dT_m/dp = 29 atm/deg at the triple point). If $dT_\lambda/dp \sim -100$ atm/deg for H_2, just as for He, then, in view of the approximate character of the estimates, we can hope to raise T_λ to 6 - 8°, and T_m also to 6 - 8°. In other words, attainment of a λ transition in liquid H_2 is not excluded (particularly if account is taken of the decrease of the derivative dT_m/dp with decreasing temperature). On the basis of calculations similar to those in [6] we can hope to obtain a more accurate lower limit for T_m in pure H_2, but we cannot dispense with experiments in any case, particularly when account is taken of the possible influence of He impurities, vacancies, etc.

The results of [7] point to the possibility of obtaining non-dense homogeneous He films on sufficiently smooth surfaces. It is of interest in this connection to ascertain the possibility of obtaining analogous H_2 films.[3]) This way, if the density of H_2 can be made noticeably smaller than the density of ordinary liquid hydrogen, there are grounds for expecting the appearance of a λ transition, and perhaps also superfluidity of the quasi-two-dimensional type. One can hardly doubt that the problems touched upon here are worthy of study.

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EXCITATION OF A REGULAR PLASMA WAVE BY A MODULATED BEAM WITH HIGH ENERGY DENSITY

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Submitted 27 January 1972
ZhETF Pis. Red. 15, No. 6, 346 - 349 (20 March 1972)

As shown in [1, 2], the excitation of one-dimensional plasma oscillations by a monoenergetic relativistic electron beam is characterized by the fact that an appreciable fraction of the beam energy is transformed into the energy of the oscillation field. This produces in the plasma large field intensities, so that an important role may be assumed by the effect of variation of the waveguide properties of the plasma [3] as a result of the dependence of the electron density on the electric field amplitude [4]. We report here investigations of the interaction of a relativistic beam with a nonlinear plasma, which point to the possibility of synchronism between the beam and the wave during the nonlinear stage of instability development if the beam and plasma parameters are so chosen that the beam velocity and the phase velocity of the wave decrease with time in accordance with an identical law. In this case the energy transferred from the beam to the field greatly exceeds the value obtained in [1, 2].

[3])Interest attaches to both films and macroscopic volumes of H_2, D_2, T_2, HD, HT, and DT, for this would reveal the influence of both the mass and of the statistics. This pertains, of course, also to the problem of hydrogen metallization.

Let a sequence of electron bunches spaced a distance ℓ apart move through a plasma with initial velocity v_0. Assuming that the dimensions of each bunch are small compared with the distance between them, $\delta\ell \ll \ell$, and replacing the bunches with charged layers having a surface charge density σ, we obtain the following system of equations describing the beam-plasma interaction:

$$\frac{\partial}{\partial x}\left(\frac{\partial^2 E}{\partial t^2} + \omega_p^2 E\right) = -4\pi\sigma e \frac{\partial^2}{\partial t^2}\sum_{s=-\infty}^{\infty} \delta[x - s\ell - x_s(t)], \tag{1}$$

$$\frac{d}{dt}\gamma_s \dot{x}_s = -\frac{e}{m}\mathrm{Re}\,E[t, x_s(t)], \quad \gamma_s = \left(1 - \frac{\dot{x}_s^2}{c^2}\right)^{-\frac{1}{2}}.$$

Here E is the self-consistent electric field and $x_s(t)$ the coordinate of each layer; the plasma frequency and field amplitude are connected by the relation [4]:

$$\omega_p^2(E) = \omega_0^2 \exp\left(-\frac{|E|^2}{E_0^2}\right), \quad E_0^2 = \frac{8m\omega_0^2 T}{e^2}, \tag{2}$$

where T is the plasma temperature in energy units and $\omega_0 \equiv \omega_p(0)$.

Taking the spatial periodicity of the system into account, we seek a solution of (1) in the form

$$E(t, x) = E(t)\exp[i\theta(t) + i(\omega_0 t - kx)] \quad , \tag{3}$$

where $k = 2\pi/\ell = \omega_0/v_0$, and E(t) and θ(t) are slowly varying functions: $\dot{E} \ll \omega_0 E$ and $\dot{\theta} \ll \omega_0$.

Substituting (3) in (1) and averaging the first equation over the spatial period, we obtain

$$\frac{d}{dt}\gamma\dot{x} = -\frac{e}{m}E\cos(\psi - \theta),$$

$$\dot{E} = 2\pi e n_1 v_0 \cos(\psi - \theta), \tag{4}$$

$$E\dot{\theta} = -\frac{\omega_0}{2}\left[1 - \exp\left(-\frac{E^2}{E_0^2}\right)\right]E + 2\pi e n_1 v_0 \sin(\psi - \theta),$$

where $\psi = -\omega_0(t - x/v_0)$ and $n_1 = \sigma/\ell$.

From the first and second equations of (4) we get the law of momentum conservation in the plasma-beam system:

$$m\gamma\dot{x} + \frac{E^2}{4\pi n_1 v_0} = m\gamma_0 v_0 \ , \tag{5}$$

which makes it possible to introduce the variable $\eta = \theta - \psi$ and to lower the order of the system (4):

$$\dot{w} = \frac{\omega_1}{2}\cos\eta \ ,$$

$$\dot{\eta} = -\frac{\omega_1}{2w}\sin\eta - \frac{\omega_0}{2}\left[1 - \exp\left(-\frac{w^2}{w_0^2}\right)\right] + \omega_0\left[1 - \frac{1 - w^2}{(1 - 2\beta_0^2 w^2 + \beta_0^2 w^4)^{1/2}}\right]. \tag{6}$$

We have introduced here the notation:

$$w^2 = \frac{E^2}{4\pi n_1 m v_o^2 \gamma_o}, \quad w_o^2 = \frac{E_o^2}{4\pi n_1 m v_o^2 \gamma_o}, \quad \omega_1^2 = \frac{4\pi e^2 n_1}{m}, \quad \beta_o = \frac{v_o}{c}.$$

Integrating the equations in (6) with initial conditions $\eta(0) = w(0) = 0$, we find the dependence of the phase of the beam on the field amplitude w:

$$\left(4\frac{n_1}{n_o \gamma_o}\right)^{1/2} w \sin\eta = w_o^2\left[1 - \exp\left(-\frac{w^2}{w_o^2}\right)\right] + w^2 + \frac{2}{\beta_o^2}\left(1 - \beta_o^2 w^2 + \beta_o^2 w^4\right)^{1/2} - \frac{2}{\beta_o^2}. \tag{7}$$

Expressing $\cos\eta$ in terms of w from relation (7) and substituting in the first equation of (6), we arrive at a first-order nonlinear equation for the field amplitude, according to which the function w(t) varies periodically with time. To find the analytic solutions we simplify (7), expanding the first part in powers of w and retaining the higher-order terms of the expansion:

$$\left(4\frac{n_1}{n_o \gamma_o}\right)^{1/2} \sin\eta = \left(\frac{1}{\gamma_o^2} - \frac{1}{2w_o^2}\right)w^3 + \left(\frac{1}{6w_o^4} + \frac{\beta_o^2}{\gamma_o^2}\right)w^5. \tag{8}$$

Let us consider the most interesting limiting cases. For a beam whose energy density is small compared with the density of the plasma thermal energy ($w_0^2 \gg \gamma_0^2$) we have [2]

$$\sin\eta = \frac{w^3}{w_1^3}, \quad w_1 = \left(4\frac{n_1}{n_o}\gamma_o^3\right)^{1/6}. \tag{9}$$

Substituting (9) in the first equation of (6) we arrive at the formulas

$$\frac{\dot{w}}{w_1} = \frac{1}{\tau_1}\left[1 - \left(\frac{w}{w_1}\right)^6\right]^{1/2}, \quad \tau_1 = 2^{4/3}\left(\frac{n_o}{n_1}\gamma_o^3\right)^{1/3}\omega_o^{-1}, \tag{10}$$

according to which the maximum field amplitude reaches the value w_1 within a time on the order of τ_1.

If the nonlinear resonance conditions $\gamma_0^2 = 2w_0^2$ is satisfied, then the equation for the function w(t) takes the form

$$\frac{\dot{w}}{w_2} = \frac{1}{\tau_2}\left[1 - \left(\frac{w}{w_2}\right)^{10}\right]^{1/2}, \quad w_2 = \left(4\frac{n_1}{n_o}\gamma_o^3\right)^{1/10}, \quad \tau_2 = 2^{6/5}\gamma_o^{4/5}\left(\frac{n_o}{n_1}\right)^{2/5}\omega_o^{-1} \tag{11}$$

(it is assumed that $\gamma_0 \gg 1$ and $\beta_0 = 1$).

We find analogously the maximum field amplitude w_3 in the case when the beam energy density greatly exceeds the plasma energy density:

$$w_3 = (2w_o)^{2/3}(n_1/n_o\gamma_o)^{1/6}. \tag{12}$$

In conclusion, we present the foregoing results in terms of dimensional variables:

$$\frac{E_{max}^2}{4\pi} = \begin{cases} \left(4\frac{n_1}{n_o}\gamma_o^3\right)^{1/3} n_1 m v_o^2 \gamma_o, & n_1 m v_o^2 \gamma_o^3 << n_o T \\ \left(4\frac{n_1}{n_o}\gamma_o^3\right)^{1/5} n_1 m c^2 \gamma_o, & n_1 m c^2 \gamma_o^3 = 16 n_o T \\ (2 w_o)^{4/3}\left(\frac{n_1}{n_o \gamma_o}\right)^{1/3} n_1 m v_o^2 \gamma_o, & n_1 m c^2 \gamma_o^3 >> n_o T. \end{cases} \quad (13)$$

It follows from (13) that the field energy density increases when the beam energy density approaches resonance. After going through resonance, the energy transferred by the beam to the field decreases.

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VISCOUS MOTION OF VORTICES IN TYPE-II SUPERCONDUCTORS

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Submitted 27 January 1972
ZhETF Pis. Red. 15, No. 6, 349 - 353 (20 March 1972)

1. Gor'kov and Kopnin [1] were the first to develop a consistent approach to the problem of viscous vortex motion in type-II superconductors at $T = T_c$, using the Ginzburg-Landau equations generalized to include the nonstationary case [2, 3]. This enabled Gor'kov and Kopnin to calculate that part of the friction coefficient which is connected with the finite time of relaxation of the ordering parameter to the equilibrium value in superconductors with paramagnetic impurities.

No account was taken in [1], however, of the contribution made to the energy dissipation, and consequently to the friction coefficient, by the flow of the normal current component through the core of the vortex. The order of magnitude of such a contribution was estimated by Bardeen and Stephen.

The purpose of the present work was to determine this part of the friction coefficient η within the framework of the model developed in [1], and also to discuss the question of the value of η in a superconductor without paramagnetic impurities.

We consider first the case of a superconductor with a large concentration of paramagnetic impurities ($\tau_s^{-1} >> T_c, \Delta_\infty$)[1]. In this case the equations can be reduced to the simple form written out in [1] even if the volume density of the electric charge λ differs from zero. In fact, if we introduce a gauge invariant potential μ defined by

$$\mu = \dot{\theta} + 2e\psi, \quad (1)$$

[1]) Unless otherwise stipulated, the notation is the same as in [1].