The first term in (5) has a trivial meaning. The second term depends on the frequency ω as a parameter. Obviously, the time of establishment of the stimulated scattering is obtained by taking the value of the second term at the point $\omega = \omega_m$, where the gain $g(\omega)$ has a maximum. As a result we obtain for the establishment time the expression

$$\tau_{est} = \frac{1}{2} g_{max} z \frac{1}{\pi} \int_{-\infty}^{+\infty} d\omega \left[1 - \frac{g(\omega)}{g(\omega_m)}\right] (\omega - \omega_m)^{-2}. \qquad (6)$$

Substitution of the Lorentz line shape (1) in (6) gives the well-known result (2).

The foregoing lucid considerations can be confirmed by a calculation of the Green's function

$$G(z, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \exp\{-i\omega t + i[k_0(\omega) + \delta k(\omega)]z\} \qquad (7)$$

with the aid of the saddle-point method, used in the asymptotic limit $g_{max}z \gg 1$.

By way of a concrete example, we present an expression for τ_{est} in the case when the gain line contour is given by

$$g(\omega) = \text{const}\,(\omega_1 - \omega) \exp\{-(\omega - \omega_1)^2/\Delta^2\} \qquad (8)$$

(such a gain-line shape corresponds to the Doppler or Gaussian spontaneous-scattering line contour, see, for example, [4]). In this case, $\omega_m = \omega_1 - (\Delta/\sqrt{2})$, and formula (6) yields

$$\tau_{est} = g_{max} z \frac{1}{\Delta} \sqrt{\frac{e}{\pi}}. \qquad (9)$$

We have thus shown that the time of establishment of stimulated scattering can be treated as the group-delay time of a pulse signal propagating in a medium in which the group velocity is altered by the reactive component ($\mathrm{Re}\,\delta k(\omega)$) of the effective dielectric constant, due to the action of the pump field.

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TOTAL CROSS SECTIONS FOR THE PRODUCTION OF ELECTRONIC AND MUONIC PAIRS IN COLLIDING ELECTRON BEAMS

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The problem of calculating the asymptotic total cross section for the production of e^+e^- pairs in collisions of fast charged particles was first

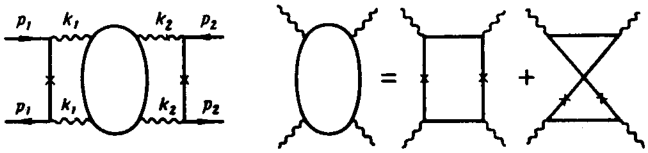


Fig. 1

considered by Landau and Lifshitz in 1934 [1]. It was shown that at ultrahigh energies, $\sqrt{s} = 2E$, the cross section increases like $(\alpha^2 r_0^2/\pi)(28/27)\ln^3(s/m_e^2)$. A recently developed mathematical formalism [2] permits a more accurate calculation of the total and differential cross sections at high energies. The derivation of more exact expressions is of interest for colliding-beam experiments. A number of workers (see [3, 4]), have recently obtained corrections $\sim\alpha^2 r_0^2 \ln^2(s/m_e^2)$ and $\alpha^2 r_0^2 \ln(s/m_e^2)$ to the total cross section. In the present paper we present the calculated contribution to the total cross section, accurate to terms $\sim\alpha^2 r_0^2$, from the diagram of Fig. 1 for e^+e^- pair production, and the contribution from all the diagrams for the case of $\mu^+\mu^-$ pair production.

The calculations were performed by a Sudakov technique [5] recently developed in connection with the search for the principal logarithmic asymptotic expressions for various processes in quantum electrodynamics [2].

We break up the region of integration with respect to the perpendicular components of the momentum transfers k_1 and k_2 (see Fig. 1a) into four regions, $|k_{1,2}^{\perp 2}| < \sigma$ and $|k_{1,2}^{\perp 2}| > \sigma$, where σ is a quantity much smaller than the square of the mass of the produced particles, and its smallness is not connected with the energy. Each of the regions $k_i^{\perp 2} > \sigma$ is broken up into two subregions, $\sqrt{\varepsilon} < \beta_i < 1$ and $\sqrt{\varepsilon} > \beta_i$, where β_i is the fraction of the energy transferred by the corresponding initial particle to the pair, and ε is a number much smaller than σ/m_e^2. The cross section calculated by us in each of these regions is of independent interest and can be separated in principle experimentally by measuring the energies and the scattering angles of the initial particles. For lack of space, we cannot present these partial cross sections here.

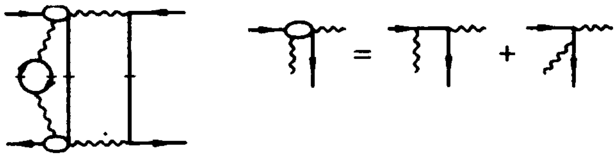
By comparing the expression obtained by summing all the partial cross sections with $k_{2\perp}^2 < \sigma$ with the expression obtained for the cross section by the Weizsacker-Williams method and the Borsellino formula [6] for the cross sections $\sigma_B(s_1)$ for pair production on an electron, we obtain the following sum rules for the Borsellino formula:

$$\int_{8m_e^2}^{\Lambda} \frac{ds_1}{s_1}\,\sigma_B(s_1) = \alpha r_0^2\left[\frac{14}{9}\ln^2\frac{\Lambda}{m_e^2} - \frac{218}{27}\ln\frac{\Lambda}{m_e^2} - 13\frac{\pi^2}{12} + \frac{418}{27}\right] + O\left(\frac{m_e^2}{\Lambda}\right), \qquad (1)$$

$$\int_{8m_e^2}^{\Lambda} \frac{ds_1}{s_1}\,\sigma_B(s_1)\ln\frac{s_1}{m_e^2} = \alpha r_0^2\left[\frac{28}{27}\ln^3\frac{\Lambda}{m_e^2} - \frac{109}{27}\ln^2\frac{\Lambda}{m_e^2} - \frac{217}{24}\zeta(3) - \right.$$

$$\left. - \frac{503}{36}\frac{\pi^2}{6} \cdot \frac{61}{6}\frac{\pi^2}{6}\ln 2 + \frac{22714}{4\cdot 243}\right] + O\left(\frac{m_e^2}{\Lambda}\right), \qquad (2)$$

where $s_1 = 2kp$, k and p are the momenta of the initial photons and of the electron, $\Lambda \gg m_e^2$, and $\xi(3) = \sum_1^\infty n^{-3}$.



a b

Fig. 2

A numerical verification of relations (1) and (2) with the aid of the Maksimov reduction formulas [7] confirms our calculations. Analogous sum rules were obtained for the case of muon pair production. By taking the diagram of Fig. 2 into account we were able to write down relations of the type (1), in which the total cross section for muon pair production is under the integral sign.

The dependence of the parameters ε and σ drops out when the contributions of all the regions are calculated. We present expressions for the contribution made to the total cross sections for e^+e^- pair production

$$\sigma_I = \frac{\alpha^4}{\pi m_e^2}\left[\frac{28}{27}\ln^3\frac{s}{m_e^2} - \frac{178}{27}\ln^2\frac{s}{m_e^2} - \left(\frac{164}{9}\frac{\pi^2}{6} - \frac{490}{27}\right)\ln\frac{s}{m_e^2} - \frac{571}{9}\xi(3) + \frac{52}{3}\frac{\pi^2}{6}\ln 2 + \frac{1045}{108}\frac{\pi^2}{6} + \frac{70515}{486}\right] \quad (3)$$

and muon pair production

$$\sigma_I = \frac{\alpha^4}{\pi m_\mu^2}\left[\frac{28}{27}\ln^3\frac{s}{m_\mu^2} - \frac{178}{27}\ln^2\frac{s}{m_\mu^2} - \left(\frac{535}{81} + \frac{14}{3}\frac{\pi^2}{6}\right)\ln\frac{s}{m_\mu^2} - \frac{28}{9}\ln^2\frac{s}{m_\mu^2}\ln\frac{m_e^2}{m_\mu^2} + \frac{14}{9}\ln\frac{s}{m_\mu^2}\ln^2\frac{m_e^2}{m_\mu^2} + \frac{562}{27}\ln\frac{s}{m_\mu^2}\ln\frac{m_e^2}{m_\mu^2} - \frac{64}{9}\ln^2\frac{m_e^2}{m_\mu^2} + \left(\frac{56}{9}\frac{\pi^2}{6} - \frac{5855}{162}\right)\ln\frac{m_e^2}{m_\mu^2} - 115\,\xi(3) - \frac{1763}{108}\frac{\pi^2}{6} + \frac{67043}{243}\right] \quad (4)$$

accurate to terms $\sim m^2/s$, m_e^2/s, and m_e^2/m^2.

For the case of electron pair production (3) we estimated, using the Weizsacker-Williams method, the contribution of the diagrams of Fig. 2 and of the diagrams that result from interference of the Bethe-Heitler mechanism (Fig. 1) and the bremsstrahlung mechanism (Fig. 2). To this end we used the results of Motz [7], who found the correction to the Borsellino formula resulting from all the other diagrams for the total pair production cross section in γ-ℓ collisions. Allowance for all other diagrams changes the coefficient of $\ln(s/m_e^2)$ in (3) by 2% and the constant in (3) by 1%. We hope to present in our next paper the results of an exact calculation of the contribution of these diagrams.

For the case of muon pair production, the total cross section is determined only by the contribution of the diagrams of Figs. 1 and 2. The result of our calculation of the contribution of the diagram of Fig. 2 coincide with the calculation of Baier and Fadin [8]. Thus, the total cross section for muon production in ee and e^+e^- collisions is

$$\sigma = \sigma_I + 2\sigma_{II},$$

$$\sigma_{II} = \frac{4\alpha^4}{\pi m_\mu^2}\left\{\left(\frac{52}{27\cdot 25}\right)\ln\frac{s}{m_\mu^2} + \frac{1}{90}\ln^2\frac{m_e^2}{m_\mu^2} + \frac{39}{450}\ln\frac{m_e^2}{m_\mu^2} + \frac{1}{45}\frac{\pi^2}{6} - \frac{3187}{250\cdot 27}\right\}. \quad (5)$$

Calculation in accordance with this formula for the energies $s \lesssim 1\ \mathrm{GeV}^2$ leads to a negative value for the total cross section, thus evidencing the need for taking into account correction terms $\sim m^2/s$ at these energies. It is therefore of interest to obtain an expression for the cross section in the region $E \sim m_\mu >> m_e$. In this energy region, the cross section contains only two "large" logarithms, $\ln(m^2/m_e^2) \sim \ln(s/m_e^2)$ [9]. The calculation of the diagram of Fig. 1 at $k_1^{\perp 2}$, $k_2^{\perp 2} < \sigma < m_\mu^2$ gives the following results for the total cross section

$$\sigma_{ee\to ee\mu\mu} \sim \left(\frac{\alpha}{\pi}\right)^2 \int_{s_{th}}^{s} \frac{ds_1}{s_1}\sigma_\gamma(s_1)\left[\ln^2\frac{\sigma}{m_e^2} I_1(\epsilon) + \ln\frac{\sigma}{m_e^2} I_2(\epsilon) + I_3(\epsilon)\right],$$

where $\varepsilon = \sqrt{s_1}/s$

$$I_1(\epsilon) = -(2+\epsilon)^2\ln\epsilon - (1-\epsilon^2)(3+\epsilon^2),$$

$$\tilde{I}_2(\epsilon) = 2(2+\epsilon)^2\ln^2\epsilon + 4(\epsilon^2-5)\ln\epsilon + 2(1-\epsilon)(\epsilon+3),$$

$$\tilde{I}_3(\epsilon) = -\frac{2}{3}\ln^3\epsilon - 8(1+\epsilon)\ln^2\epsilon + (16\epsilon^2 + 60\epsilon + 8)\ln\epsilon + (1-\epsilon)(34\epsilon + 50)$$

$\sigma_\gamma(s_1)$ is the cross section for the production of a muon pair by two photons [10]. Formula (6) is valid also for the case of hadron production. To estimate the order of magnitude of the cross section, we can put in (6) $\sigma \sim m^2$ (or E), and then the first term in (6) coincides with the result of Brodsky et al. [9].

In conclusion we note that the radiative corrections to the total cross section for pair production turn out to be of the order of $(\alpha^4/m^2\pi)\ln^3(s/m_e^2)$, which is comparable with the contribution of the constant term in (3) or (4) for $\ln(s/m_e^2) \sim 20$.

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