of the spin S, whereas for the Ising model we have a decrease of the Curie temperature at $S = 1/2$, and an increase at $S \geq 1$.

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ANOMALOUS PLASMA RESISTANCE DUE TO INSTABILITY AT CYCLOTRON HARMONICS

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The appearance of a high anomalous plasma resistance in a large number of experiments (particularly in collisionless shock waves [1]) is attributed to the appearance of ion-acoustic instability [2]. The ion-acoustic instability can arise, however, only in the case of sufficiently strong anisothermy, $T_e \gg T_i$. Yet an anomalous resistance is observed also when this condition is not satisfied [1, 3].

Recently, in connection with the problem of anomalous resistance, the instability at electron cyclotron oscillations (Bernstein modes) has been under discussion [4 - 9]. The Bernstein modes are oscillations with a wave vector that is strictly or almost strictly perpendicular to the magnetic fields, and have frequencies on the order of $n\omega_{He}$. If current flows through the plasma, then the oscillation frequency turns out to be, owing to the Doppler effect, in the laboratory frame where the ions are at rest, $\omega' = n\omega_{He} - \vec{k}\cdot\vec{v}_d$, where $\vec{v}_d$ is the drift velocity. At sufficiently large k and v_d it is possible to decrease strongly the frequency in the ion reference frame, to make these oscillations interact with the ions at $\omega' \sim kv_T$. An effect such as wave instability with negative energy sets in. These oscillations have a rather large growth increment, $\gamma \sim \omega_{He}(v_d/v_{Te})$. Unlike in ion sound, the instability sets in also at $T_i \gtrsim T_e$.

The purpose of the present article is to derive an expression for the anomalous resistance that results from the buildup of Bernstein modes. The main question is the explanation of the nonlinear mechanism that leads to saturation of the oscillation growth. For ion-acoustic instability, such a mechanism would be induced scattering of waves by ions (nonlinear Landau damping by ions) [2]. For the Bernstein modes, the principal role is played by electronic nonlinearity. We use, just as in the theory of strong turbulence [10], the fact that the resultant turbulent transport coefficients play a stabilizing role. In this case the appearance of the anomalous resistance itself (anomalous scattering of electrons by oscillations) will lead to stabilization of the instability. From the condition that the instability increment in the turbulent plasma should vanish we obtain the value of the anomalous resistance. Electron scattering by oscillations with a still-unknown frequency ν_{eff} will be taken into account by introducing into the kinetic equation for the electrons a collision integral having the same form as the Coulomb integral. For the correction to the distribution function $f \sim \mathrm{esp}[i(kx - \omega t)]$ we obtain the equation

$$-i(\omega - kv_{\perp}\cos\phi) f - \omega_{He}\frac{\partial f}{\partial \phi} + \frac{e}{m}E\frac{\partial f_0}{\partial v_x} = Stf, \tag{1}$$

where ϕ is the azimuthal angle in velocity space, $v_{\perp}$ is the component of the velocity V perpendicular to the magnetic field, and the ordinary frequency of the Coulomb collision is replaced by the effective one, so that the plasma resistance is $\sigma = ne^2/m\nu_{eff}$. The dispersion equation for short-wave ($k\rho_e >> 1$) cyclotron oscillations is

$$\epsilon = 1 + \frac{\omega_{pi}^2}{k^2 v_{Ti}^2}[1 + i\sqrt{\pi}Z_i e^{-Z_i^2} - \psi(Z_i)] + \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left[1 - \frac{\omega}{\sqrt{2\pi}\,k\rho_e(\omega - \omega_{He})} + i\,16{,}5\,\nu_{eff}\frac{k\rho_e\omega}{\sqrt{2\pi}(\omega - n\omega_{He})^2}\right] \tag{2}$$

where

$$\psi(Z) = 2Ze^{-Z^2}\int_0^Z e^{t^2}dt, \quad Z_i = \frac{\omega - kv_d}{\sqrt{2}kv_{Ti}}, \quad v_{Te} = \sqrt{\frac{T_e}{m_e}}, \quad \rho_e = \frac{v_{Te}}{\omega_{He}}.$$

Let us consider the case when the drift velocity is much less than the electron thermal velocity, $v_d^2 < 3(v_{Te}v_{Ti}/n)(1 + k^2r_{de}^2)$, $r_{de}^2 = T_e/4\pi n_0 e^2$. The real part of the frequency is equal to $\omega = n\omega_{He} + \Delta\omega$, where

$$\Delta\omega = \frac{1 + k^2r_{de}^2 + \Theta(1 - \psi) - i\,\Theta\sqrt{\pi}Z_i\exp(-Z_i^2)}{\sqrt{2\pi}\,k\rho_e\{[1 + k^2r_{de}^2 + \Theta(1 - \psi)]^2 + \Theta\sqrt{\pi}Z_i\exp(-Z_i^2)\}} \tag{3}$$

here $\Theta = T_e/T_i$, $Z_i = (n\omega_{He} - \vec{k}\cdot\vec{v}_d)/\sqrt{2}kv_{Ti}$. The imaginary part of the frequency, under the condition $T_i > T_e$, is equal to

$$\gamma_m = 0.3\,\omega_{He}(v_d/v_{Te})(T_e/T_i)(1 + k^2r_{de}^2)^{-2} - 11.4(T_e/T_i)\nu_{eff}(v_T/v_d)^2. \tag{4}$$

Owing to the presence of the large "Pitaevskii factor" $k^2\rho_e^2$ [11], the stabilizing role of the collision becomes appreciable already at small ν_{eff}. In the stationary state, when the exponential growth of the oscillation stops, ν_{eff} is determined from the condition $\gamma = 0$. The most dangerous modes are those with $k = n\omega_{He}/v_d$ and $n = 1$. With increasing oscillation amplitude, the value of ν_{eff} increases. The harmonics $\omega = n\omega_{He}$ become stabilized when a value on the order of $\nu_{eff} \sim \gamma(v_d)/k^2\rho_e^2 \sim (\gamma/n^2)(v_d/v_{Te})^2$ is reached, where $\gamma(v_d)$ is the linear increment, i.e., the high-order harmonics are the first to be stabilized. When ν_{eff} reaches the limiting value (see (5)) all the modes reach saturation.

We thus find that

$$\nu_{eff} = 0.02\left(\frac{v_d}{v_{Te}}\right)^3\left(1 + \frac{\omega_{He}^2}{\omega_{pe}^2}\,\frac{v_{Te}^2}{v_d^2}\right)^{-2}\omega_{He}. \tag{5}$$

ν_{eff} turns out to be small owing to the "Pitaevskii factor" $k^2\rho_e^2$. The resistance determined by (5) is much lower than the resistance due to the ion-acoustic instability [2], but on the other hand the condition $T_e \gg T_i$ is not needed here. The anomalous resistance of the Bernstein modes leads to a shock-wave thickness on the order of

$$\begin{aligned} \Delta &= (c/\omega_{pe})(M/m)^{1/8}, \qquad (T_i/T_e) > (M/m)^{1/6}, \\ \Delta &= (c/\omega_{pe})(M/m)^{1/6}, \qquad (T_i/T_e) << (M/m)^{1/6}. \end{aligned} \tag{6}$$

The last estimate agrees well with the experimental data [3]. In addition to the cyclotron instability at $T_i \geq T_e$ there can develop in the plasma an instability of the electron-sound type [12]. For this instability, the quantity $\nu_{eff} \sim \omega_{He}(m/M)^{1/2}$ is larger than (5) [13]. However, in those cases, when the electron-acoustic instability does not have time to develop and go over to the nonlinear regime, owing to the smallness of the growth increment ($\gamma \sim \omega_{He}(m/M)^{1/2}$) (for example, in collisionless shock waves), the principal role will be played by the anomalous resistance considered above.

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