SCALE RELATION FOR PROTON FORM FACTORS

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The authors of [1 - 3] conclude on the basis of an analysis of data on e-p scattering at $q^2 \leq 2\ (\mathrm{GeV}/c)^2$ (q^2 is the square of the momentum transfer) that in the range of q^2 from 1 to $2(\mathrm{GeV}/c)^2$ there is a considerable deviation from the so-called scale relation

$$G_M(q^2) = \mu G_E(q^2). \tag{1}$$

Here $G_E(q^2)$ and $G_M(q^2)$ are the charge and magnetic form factors of the proton and μ is the total magnetic moment of the proton (in nuclear magnetons). On the other hand, the data in the q^2 interval from 1 to 3.75 $(\mathrm{GeV}/c)^2$, obtained in [4], agree with (1) within the limits of errors.

The present article is devoted to a verification of the scale relation (1) on the basis of an analysis of all the available data on the e-p scattering cross sections. The data of [1] were also processed separately. We used the data reduction method described in [5, 6]. This method differs from the customary one (construction of the Rosenbluth line at fixed q^2). The electromagnetic form factors of the proton are extracted directly from the data on the differential e-p scattering cross section. To this end, a certain functional dependence of the form factors on q^2 is assumed, and the values of the corresponding parameters are obtained by minimizing the functional χ^2. The experimental data of different groups are reduced by introducing normalization factors that take into account the possible errors in the normalization of the data.

We write

$$G_M(q^2) = S(q^2)\mu G_E(q^2). \tag{2}$$

For the form factor $G_E(q^2)$ we have the expression

$$G_E(q^2) = \frac{a_1}{1 + a_2 q^2} + \frac{1 - a_1}{1 + a_3 q^2}, \tag{3}$$

which, as shown in [6], describes satisfactorily all the known experimental data on e-p scattering. We shall make various assumptions concerning the function $S(q^2)$. From the normalization conditions $G_E(0) = 1$ and $G_M(0) = \mu$ it follows that

$$S(0) = 1. \tag{4}$$

Further, as is well known, at the threshold of the processes $e + \bar{e} \leftrightarrows p + \bar{p}$ at $q^2 = -4M_p^2$ (M_p is the proton mass) we have

$$G_M(-4M_p^2) = G_E(-4M_p^2). \tag{5}$$

If the form factors G_M and G_E are not equal to zero at $q^2 = -4M_p^2$ (we note that the first experimental data [7] on the process $e + \bar{e} \leftrightarrows p + \bar{p}$ favor this assumption), then we obtain from (2) and (4)

$$S(-4M_p^2) = \frac{1}{\mu}. \tag{6}$$

We assume first that the function $S(q^2)$ is a ratio of polynomials of equal degree, i.e., that the form factors $G_M(q^2)$ and $G_E(q^2)$ behave in the same manner when $q^2 \to \infty$. We confine ourselves to first-degree polynomials. We have

$$S(q^2) = \frac{a + b\tau}{1 + c\tau}, \tag{7}$$

where $\tau = q^2/4M_p^2$.

From (4) and (6) we find that the function $S(q^2)$ is characterized by one parameter and is of the following form:

$$S(q^2) = \frac{1 + \left[1 - \frac{1}{\mu}(1 - c)\right]\tau}{1 + c\tau}. \tag{8}$$

As a result of the reduction of all the available data on the e-p scattering cross sections[1)] it was found that

$$c = 1.05 \pm 0.09. \tag{9}$$

For the parameters a_1, a_2, and a_3, we obtained the following values:

$$a_1 = -0.48 \pm 0.08; \quad a_2 = 0.69 \pm 0.05\ (\mathrm{GeV/c})^{-2}; \quad a_3 = 2.18 \pm 0.08 (\mathrm{GeV/c})^{-4} \tag{10}$$

The quality of the description of the experimental data in this case ($\chi^2 = 396$ at $\bar{\chi}^2 = 313$) is practically the same as in the parametrization of the form factors G_M and G_E in [6] by a sum of two poles with independent parameters. We note that within the limits of errors the values in (10) coincide with the values of the parameters a_1, a_2, and a_3 obtained in this last case. From (8) and (9) we find that $\mu G_E/G_M = S^{-1}$ is equal to 1.003 ± 0.008, 1.018 ± 0.036, 1.023 ± 0.045, and 1.027 ± 0.054 at q^2 values of 1, 5, 10, and 25 $(\mathrm{GeV/c})^2$, respectively.

Thus, within the limits of errors, the quantity $\mu G_E/G_M$, obtained by reducing all the available experimental data on e-p scattering under the assumptions that $G_E(q^2)$ and $S(q^2)$ are given by expressions (3) and (8), does not differ from unity in the entire experimentally investigated interval of q^2. We emphasize that this conclusion was obtained under the condition that the form factors must be equal at the point $q^2 = -4M_p^2$.

Using the relation (2), (3), and (8), we have reduced separately the experimental data in the interval $q^2 \leq 2\ (\mathrm{GeV/c})^2$, obtained in [1], in which a deviation from (1) is reported. For the parameter c we obtained in this case a value $c = 0.85 \pm 0.19$ ($\chi^2 = 26$ at $\bar{\chi}^2 = 49$). The quantity $\mu G_E/G_M$ is equal to 0.988 ± 0.016, 0.978 ± 0.028, and 0.964 ± 0.047 for q^2 equal to 0.5, 1, and 2 $(\mathrm{GeV/c})^2$, respectively.

Further, in order to obtain a better description of all the available data on e-p scattering, we have considered the case of a different behavior of the form factors $f_M(q^2)$ and $G_E(q^2)$ as $q^2 \to \infty$. Assuming that $S(q^2)$ is the ratio of a second-degree polynomial in q^2 to a first-degree polynomial, we obtain with the aid of (4) and (6)

[1)] For references see [6]. The earlier data were replaced in the reduction by the data of [1].

$$S(q^2) = \frac{1 + \left[1 + e - \frac{1}{\mu}(1-d)\right]r + er^2}{1 + dr} \quad (11)$$

It is obvious that when e = 0 the expression (11) is transformed into (8). The data reduction has shown that the parameter e is equal to zero within the limits of error ($e = -0.02 \pm 0.14$). Its introduction does not improve the quality of the description of the experimental data ($\chi^2 = 396$ at $\bar{\chi}^2 = 312$).

Finally, we reduced the data for the case when the function $S(q^2)$ is a ratio of polynomials of first degree and the condition (6) is not imposed on this function. The two parameters characterizing $S(q^2)$ turn out to be strongly correlated in this case and are determined with large errors. The quality of the description remains at the same level as before.

We note in conclusion that we have reduced also the data of [1] in the interval $q^2 \leq 2$ (GeV/c)^2, assuming also, just as in that reference, the following expression for $S(q^2)$:

$$S(q^2) = \frac{1}{1 + \beta q^2}. \quad (12)$$

At $\chi^2 = 29$ and $\bar{\chi}^2 = 49$, we found that $\beta = -0.026 \pm 0.028$ (GeV/c)^2. Describing the values of $\mu G_E/G_M$ obtained from the Rosenbluth line by expression (12), the value obtained in [1] for the parameter β was $\beta^B = -0.059 \pm 0.029$ (GeV/c)$^{-2}$. The value obtained in [4] was $\beta^{SL} = -0.051 \pm 0.036$ (GeV/c)$^{-2}$.

Thus, an analysis of all the data available on e-p scattering shows that in the entire investigated range of momentum squared the ratio $\mu G_E/G_M$ does not differ from unity, within the limits of errors. It is of interest to note that this agrees with the requirement of equality of the charge and magnetic form factors in the time-like region at $q^2 = -4M_p^2$.

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