in the region $s < s_1 \simeq 200\ (\mathrm{GeV}/c)^2$ and ceases to narrow down at higher energies [6]).

We substitute the partial-wave projection of the quasipotential (4, 5) in (3) and continue the asymptotic expression for the partial amplitudes into the region of low energies. The figure shows the behavior of the lower partial wave (the parameters are taken from [5]).

We present also the partial amplitudes in the case when the pomeranchon is a standing pole. In our case this corresponds to continuation into the region of low energies from the asymptotic region $s > s_1$.

It is seen from the figure that the model under consideration, while ensuring narrowing of the cone at $s < s_1$, does not lead, in contrast to [7], to the appearance of loops on the Argand diagrams for pp scattering.

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ELECTROMAGNETIC POLARIZABILITY OF THE PION

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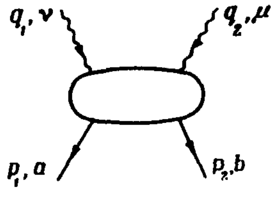
The possibility of experimentally measuring the electro-magnetic polarizability of the pion by determining the level shift in π-mesic atoms was pointed out recently in a number of papers [1, 2]. In the present paper we obtain the theoretical value of the polarizability within the framework of the low-energy π-meson technique.

The electromagnetic polarizability k_e determines the dipole moment $\vec{d} = 2k_e\vec{E}$ of the meson in an external electric field, and leads to an interaction energy in the form

$$H_{int} = -k_e E^2. \tag{1}$$

To calculate k_e, we consider the Compton effect on the pion at low energies. The notation for the momenta is shown in the figure (ν and μ are the polarization indices of the photons and a and b are the isotopic indices of the mesons).



We assume, as is customary in the low-energy technique, that it is possible to expand in powers of the momentum in the non-pole terms of the amplitude of the process in the figure. It follows then from the gauge invariance (the Ward identity) that, accurate to second order in the momenta, the amplitudes of the Compton effect for real quanta is given by

$$T^{ab}_{\nu\mu}(p_1, p_2;\ q_1, q_2) = (T^{ab}_{\nu\mu})_{\mathrm{pol}} + (T^{ab}_{\nu\mu})_{\mathrm{cont}}, \tag{2}$$

$$(T^{ab}_{\nu\mu})_{\mathrm{pol}} = 2e^2(\delta_{ab} - \delta_{3a}\delta_{3b})\left[\delta_{\nu\mu} + \frac{2p_{1\nu}p_{2\mu}}{(p_1 - q_1)^2 - \mu^2} + \frac{2p_{1\mu}p_{2\nu}}{(p_1 - q_2)^2 - \mu^2}\right], \tag{3'}$$

$$(T^{ab}_{\nu\mu})_{\mathrm{cont}} = 2e^2[\beta(\delta_{ab} - \delta_{3a}\delta_{3b}) + \beta_0\delta_{3a}\delta_{3b}](q_1q_2\delta_{\nu\mu} - q_{1\mu}q_{2\nu}) \tag{3"}$$

$$e^2 = 4\pi\alpha, \quad p_i^2 = \mu^2, \quad q_i^2 = 0.$$

The amplitude $T_{\nu\mu}$ corresponds to an effective Lagrangian

$$L = -ieA_\mu\left(\frac{\partial\phi^+}{\partial x_\mu}\phi - \frac{\partial\phi}{\partial x_\mu}\phi^*\right) + 2e^2A_\mu^2\phi^*\phi - \frac{e^2}{2}(\beta\phi^*\phi + \frac{\beta_0}{2}\phi_0^2)F_{\nu\mu}^2, \tag{4}$$

where ϕ and ϕ_0 are the fields of the π^- and π_0 meson. From a comparison of (1) and (4) we get

$$k_e = e^2\beta/2\mu. \tag{5}$$

The constant β_0 determines the electric polarizability of the π^0 meson: $k_e^0 = e^2\beta_0/2\mu$. Equation (4) contains also the contribution of the magnetic polarizability $k_h = -k_e$.

To calculate β and β_0 we consider $T^{ab}_{\nu\mu}$ as $p_1 \to 0$. The use of current algebra makes it possible to obtain in standard fashion (see [3])

$$T^{ab}_{\nu\mu}(p_2, 0;\ q_2, q_1) = -\frac{e^2\epsilon_{3ac}}{F_\pi}\left[\tau^{bc}_{\nu\mu}(p_2, q_2) + \tau^{bc}_{\mu\nu}(p_2, q_1)\right], \tag{6}$$

where

$$\tau^{bc}_{\nu\mu}(p, q) = i\int dx\, e^{-iqx} < \pi^b(p)\,|(a_\nu^c(0)\,v_\mu^3(x))_+|\,0 > \tag{7}$$

$F_\pi = 0.83\ \mu/\sqrt{2}$, and μ is the pion mass.

Here a_ν^b and v_ν^b are the axial and vector currents (in the quark model, $a_\nu^b = (1/2)\bar\psi\tau^b\gamma_\nu\gamma_5\psi$ and $v_\nu^b = (1/2)\bar\psi\tau^b\gamma_\nu\psi$).

A phenomenological expansion of $\tau_{\nu\mu}(p, q)$, using conservation of the vector current and partial conservation of the axial current, is

$$\tau^{bc}_{\nu\mu}(p, q) = -\epsilon_{3bc}\left[F_\pi\left(\delta_{\mu\nu} - \frac{q_\nu p_\mu}{pq} + \frac{p_\nu p_\mu}{pq}\right) + h_A(pq\delta_{\mu\nu} - q_\nu p_\mu)\right]. \tag{8}$$

Taking (8) and (6) into account and comparing with (3') and (3") as $p_1 \to 0$, we obtain:

$$\beta = h_A/F_\pi, \quad \beta_0 = 0 \tag{9}$$

whence, in particular, it follows that the polarizability of the neutral pion is equal to zero.

The quantity $\tau_{\nu\mu}$ determines the contribution of the axial current in the $\pi \to e\nu\gamma$ decay. The amplitude of this decay is

$$T_{\nu}(\pi(p) \to e + \nu + \gamma(q)) = i\,eG\cos\theta\,(M_{\nu\mu}\ell_{\mu} + F_{\pi}\bar{\nu}_e\gamma_{\nu}(\hat{k} + \hat{q} - m)^{-1}\hat{p} \times \times (1 - \gamma_5)u_{\nu})\ldots, \tag{10}$$

where ℓ_{μ} is the lepton current, and m the momentum and mass of the electron,

$$M_{\nu\mu} = + ih_V\epsilon_{\nu\mu\alpha\beta}p_{\alpha}q_{\beta} - h_A(pq\delta_{\mu\nu} - p_{\nu}q_{\mu}) - F_{\pi}\left(\delta_{\mu\nu} - \frac{q_{\mu}p_{\nu}}{pq} + \frac{p_{\mu}p_{\nu}}{pq}\right)\ldots, \tag{10'}$$

$$h_V = f/2e^2. \tag{10''}$$

The constant f in (10") is connected with lifetime of the π^0 meson (see [4]), $\tau(\pi^0) = 64\pi/f^2\mu^3$, and the constant h_A is expressed as follows from (5) and (9), in the terms of the polarizability of the π^- meson.

From the data on the $\pi \to e\nu\gamma$ decay probability we know two solutions for the ratio h_A/h_V [5]:

$$\gamma \equiv h_A/h_V = 0.4 \quad \text{or} \quad -2.1^{1)}. \tag{11}$$

The constant h_A can be expressed in terms of the spectral functions of the vector and axial currents (see [6]):

$$h_A = \frac{1}{F_{\pi}}\left\{\frac{\langle r^2\rangle}{3} + \frac{1}{F_{\pi}^2}\int\frac{\rho^A(\kappa^2) - \rho^V(\kappa^2)}{\kappa^4}\,d\kappa^2\right\}, \tag{12}$$

where $\langle r^2\rangle^{1/2}$ is the pion radius. (We note that $F_{\pi} = F'_{\pi}/\sqrt{2}$ and $\rho^{AV} = \rho'_{A,V}/2$, where F'_{π} and $\rho'_{A,V}$ are respectively the constant of the $\pi \to e\nu$ decay and the spectral functions used in [6].) The value 0.4 in (11) is preferable if vector dominance, which yields $\gamma \simeq 0.6$, holds in the spectral integrals for vector and axial currents.

Taking (5), (9), and (11) into account we obtain the final result

$$k_e = e^2h_A/2\mu F_{\pi} = f\gamma/4\mu F_{\pi}. \tag{13}$$

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1) It should be noted that γ was calculated in [5] by using the value $\tau(\pi^0) \simeq 10^{-16}$ sec (this corresponds to $\Gamma \simeq 6.6$ eV and $f \simeq 0.45\alpha/\mu$). If we assume, following the data of [7], that $\tau(\pi^0) \simeq 0.56 \times 10^{-16}$ sec (this corresponds to $\Gamma \simeq 11.8$ eV and $f \simeq 0.6\alpha/\mu$), then the two solutions for γ will be respectively 0.035 and -1.8. Then $k_e \simeq 10^{-2}\alpha/\mu^3$ or $0.45\alpha/\mu^3$. It is thus desirable to have independent information on h_A, which can be obtained by polarization measurements of the $\pi \to e\gamma\nu$ decay. It is desirable to have also more reliable information on the quantity $\tau(\pi^0)$.

The numerical value of k_e turns out to be $\sim 0.1\alpha/\mu^3$ if we use $\gamma = 0.4$ in (11). This value of the polarizability leads to an energy shift ~ 2 eV in the 6h - 5g transition in Tl^{81}. The measurement of such effects calls for a relative accuracy $\sim 10^{-5}$ in the determination of the transition energy, which apparently is feasible (see [8, 9]). An accuracy $\sim 6 \times 10^{-5}$ has already been attained for the same mesic atom Tl^{81} (see [9]).

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ERRATA

In the article by S. N. Bagaev et al., Vol. 15, No. 2, p. 64, references [9] and [10] should be interchanged.