from the thermal conductivity for the given sample, $\tau_p^R(T) \sim T^{0.4}$ in the interval $1.3 \le T \le 2.3$°K).

It is interesting to note that since the contribution of the phonon-phonon carrier dragging becomes noticeable only in sufficiently bulky samples, a simultaneous investigation of the dependence of the thermoelectric power and of the thermal conductivity on the sample dimensions makes it possible to determine the value of $\tau_p^N(e)$ from the appearance of the size effect in the thermoelectric power. Phonon-phonon dragging can lead to an appreciable growth of the thermoelectric power at low temperatures; bismuth offers advantages in this respect over semiconductors, since even in the purest of the latter it has been practically impossible so far to observe the exponential variation in the thermal conductivity at low temperatures, owing to the strong isotopic scattering.

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SINGULARITIES OF THE DIAGRAM OF STATE OF AN He^3-He^4 MIXTURE DUE TO THE HYDROSTATIC EFFECT

A.T. Berestov, A.V. Voronel', and M.Sh. Giterman
All-union Research Institute for Physico-technical and Radio Measurements
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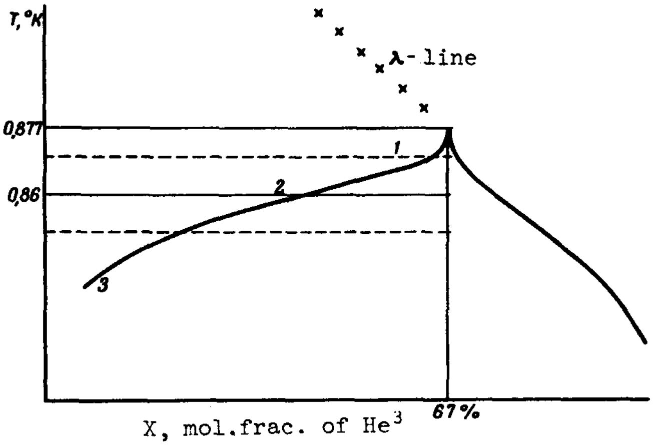
The diagram of state of the He^3-He^4 mixture has a region of separation into two liquid phases, the phase with the lower He^3 concentration remaining superfluid [1 - 4]. The coexistence curve of the liquid phases, when plotted in coordinates T and X, has according to the latest experiments [4] a very unique form (see the figure), which has induced a number of workers to construct specific theories of this phenomenon [5, 6].

We shall show that the form of the coexistence curve of this mixture, under the conditions of the experiment [4], is determined by the hydrostatic effect, i.e., by the concentration inhomogeneity due to the gravitational field [7, 8], and remains therefore a function of the experimental conditions. This explains the discrepancies between the data of different authors [1 - 4] and the resultant polemics.

Let us verify first that the parameters determining the interval of action of the hydrostatic effect near the critical point of the He^3-He^4 mixture are considerably larger than in other mixtures. In [7, 8], such parameters were calculated on the basis of the classical theory of critical phenomena, but their calculation in terms of the scaling indices has practically no effect on the result.

As follows from [8], the influence of the hydrostatic effect becomes manifest in the vicinity of the critical point $(T - T_c)/T_c < t_0 \sim (mgH/RT_c)^{2/3}$, or, at $T = T_c$, in the vicinity of the critical concentration $(X - X_c)/X_c < x_0 \sim (mgH/IT_c)^{1/3}$. For ordinary mixtures, $T_c \sim 10^2$ °K and t_0 and x_0 are sufficiently small. For the He^3-He^4 mixture, where $T_c \simeq 0.86$°K, practically the entire measurement region lies in the zone of the hydrostatic effect.



In the case when the scaling theory is applicable [9], the equation of state of the mixture in the presence of a gravitational field takes the form

$$\Delta\mu + mgH \equiv x^{\delta} f(z) + mgH = 0, \qquad (1)$$

where μ is the chemical potential of the mixture, $z = t/x^{1/\beta}$, $t = (T - T_c)/T_c$, $x = (X - X_c)/X_c$ and the height H is measured from the critical-concentration level. Asymptotic forms of the function $f(z)$ are well known [9].

As $z \to \infty$, i.e., for $T > T_c$ near the critical concentration, we have

$$x \sim \frac{h}{t^{\gamma}}, \qquad t > t_0 \sim h^{1/\beta\delta} (h = mgH/RT_c), \qquad (2)$$

which corresponds to region 1 in the figure.

Near the coexistence curve, i.e., for $T < T_c$ at $|t| > t_0$, we have $x \sim t^{\beta}$, which gives a continuation of the coexistence curve into region 3 of the figure.

Finally, in an immediate vicinity of the critical point on either side ($|t| < t_0$), where the expansion of the function $f(z)$ is of the form $f(z) = f_0 + f_1 z + \ldots$, we obtain from (1)

$$x = \pm\left(\frac{|h|}{f_0}\right)^{1/\delta} \left[1 - \frac{f_1 t}{\delta f_0 (|h|/f_0)^{1/\beta\delta}}\right], \qquad (3)$$

which corresponds to region 2 in the figure.

For comparison with experiment, it is convenient to introduce three dimensionless parameters: the width Δ_x of the two-phase region at $T = T_c$, the slope dx/dt of the linear section in region 2, and the limit t_0 of this section. The obvious relation between these quantities, $(dx/dt)t_0 = \Delta_x$, is the condition for choosing the critical parameters of the mixture. The figure shows that the coexistence curve is asymmetrical, which apparently is a fact common to critical phenomena, and in the case of the He^3-He^4 mixture this asymmetry becomes aggravated by the fact that the left-hand branch of the coexistence curve is superfluid, i.e., the temperature gradients that are produced in it relax much more rapidly than in the nonsuperfluid phase[1].

Bearing this in mind, we shall carry out a comparison with the experiment of [4] for the left-hand branch of the coexistence curve, from which it follows that $\Delta_x = (2.1 \pm 0.2) \times 10^{-2}$, $t_0 = (0.9 \pm 0.1) \times 10^{-2}$ and $dx/dt = 2.4 \pm 0.2$. Then $X_c = 67\%$ and $T_c = 0.86°K$. Contrary to the customary assumption, the critical temperature does not lie at the vertex of the curve on the figure.

Region 1, which is adjacent to the vertex, is "far" from the critical point ($t > t_0$), and the coexistence curve is described in this region by formula (2). The general character of the experimental curve is represented correctly by this formula, but there are not enough experimental data to determine the critical index γ.

We note that the influence of the hydrostatic effect on the coexistence curve is more strongly pronounced in the experiments of [4] than in other investigations, owing to the construction of the setup, namely, the concentration of the mixture was determined independently at two fixed levels spaced ∿4 cm apart. In other cases [1 - 3], the concentration was determined as before from the height of the vessel. The characteristics of the parameters Δ_x, t_0, and $(dx/dt)^{-1}$ are decreased thereby at least several times, and the experimental coexistence curves turn out to be smoother and more similar to the coexistence curves of other mixtures.

From Fig. 3 of [4] it is possible to assess the extent to which the temperature at which the meniscus is at the level of the resistance pickup ($H_1 \simeq 0.8$ cm) differs from the temperature corresponding to the location of the meniscus at the capacitance pickup ($H_2 = 2 \pm 0.5$ cm). For the experimental conditions $x = 2.4 \times 10^{-2}$ and $(H_2 - H_1)/H_m = 0.3 \pm 0.1$ given in [4], it turned out that $\Delta t = (1 - 1.2) \times 10^{-2}$. The law of motion of the meniscus with changing temperature was investigated in [10], according to which

$$\frac{H_2 - H_1}{H_m} = \frac{\bar{x}\,\Delta t}{2(\Delta_x)^2}\,\frac{2-\alpha}{1-\alpha}\,\frac{dx}{dt}\left[1 + \frac{2-\alpha}{1-\alpha}\,\frac{\Delta t}{\Delta_x}\,\frac{dx}{dt}\right]^{-1}, \tag{4}$$

where H_m is the height of the vessel, $\bar{x}$ the average He^3 concentration in the vessel, and α an index determining the singularity of the specific heat at the critical point. Substituting in (4) the values of the parameters $\bar{x}$, dx/dt and Δ_x, and putting $\alpha \simeq 0$, we obtain good agreement between formula (4) and experiment. This serves as an independent verification of the influence of the hydrostatic effect on the properties of the He^3-He^4 mixture.

There is one more possibility of estimating the influence of the hydrostatic effect, by measuring the discontinuity of the specific heat ΔC_{PX} on crossing the coexistence curve. Calculation shows [11] that in the presence of the hydrostatic effect ΔC_{PX} decreases on approaching the critical point, and should vanish in the concentration interval $|X - X_c| < \Delta_x X_c$, which equals

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[1] It should be noted that even a stable gradient ∿10^{-5} deg/cm may suffice to produce a noticeable asymmetry.

$1.1 \times 10^{-2} H_m^{1/3}$ for the He^3-He^4 mixture. The existing experimental data, given in [2] only in the form of a small-scale plot, apparently do not contradict the predictions of the theory, but do not suffice for quantitative conclusions.

Let us mention in conclusion a certain specific feature of the He^3-He^4 mixture. The smallness of the critical temperature ($T_c \simeq 0.86°K$), which is the lowest of all the investigaged critical points, is connected with the smallness of the characteristic energy parameter $\varepsilon_{11} + \varepsilon_{22} - 2\varepsilon_{12}$, where ε_{11} and ε_{22} are the interaction energies of like atoms, while ε_{12} are the energies for unlike atoms. Critical points of just this type were given in [13] as examples of a superposition of singularities at $T = 0$ and $T = T_c$ within the framework of two-dimensional model theories. One of the conclusions is the possible reduction of the region of applicability of the scaling relations to an exponentially narrow region. It is possible that the situation is the same in the He^3- He^4 mixture and the critical indices may turn out to be unusual.

We have not considered here the λ curve of the He^3-He^4 mixture, which starts in pure He^4 and terminates on the coexistence curve of the He^3-He^4 mixture, possibly at the critical point. The self-consistent-field theory does not hold in the immediate vicinity of the λ point, which is connected with the transition temperature at least in pure He^4, by the relation [14, 15] $(T - T_c)/T_c < t_1 \sim T_c^3$. If it is assumed that a similar conclusion holds true for the entire λ curve, then at the end of this curve the region of applicability of the theory of the self-consistent field is larger by approximately 1.5 orders than in pure He^4. Thus, it is quite probable that it is precisely the He^3-He^4 mixture which has a sufficiently wide range of applicability for the self-consistent field theory.

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