RELAXATION OF INTERNAL STRESSES IN HETEROPHASE SYSTEMS AND PHASE NUCLEATION IN SOLIDS

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During the course of phase transitions of real solids, heterophase states are produced as a rule, and are characterized by the presence of internal stresses. The source of the stresses is the incompatibility of the intrinsic strains of the contiguous phases at the interphase boundaries. If the microscopic continuity of the crystal lattice[1] is conserved at all points during the transition, including the interphase surfaces, then the result is a system of phases that are usually called coherent. The level of the internal stresses in the system of coherent phases is maximal. These stresses decrease when the continuity of the phases is disturbed at the interphase boundaries and the coherence is lost[2]. The relative stabilities of the coherent and incoherent phases (the latter corresponding to loss of continuity in the entire interphase surface) is determined by the specific area of interphase surface.

In the present article we wish to call attention to certain hitherto unnoticed thermodynamic aspects of the problem of coherence loss.

1. In a crystalline body, the incoherent state is apparently stable only in exceptional cases. Actually, the relaxation of the internal stresses can be attained not only as a result of the exclusion of the stress sources by destroying the contact between the phases, but also by cancelling out these stresses by fields of suitable distributed defects. In particular, complete relaxation of the stresses is attained by producing on the interphase surface a dislocation grid with linear density $\rho \sim \varepsilon_0/\vec{b}$, where $\vec{\varepsilon}_0$ is the jump of the intrinsic deformations on the phase boundary, and $\vec{b}$ is the Burgers vector of the dislocations[3].

The microdistortions in the dislocation nuclei, being incoherent with the remaining stress field, make an additive contribution to the energy of the system. This contribution is proportional to $\alpha G b^2$ (G is the shear modulus and α is a factor on the order of unity). The elastic energy of the internal stresses is thus "pumped over" into the microdistortion energy, which makes up, together with the surface energy σ_0 of the coherent phases, the effective surface interphase energy $\sigma_0 + \alpha G b^2 \rho \simeq \sigma_0 + \alpha G b \varepsilon_0$. A comparison of this quantity with the surface energy of the noncoherent phases ($\sigma_n \sim Gb$) shows that under typical conditions, when $\varepsilon_0 < 10^{-1} - 10^{-2}$ and $\sigma_0 << \sigma_n$, the relaxed state at which the coherence of the phases is conserved over a considerable part of the interphase boundary is more stable than the incoherent state. The composite heterophase systems and polycrystalline aggregates should apparently tend

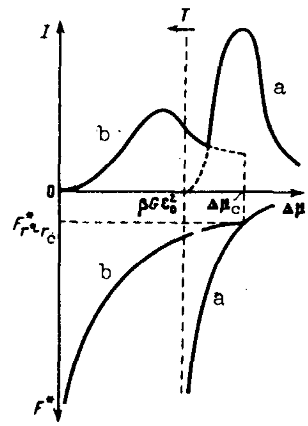
[1]I.e., any closed contour drawn through the lattice points prior to the transformation remains closed after the transformation (condition of compatibility of the total deformations of the lattice).

[2]Loss of coherence can be regarded as the result of passing internal cuts through the interphase boundaries at a relative free displacement of the edges of the cuts. If the produced phase has a smaller specific volume than the initial phase, then the loss of coherence leads to formation of an interphase "gap" and to a separation of the phases, so that the stresses vanish.

[3]The local value of ρ is determined from the equation $\hat{\varepsilon}_0 \times \vec{n} = \rho \vec{b} \vec{\tau}$, where $\vec{n}$ is the normal to the interphase boundary and $\vec{\tau}$ is the direction of the dislocation line.

towards a partial establishment of coherence, viz., a partial transformation of the phase surface energy proper into the energies of the micro- and macrodistortions apparently takes place in the equilibrium state.

2. If the relaxation of the internal stresses is the result of dislocation formation, then the free energy of a heterophase system is a functional of the dislocation distribution. A solution of the variational problem for the free-energy minimum at given configurations of the interphase boundaries and of the type of dislocations should yield the equilibrium surface density of the dislocations. By way of example, let us consider a very simple heterophase system, consisting of a spherical inclusion of radius r in an unbounded matrix; the elastic and surface properties of the phases are isotropic, and the intrinsic deformation of the phase is included in the form of dilatation $\hat{\varepsilon}_0 = \varepsilon_0 \hat{I}$. The stresses are relaxed by the dislocations with Burgers vector $\vec{b}$ lying in the interphase plane; the distribution of the dislocations is described by a single parameter ρ. The free energy of such a system is reckoned from the state of the homogeneous matrix phase and is equal to



Barrier for nucleation and rate of nucleation of partially-coherent (a) and coherent (b) phases.

$$F = [-\Delta\mu + \beta G(\epsilon_0 - b\rho)^2]\frac{4}{3}\pi r^3 + (\sigma_0 + aGb^2\rho)\,4\pi r^2, \quad (1)$$

where Δμ is the difference between the specific free energies of the undistorted phases, and β is a factor on the order of unity (β = 4 if the elastic properties of the phases are equal). *The minimum of* F(ρ) *is reached at* equilibrium dislocation density

$$\rho_0 = \frac{\epsilon_0}{b}\left(1 - \frac{r_c}{r}\right), \quad r_c = \frac{3}{2}\,\frac{a}{\beta}\,\frac{b}{\epsilon_0} \sim \frac{b}{\epsilon_0}. \quad (2)$$

The coherent state corresponding to complete conservation of the internal stresses is stable when $r < r_c$. With increasing r, the equilibrium dislocation density increases, as does the degree of relaxation of the stresses, which tends asymptotically to eliminate the stresses completely.

3. The foregoing considerations concerning the equilibrium degree of relaxations of the internal stresses in a heteorphase system are significant when it comes to a study of the nucleation of new phases in solids. In the model under consideration, the nucleation of a new phase corresponds to passage of the system through a saddle point (r*, ρ*) on the surface F(r, ρ) (in (1), Δμ > 0 and increases with increasing deviation from the point of thermodynamic equilibrium of the phases, while r* is connected with ρ* by the relation (2)). At $r^* = r_c$ the saddle point degenerates into a maximum, and the energy barrier F(r*, ρ*) for the nucleation of the relaxed phase goes over into the barrier $F(r^*) = (16\pi/3)\sigma_0^3/(\Delta\mu - \beta G\varepsilon_0^2)2$ for coherent nucleation (Fig. a).

When $\Delta\mu < \beta G\varepsilon_0^2$, the formation of a coherent phase is thermodynamically not convenient, and only a relaxed phase can arise; when $\Delta\mu > \Delta\mu_c (r^*(\Delta\mu_c) \equiv r_c)$ the phase nucleation occurs coherently. In the intermediate region $\beta G\varepsilon_0^2 < \Delta\mu < \Delta\mu_c$, nucleation takes place in a manner that ensures the maximum rate of transformation. Thus, the plot of the rate of nucleation against $\Delta\mu$ should have two branches corresponding to the formation of a relaxed (partially coherent) phase[4]. Since the rate of nucleation is proportional to $\exp(-F^*/kT)$, it follows that the rate of nucleation during the formation of a low-temperature phase from a high-temperature one should be nonmonotonic (Fig. b).

The absolute values of the rates of nucleation of coherent and partly-coherent phases may differ significantly. For example, at small $\Delta\mu$, when $\tau^* \gg r_c$ and $\rho^* \simeq \varepsilon_0/b$, $F^* = (16\pi/3)(\sigma^0 + \alpha Gb\varepsilon_0)^3/\Gamma\mu^2$ and, if $\sigma^0 < \alpha Gb\varepsilon_0$, the rate of nucleation of the relaxed phase is lower by several orders of magnitude than the rate of coherent nucleation at equal values of the moving force. It is necessary to add to the already noted difference between the barriers to nucleation also the difference between the mobilities of the interphase boundaries [1], which makes the rate of transition with formation of a coherent phase much larger than for the transition with formation of a relaxed phase. The formation of a coherent or partly coherent phase corresponds apparently to two known kinetic types of transition in a single-component system, viz., normal and martensitic [2].

[1] A.L. Roitburg, ZhETF Pis. Red. 13, 95 (1971) [JETP Lett. 13, 66 (1971)].
[2] G.V. Kurdyumov, Problemy metallovedeniya i fiziki metallov (Problems of Metallography and Metal Physics), M., Vol. 3, 9 (1952).

$\rho^0 - \omega$ INTERFRENCE IN THE REACTIONS $\gamma N \to \pi^+\pi^-\Delta$

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1. At the present time, the main information concerning the magnitude and phase of the $\omega \to \pi^+\pi^-$ decay due to electromagnetic $\rho^0 - \omega$ mixing [1 - 3] is obtained from experiments on the study of the effects of $\rho^0 - \omega$ interference in $\pi^+\pi^-$ mass spectra [4 - 6]. For a quantitative determination of the parameters of the electromagnetic $\rho^0 - \omega$ mixing it is important to know the relative magnitude, the relative phase, and the degree of coherence of the amplitudes of ρ^0- and ω-meson production reactions. Therefore reactions with known simple mechanisms of ρ^0 and ω production are most convenient for the study of the $\rho^0 - \omega$ mixing.

At the same time, the $\rho^0 - \omega$ interference in the $\pi^+\pi^-$ system can be used to verify and refine the theoretical mechanisms of ρ^0 and ω production [4].

In both cases, special interest attaches to processes in which the cross sections for ω-meson production are much larger than ρ^0-production cross sections, since it is necessary to expect for them a considerable enhancement of the $\rho^0 - \omega$ interference.

We wish to note in the present article that examples of such reactions are the reactions $\gamma N \to \rho^0\Sigma$ and $\gamma N \to \omega\Delta$, experiments on which are being performed at present and for which the first data have already been obtained [7 - 12].

[4] If the relaxation takes place with participation of more than one type of dislocation, then their sequential inclusion in the process may become manifest by the appearance of new kinetic branches of the transformation.