$$\frac{E^2_{max}}{4\pi} = \begin{cases} \left(4\frac{n_1}{n_o}\gamma_o^3\right)^{1/3} n_1 m v_o^2 \gamma_o, & n_1 m v_o^2 \gamma_o^3 << n_o T \\ \left(4\frac{n_1}{n_o}\gamma_o^3\right)^{1/5} n_1 m c^2 \gamma_o, & n_1 m c^2 \gamma_o^3 = 16 n_o T \\ (2w_o)^{4/3}\left(\frac{n_1}{n_o \gamma_o}\right)^{1/3} n_1 m v_o^2 \gamma_o, & n_1 m c^2 \gamma_o^3 >> n_o T. \end{cases} \quad (13)$$

It follows from (13) that the field energy density increases when the beam energy density approaches resonance. After going through resonance, the energy transferred by the beam to the field decreases.

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VISCOUS MOTION OF VORTICES IN TYPE-II SUPERCONDUCTORS

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1. Gor'kov and Kopnin [1] were the first to develop a consistent approach to the problem of viscous vortex motion in type-II superconductors at $T = T_c$, using the Ginzburg-Landau equations generalized to include the nonstationary case [2, 3]. This enabled Gor'kov and Kopnin to calculate that part of the friction coefficient which is connected with the finite time of relaxation of the ordering parameter to the equilibrium value in superconductors with paramagnetic impurities.

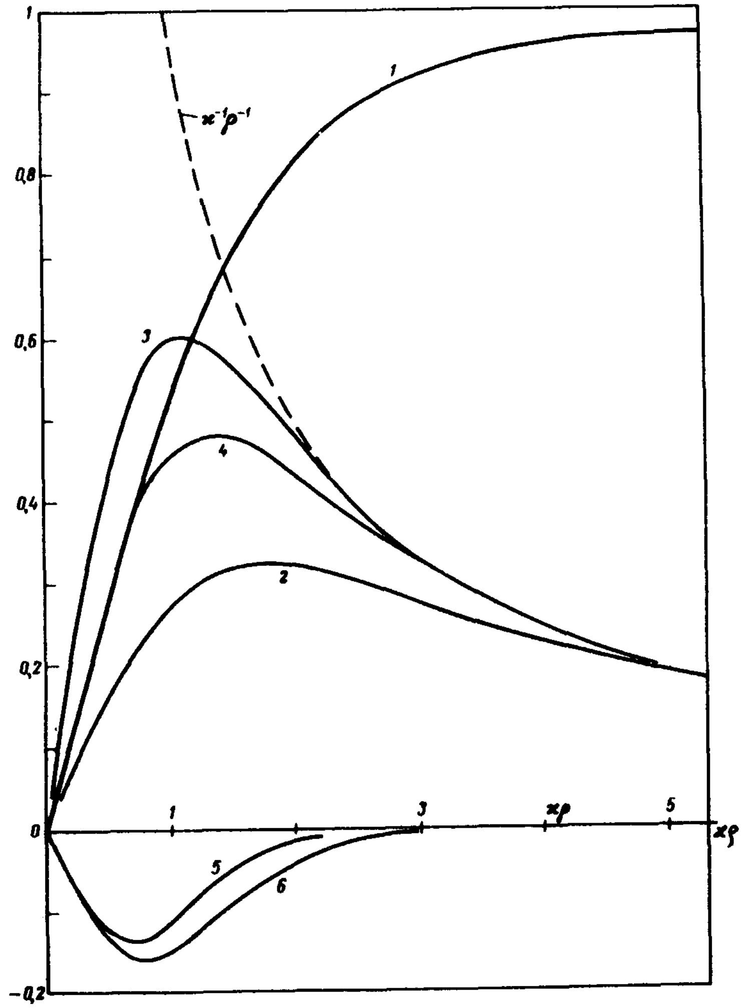
No account was taken in [1], however, of the contribution made to the energy dissipation, and consequently to the friction coefficient, by the flow of the normal current component through the core of the vortex. The order of magnitude of such a contribution was estimated by Bardeen and Stephen.

The purpose of the present work was to determine this part of the friction coefficient η within the framework of the model developed in [1], and also to discuss the question of the value of η in a superconductor without paramagnetic impurities.

We consider first the case of a superconductor with a large concentration of paramagnetic impurities ($\tau_s^{-1} >> T_c, \Delta_\infty$)[1]. In this case the equations can be reduced to the simple form written out in [1] even if the volume density of the electric charge λ differs from zero. In fact, if we introduce a gauge invariant potential μ defined by

$$\mu = \dot{\theta} + 2e\psi, \quad (1)$$

[1] Unless otherwise stipulated, the notation is the same as in [1].



Radial distributions of the following quantities: 1) modulus of the ordering parameter f, 2) current density $j = -f^2 Q_0$, 3 and 4) amplitudes of the potential $i\mu/\dot{u} + r^{-1}$ for $\tau_s^{-1} \gg T_c$ and $\tau_s^{-1} \ll T_c$, respectively, and 6) amplitudes of the charge $i\mu f^2/\dot{u}$ for $\tau_s^{-1} \gg T_c$ and $\tau_s^{-1} \ll T_c$, respectively.

where ψ is the anomalous term introduced in [2], then the system of equations (1) - (3) of [1] for the quantities j, Q, Δ, and μ turns out to be valid and closed. On the other hand, Eq. (19') of [2] only determines the (very small) deviation of the scalar potential from ψ for a known λ. Further, the right-hand side of Eq. (4) of [1] contains at $\lambda \neq 0$ a term $\dot{\lambda}$, but this term, like the term div $\dot{Q}$, is proportional to the square of the velocity $\dot{u}$ of the vortex and can be neglected in the first approximation in $\dot{u}$.

Thus, Eqs. (1) - (4) of [1] are perfectly adequate, but it does not follow from them that the potential μ vanishes identically, and with it the volume charge density $\lambda \sim \Delta^2\mu$. Indeed, the equation for μ

$$12\Delta^2\mu - \frac{1}{\kappa^2}\nabla^2\mu = 0 \tag{2}$$

should be solved with the boundary condition $(\kappa^{-1}\nabla\mu + \dot{Q}_0) \to$ const as $\rho \to 0$, as is required if the density of the normal current component is to be finite as $\rho \to 0$. This yields for the amplitude of μ ($\mu(\rho, \phi) = \mu(\rho)e^{i\phi}$) the equation

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\mu}{d\rho}\right) - \frac{1}{\rho^2}\mu - 12\kappa^2 f^2\mu = 0 \tag{3}$$

with the boundary condition $\mu(0) \to i\dot{u}\kappa^{-1}\rho^{-1}$.

The figure shows the numerically calculated amplitude of the potential and charge density as functions of the distance from the center of the moving vortex. The vortex motion thus produces on the "boundary" of the core (at a distance $\sim\xi$) charges with a dipole moment proportional to the vortex velocity $\dot{u}$ and directed perpendicular to this velocity. According to the equation for $\mu(\theta, \phi)$ (Eq. (2) of [1]), the value of $f^2\mu$ at a given point determines also the density of the points were the superconducting-current lines are transformed into normal and vice versa (depending on the sign of $f^2\mu$). Such a picture differs from that described in [4] only in that the "surface" charge of the core is smeared out over the radius.

To determine the friction coefficient it is necessary to repeat the procedure given in [1] for summing the equations, without omitting the term with μ. It turns out that the results of [1] are valid if the coefficient γ is replaced by the sum

$$\gamma = \gamma_R + \gamma_N \approx 0.438. \tag{4}$$

Here $\gamma_R \simeq 0.279$[2)] is the part of γ connected with the relaxation of the ordering parameter, while the part

$$\gamma_N = -\frac{i}{\dot{u}}\int_0^\infty f^2\mu(r)\,dr \approx 0.159 \tag{5}$$

is connected with the normal currents in the core. The contributions of these two processes to γ, and consequently also to the effective friction coefficient $\eta = 6\gamma\sigma\Phi_0 c^{-2}H_{c2}$ (in dimensional units), are thus of the same order of magnitude.

3. For a superconductor with low paramagnetic-impurity concentration, the equations can be written in explicit form [3] only if the bilateral condition $\Delta \ll \tau^{-1} \ll T_c$ is satisfied; this is possible only in a very small temperature interval near T_c. Introducing μ in accordance with (1), where now

$$\psi = -\frac{\tau_1}{4ie}\int \gamma_\epsilon\, d\epsilon \tag{6}$$

we can again write down a system of equations at $\lambda \neq 0$ in a gauge-invariant form. For μ we obtain Eqs. (2) - (3), accurate to the extent that the coefficient 12 is replaced by $a \simeq 5.86$. The corresponding solutions are also shown in the figure. Integrating the system of equations, we obtain for the friction coefficient (in dimensional units)

$$\eta = \frac{a}{2}(\gamma_N + \gamma_R)\sigma\Phi_0 H_{c_2} c^{-2}, \tag{7}$$

where $\gamma_N \simeq 0.233$. The relaxation part γ_R is the sum of the terms $\gamma_{R_0} \simeq 0.279$ and of the anomalous term $\nu\gamma_\alpha$, where $\gamma_\alpha \simeq 0.566$.

4. Of greatest interest is, naturally, the determination of η for superconductors without paramagnetic impurities. As $\tau_s^{-1} \to \Delta$, the term in the right-hand side of Eq. (7) in [3] for the anomalous term u_1 is no longer correct. Since this term obviously should not depend on τ_s when $\tau_s^{-1} \ll \Delta$, its order of magnitude can be estimated by replacing τ_s^{-1} with Δ. In this case, going

[2)] The value given in [1] is $\gamma_R \simeq 0.247$.

through all the steps mentioned in Sec. 3, we obtain for η the same expression (7), but now $\gamma = \gamma_0 + \beta T_c/\Delta$, where $\gamma_0 = \gamma_N + \gamma_{R_0}$ and $\beta > 0$. Thus the quantity

$$\eta_o = \frac{a}{2}\gamma_o \sigma \Phi_o H_{c_2} c^{-2} \approx 1.47\sigma \Phi_o H_{c_2} c^{-2} \qquad (8)$$

is the lower bound of the friction coefficient in a superconductor having a high nonmagnetic-impurity concentration.

At a certain deviation from T_c, where the applicability of the Ginzburg-Landau equations can still be expected), the second term of γ becomes practically constant. In this region the excess of the friction coefficient over the minimum value (8) is determined by the ratio of the quantities γ_0 and β, the latter being unknown. An analysis of the presently known experimental data likewise fails to answer this question, since these data give values of γ that differ greatly (by as much as a factor of 3) at the same values of T/T_c.

It can only be noted that such factors as the presence of local excitations in the vortex core [5] or inhomogeneities of the material [6] lead to a smearing of the singularity in the excitation spectrum and by the same token to a decrease of the anomalous terms that give rise to the increment of γ_0. It is possible that the already mentioned strong scatter of the experimental values of the friction coefficient is due just to the fact that these and similarly acting factors cannot be controlled.

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POSSIBLE EXISTENCE OF HADRONIC ISOMERS

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This article deals with the angular momenta of micro-objects (say molecules, atoms, atomic nuclei, and the so-called elementary hadrons), which have finite dimensions and can assume different excited states. In spite of the great differences between the dimensions of such quantum objects, there is a certain similarity between them. The angular momentum is the product of the momentum p by the radius R. Owing to the uncertainty relation we have $p \sim R^{-1}$, so that examination of the angular momenta reveals a unique similarity between quantum objects that differ strongly in their scales and in their mean energy distances between excitation levels. This similarity suggests that we might be able to consider heuristically the behavior of objects of a known structure, such as atomic nuclei to gain an idea of the behavior of objects whose structure is known and linear dimensions are smaller, such as hadrons.