)|$ in the region of the δ resonance [3, 4], was observed at energies $\sqrt{s} = 2\epsilon > 1$ GeV. In experiments [5, 6], in the same energy region, it was also established

that the photon goes over with high probability into multihadron states. In this connection, the indicated increase of the form factor $|F_\pi(s)|$ can be explained as follows: the photon goes over into multihadron states (this occurs with high probability), which are then transformed into a pair of pions¹⁾. By the same token, the intermediate states with large number of particles become significant, in other words, we are dealing with inelastic contributions to the unitarity relation for the form factor. This relation is

$$\begin{aligned} \text{Im } F_\pi(s) = & \beta f_{11} F_\pi^-(s) \theta(s - 4\mu^2) + \frac{(2\pi)^4}{2} \sum_{\substack{n \neq \pi^+\pi^- \\ (J=1)}} \delta(p_+ + p_- - p_n) \times \\ & \times \langle \pi^+\pi^- | T | n \rangle \frac{(\mathbf{p}_+ - \mathbf{p}_-)^{\mu} < 0 | \mathcal{J}_\mu(0) | n \rangle^*}{(s - 4\mu^2)} = \beta f_{11} F_\pi^-(s) \theta(s - 4\mu^2) + D. \end{aligned} \quad (1)$$

We have separated here the elastic term, which is expressed in terms of the $\pi\pi$ -scattering amplitude f_{11} in the state $I = J = 1$, $\beta = [1 - (4\mu^2/s)]^{1/2}$, μ is the pion mass, $s = (p_+ + p_-)^2$, p_\pm are the pion momenta, $\langle n | \mathcal{J}_\mu(0) | 0 \rangle$ is the amplitude for the transition of the photon into the state $|n\rangle$,

$$F_\pi^\pm(s) = F_\pi(s \pm i\epsilon), \quad F_\pi^-(s) = (F_\pi^+(s))^*, \quad \text{Im } F_\pi(s) = \frac{F_\pi^+(s) - F_\pi^-(s)}{2i}.$$

In the assumed normalization

$$\beta f_{11} = \frac{\eta e^{2i\delta_{11}} - 1}{2i}, \quad \eta \leq 1. \quad (2)$$

In the elastic region $4\mu^2 \leq s \leq 16\mu^2$, $\eta = 1$, and $D = 0$.

Taking the complex conjugate of (1), we obtain another relation for $F^+(s)$ and $F^-(s)$, which coincides with (1) in the elastic region ($s \leq 16\mu^2$). In the inelastic region, on the other hand, by solving the obtained relation together with (1), we can obtain by algebraic means an explicit expression for the form factor $F_\pi(s)$, assuming f_{11} and D to be specified

$$F_\pi^+(s) = \frac{4D}{1 - \eta^2} \frac{1}{2i} \left(\eta e^{2i\delta_{11}} \frac{D^*}{D} - 1 \right). \quad (3)$$

This representation of the form factor is convenient because, unlike the usual dispersion relations, it connects quantities obtained at the same energy.

The quantity and the right-side of (3) are not independent. The connection between them (which, of course, does not have a local character) can be obtained by considering the dispersion representations for $F_\pi(s)$. The usual procedure [7, 8] in such an analysis is to introduce the functions

$$X(s) = \exp \left[\frac{s}{\pi} \int_{4\mu^2}^{\infty} \frac{ds' \delta_{11}(s')}{s'(s'-s)} \right].$$

¹⁾ It is obvious that such a picture can take place for the electromagnetic form factors of K, p, etc.

Then the real function $\Phi_\pi(s) = F_\pi(s)/X(s)$ has a cut at $s \geq 16 \mu^2$ $\Phi_\pi(0) = F_\pi(0) = 1$, and the following relation is satisfied on the cut:

$$\Phi_\pi^+(s) = \eta(s) \Phi_\pi^-(s) + C(s), \quad C = \frac{2iD}{X^+}, \quad (4)$$

where $X^\pm(s) = X(s \pm i\epsilon)$. This relation makes it possible, in analogy with (3), to determine $\Phi_\pi(s)$ on the cut

$$\Phi_\pi^\pm(s) = \frac{\text{Re } C(s)}{1 - \eta(s)} \pm i \frac{\text{Im } C(s)}{1 + \eta(s)}. \quad (5)$$

This leads to the following dispersion relation for the form factor:

$$F_\pi(s) = X(s) \left[P_{n-1}(s) + \frac{s^n}{\pi} \int_{16\mu^2}^{\infty} \frac{\text{Im } C(s') ds'}{(1 + \eta(s')) s'^n (s' - s)} \right], \quad (6)$$

where $P_{n-1}(s)$ is a polynomial of degree $n - 1$, and n subtractions have been performed here. If we assume that $X(s)$ is given by the vector-dominance model and $F_\pi(s)$ decreases with increasing s , then we can confine ourselves to one subtraction²⁾, and

$$F_\pi(s) = X(s) \left[1 + \frac{s}{\pi} \int_{16\mu^2}^{\infty} \frac{\text{Im } C(s') ds'}{(1 + \eta(s')) s' (s' - s)} \right] \quad (7)$$

As already noted, the functions in the right-hand side of (3) are not independent. From (5) and (7) follows the connection

$$\frac{\text{Re } C(s)}{1 - \eta(s)} = 1 + \frac{s}{\pi} \mathcal{P} \int_{16\mu^2}^{\infty} \frac{\text{Im } C(s') ds'}{(1 + \eta(s')) s' (s' - s)}. \quad (8)$$

In analogy with (6) we can obtain a dispersion relation expressing $F_\pi(s)$ in terms of an integral of $\text{Re } C(s)/(1 - \eta(s))$ over the cut and a relation of the type (8).

To find the form factor $F_\pi(s)$ it is necessary to know D , for which we can obtain a rigorous inequality. We take into account the fact

$$\begin{aligned} \sigma_{e^+e^- \rightarrow h} &= \frac{8\pi^2 \alpha^2}{s^2(s - 4\mu^2)} \sum_{\substack{n \neq \pi^+\pi^- \\ (J=1)}} |\langle 0 | (p_+ - p_-)^\mu \mathcal{J}_\mu(0) | n \rangle|^2 \times \\ &\quad \times (2\pi)^4 \delta(p_+ + p_- - p_n), \\ 1 - \eta^2 &= \frac{\beta}{24\pi} \sum_{\substack{n \neq \pi^+\pi^- \\ (J=1)}} |\langle \pi^+\pi^- | T | n \rangle|^2 (2\pi)^4 \delta(p_+ + p_- - p_n), \end{aligned} \quad (9)$$

²⁾ We note that in the presence of experimental values of $|F_\pi(s)|$ for high energies ($\sqrt{s} > 1 \text{ GeV}$), it is necessary to use for their analysis formulas in which inelastic processes of the type (7) are taken into account, where it is necessary to parametrize $\text{Im } C(s)/(1 + \eta(s))$.

where

$$\sigma'_{e^+e^- \rightarrow h} = \sigma_{e^+e^- \rightarrow h} - \sigma_{e^+e^- \rightarrow \pi^+\pi^-}, \quad \sigma_{e^+e^- \rightarrow h}$$

and $\sigma_{e^+e^- \rightarrow h}$ is the total cross section for the production of hadrons in the annihilation of an electron-positron pair in the one-photon channel with $I = 1$. Then, using the Cauchy-Bunyakovskii inequality, we get from (1)

$$|D| \leq \sqrt{\frac{\sigma'_{e^+e^- \rightarrow h}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}}} \frac{(1 - \eta^2)}{\beta^3} \quad (10)$$

Here $\sigma_{e^+e^- \rightarrow \mu^+\mu^-} = 4\alpha^2\pi/3s$ is the asymptotic cross section of the process $e^+e^- \rightarrow \mu^+\mu^-$. The equality in (10) is obtained when the summation in D is coherent, as is the case, for example, in the vector-dominance model (extended also to the am-

plitude $\langle \pi^+\pi^- | T | n \rangle$). However, direct utilization of this model with fixed constants leads rapidly to violation of unitarity of the limit for the cross sections. On the other hand, if we use the fact that $\sigma_{e^+e^- \rightarrow h} < \text{const}/s$ as $s \rightarrow \infty$ [9], then Eq. (10) results apparently in too weak limitation on D in the asymptotic region.

In the region of relatively low energies we can probably assume approximately the equal sign in (10). Recognizing that in the region $\sqrt{s} < 1.4$ GeV the values of δ_{11} and η are known from experiment [10], it is of interest, by way of an illustration, to calculate directly $F_\pi(s)$ from formula (3) (assuming that D is real³⁾). Unfortunately, the cross section $\sigma^1_{e^+e^- \rightarrow h}$ is known so far with very low accuracy. Taking⁴⁾ $\sigma^1_{e^+e^- \rightarrow h} = 12n\delta$ ($\sqrt{s} = 1$ GeV) [12] and $\sigma^1_{e^+e^- \rightarrow h} = 30n\delta$ ($\sqrt{s} = 1.2 - 1.4$ GeV) [5] (events with soft pions are cut off), as well as η and δ_{11} from [10], we obtain $F_\pi(s)$. The results (marked by crosses) are given in the figure, which shows also the experimental data⁵⁾ from [1, 2], and where the curve represents the "tail" of the ρ resonance. In the energy region $\sqrt{s} > 1.4$ GeV there are no data for η and δ_{11} .

From (3) and (10) there follows also a rigorous inequality for $|F_\pi(s)|$ (for complex D), the derivation of which requires only the use of the unitarity condition

³⁾We wish to note here that for a theoretical analysis it is very important to know the phase of the form factor $F_\pi(s)$.

⁴⁾We need cross sections for $I = 1$. It must be borne in mind, however, that in the vector-dominance model (see, for example, [11]), and also in the case of exact $SU(3)$ symmetry, multiple production occurs mainly in the state $I = 1$.

⁵⁾It is curious that nowhere did we normalize to the Breit-Wigner curve. We note that the results for real D constitute a real upper bound for the form factor $|F_\pi(s)|$.

$$|F_{\pi}(s)|^2 \leq \frac{4}{\beta^3} \frac{\sigma'_{e^+e^- \rightarrow h}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}} \left(\frac{1+\eta}{1-\eta} \right). \quad (11)$$

It can be written also in the form

$$\sigma_{e^+e^- \rightarrow \pi^+\pi^-} \leq \sigma'_{e^+e^- \rightarrow h} \frac{1+\eta}{1-\eta}. \quad (12)$$

We determine the phase of the form factor $F_{\pi}^+(s) = |F_{\pi}(s)| \exp(i\delta_F)$; it then follows from the unitarity relation (1) that

$$|F_{\pi}| = \frac{2|D|}{|1 - \eta e^{2i(\delta_{11} - \delta_F)}|}. \quad (13)$$

Substituting the upper limit of $|D|$ in (10), we obtain the following inequality

$$\sin^2(\delta_{11} - \delta_F) \leq \frac{1-\eta^2}{4\eta} \left[\frac{4\sigma'_{e^+e^- \rightarrow h}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-} \beta^3 |F_{\pi}(s)|^2} - \frac{1-\eta}{1+\eta} \right]. \quad (14)$$

It is interesting that at $\sqrt{s} = 1.0$ GeV ($\eta = 0.95$, $|F_{\pi}(s)|^2 = 3$) we have $\sin^2(\delta_{11} - \delta_F) \leq 0.005$, i.e., $|\delta_{11} - \delta_F| \leq 4^\circ$.

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