TIME OF ESTABLISHMENT OF STATIONARY REGIME OF STIMULATED LIGHT SCATTERING

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The question of the time of establishment of a stationary regime of stimulated light scattering was considered in [1 - 3] for the case of a Lorentz dependence of the gain $g(\omega)$ (in terms of intensity, in cm^{-1}) on the frequency ω:

$$g(\omega) = g_{max} \operatorname{Im} \frac{-\Gamma}{\omega - \omega_m + i\Gamma} . \tag{1}$$

In these papers, on the basis of an explicit analytic expression for the Green's function of the corresponding system of equations (containing a Bessel function of imaginary argument), the following result was obtained for the establishment time:

$$\tau_{est} = \frac{1}{2} g_{max} z \frac{1}{\Gamma} . \tag{2}$$

In the present article we generalize formula (2) to the case of an arbitrary shape of the gain line $g(\omega)$, and propose a clear interpretation of formula (2), not connected with the concrete analytic expression for the Green's function.

To determine the establishment time, we follow the usual formulation of the problem (see [1]), in which the pump field is assumed to be monochromatic and constant in time, and consider propagation of a δ-function input pulse through the medium. The time required for the amplified response at the output to reach its maximum value will be taken to be the establishment time.

Since the problem in question is linear in the amplified field, the presence of a frequency-dependent gain leads, in accordance with the dispersion relations, to a real part of the increment to the wave vector. The total increment $\delta k(\omega)$ can be represented in the form

$$\delta k(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{g(\omega')d\omega'}{\omega - \omega' + i\epsilon} \quad ; \quad \epsilon \to +0. \tag{3}$$

In addition, the wave vector in the medium, in the absence of a pump field, is given by $k_0(\omega) = k_0(\omega_m) + (\omega - \omega_m)/v_{gr}^{(0)}$, where $v_{gr}^{(0)}$ is the group velocity of the light in the medium. The time delay in the arrival of the light pulse at a point z of the medium is given by

$$t_{delay} = \frac{z}{v} = z \frac{d \operatorname{Re} k(\omega)}{d\omega} . \tag{4}$$

In our case it is necessary to take into account both terms in $k(\omega)$, the one due to the initial medium, and the one due to the interaction with the pump field. As a result we obtain

$$t_{delay} = \frac{z}{v_{gr}^{(o)}} - z \frac{1}{2\pi} \operatorname{Re} \int_{-\infty}^{+\infty} \frac{g(\omega')\, d\omega}{(\omega - \omega' + i\epsilon)^2} . \tag{5}$$

The first term in (5) has a trivial meaning. The second term depends on the frequency ω as a parameter. Obviously, the time of establishment of the stimulated scattering is obtained by taking the value of the second term at the point $\omega = \omega_m$, where the gain $g(\omega)$ has a maximum. As a result we obtain for the establishment time the expression

$$\tau_{est} = \frac{1}{2} g_{max} z \frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \left[1 - \frac{g(\omega)}{g(\omega_m)} \right] (\omega - \omega_m)^{-2}. \qquad (6)$$

Substitution of the Lorentz line shape (1) in (6) gives the well-known result (2).

The foregoing lucid considerations can be confirmed by a calculation of the Green's function

$$G(z, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \exp\{ -i\omega t + i[k_0(\omega) + \delta k(\omega)] z \} \qquad (7)$$

with the aid of the saddle-point method, used in the asymptotic limit $g_{max} z \gg 1$.

By way of a concrete example, we present an expression for τ_{est} in the case when the gain line contour is given by

$$g(\omega) = \text{const} (\omega_1 - \omega) \exp\{ -(\omega - \omega_1)^2/\Delta^2 \} \qquad (8)$$

(such a gain-line shape corresponds to the Doppler or Gaussian spontaneous-scattering line contour, see, for example, [4]). In this case, $\omega_m = \omega_1 - (\Delta/\sqrt{2})$, and formula (6) yields

$$\tau_{est} = g_{max} z \frac{1}{\Delta} \sqrt{\frac{e}{\pi}}. \qquad (9)$$

We have thus shown that the time of establishment of stimulated scattering can be treated as the group-delay time of a pulse signal propagating in a medium in which the group velocity is altered by the reactive component ($\text{Re}\, \delta k(\omega)$) of the effective dielectric constant, due to the action of the pump field.

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TOTAL CROSS SECTIONS FOR THE PRODUCTION OF ELECTRONIC AND MUONIC PAIRS IN COLLIDING ELECTRON BEAMS

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The problem of calculating the asymptotic total cross section for the production of e^+e^- pairs in collisions of fast charged particles was first