$$|\mathrm{Im}\, T_2^{\mu+}| \geq \sqrt{10^{-5}\Gamma_L^\gamma}\,[1 - |\epsilon|\cos(\phi_\epsilon + \phi_{LS}^\gamma)]\sqrt{\Gamma_S^\gamma/\Gamma_L^\gamma}\,, \tag{9}$$

where $\phi_{LS}^\gamma = \arg\,[T_L(T_S)^*]$. If $\Gamma_S^\gamma = 1.2 \times 10^{-3}\Gamma_S = 0.72\Gamma_L$, then at $\phi_{LS}^\gamma = 0$ we have $|\mathrm{Im}\, T_2^{\mu+}| \geq 0.95\sqrt{(10^{-5}\Gamma_L^\gamma)}$, and at $\phi_{LS}^\gamma = -45°$ we have $|\mathrm{Im}\, T_2^{\mu+}| \geq 0.93\sqrt{(10^{-5}\Gamma_L^\gamma)}$. The first of these values of $|\mathrm{Im}\, T_S^{\mu+}|$ corresponds to $\Gamma_S^\mu \geq 2.5 \times 10^{-7}$, and the second to $\Gamma_S^\mu \geq 2.1 \times 10^{-7}$. In order to reach the latter limits it is necessary to have: a) CP violation in the $K_L \to 2\gamma$ decay, b) an anomalously large probability of $K_S \to 2\gamma$, c) an anomalously strong $2\pi - 2\mu$ interaction, d) an anomalously strong $2\pi \to 2\gamma$ interaction. The simultaneous occurrence of these four anomalies is extremely unlikely.

If $\Gamma_L^\gamma = 4 \times 10^{-4}$, which does not contradict any of the experiments [2], then the possibilities (1), (2), and (3), correspond to lower limits $\Gamma_S^\mu/\Gamma_S \geq 2.9 \times 10^{-7}$, 1.8×10^{-7}, and 1.1×10^{-7}, respectively. The best of the published results is $\bar{\Gamma}_S^\mu = 7 \times 10^{-6}\Gamma_S$ [10]. As reported by Professor Kleinknecht, a preliminary result $\bar{\Gamma}_S^\mu = 1.5 \times 10^{-6}\Gamma_S$ was obtained at CERN.

A detailed analysis, the results of which will be published separately, shows that strong $2\pi - 2\mu$ interaction (7) does not contradict the available experimental data on the verification of the quantum electrodynamics of the muon, namely $g = 2$, elastic and inelastic scattering of muons by nucleons, production of muon pairs in collisions of hadrons, and the levels of μ-mesic atoms.

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DETECTION OF FAST PARTICLES BY THE DEFLECTION OF ELECTROMAGNETIC WAVES IN A TRAIN OF EXCITED STATES OF ATOMS ACCOMPANYING THE PARTICLE

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1. A particle moving in a substance excites continuously the atoms with which it collides in the substance as it moves. The successive excitation of the immobile atoms along the particle path is equivalent to motion of a "train" of excited atoms. The longitudinal dimensions of the train are determined by the lifetime τ of the excited state, and are macroscopic for relativistic

particles. The transverse dimensions of the train for ultrarelativistic particles are also macroscopic [1]. Thus, a train of excited states follows the particle at the same velocity, and this circumstance can be used to register the particle.

The dielectric properties of the excitation train differs from the dielectric properties of the rest of the substance. This difference is appreciable, for example, for frequencies that are close to the difference between the energies of two excited states of the atom (molecule) of the substance, $\omega_{21} = E_2 - E_1$ ($\hbar = c = 1$). In this frequency region, the dielectric constant of the unexcited substance ε does not have any natural frequencies, and the dielectric constant of the excited substance does. Owing to the difference between the dielectric constants and owing to the train motion, a plane electromagnetic wave passing through the excitation train is deflected and changes its frequency; this can be used to measure the particle velocity.

2. The macroscopic dimensions of the train of excitations and the large wavelength of the field make it possible to use macroscopic electrodynamics for an estimate of the effect. Let $\vec{P}_1(\vec{r}, t)$ be the change of the polarization of the excited matter compared with the unexcited one, and let $\vec{j}$ be the current density of the moving particle. The total magnetic field is given by the equation

$$\Delta \mathbf{H}(\mathbf{R}, \omega) + \omega^2 \epsilon \mathbf{H}(\mathbf{R}, \omega) + 4\pi \operatorname{rot} \mathbf{j}(\mathbf{R}, \omega) = 4\pi i \omega \operatorname{rot} \mathbf{P}_1(\mathbf{R}, \omega). \tag{1}$$

In the first approximation, we neglect the excitation of the substance and drop the right-hand side of (1). The solution is then the sum of the self-field of the charge and the unexcited substance, $\vec{H}_1(\vec{R}, \omega)$, and of the field of the incident plane wave $\vec{H}_0(\vec{R}, \omega)$. The second approximation is given by the field deviation $\vec{H}_2(\vec{R}, \omega)$

$$\Delta \mathbf{H}_2(\mathbf{R}, \omega) + \omega^2 \epsilon \mathbf{H}_2(\mathbf{R}, \omega) = 4\pi i \omega \operatorname{rot} \mathbf{P}_1^{\circ}(\mathbf{R}, \omega),$$

where the quantity H_2 is neglected in the calculation of the right-hand side.

By solving the equation, it is easy to find the energy flowing in a solid angle $d\Omega$ in the frequency interval $d\omega$ ($\vec{k} \equiv \vec{n}\omega\sqrt{\varepsilon}$)

$$d\mathcal{E}(\mathbf{n}, \omega) = \frac{\omega^2 d\omega d\Omega}{4\pi^2 \sqrt{\epsilon}} \left| \int e^{i\omega t} dt \int d^3 r \, e^{-i\mathbf{k}\mathbf{r}} \operatorname{rot} \mathbf{P}_1(\mathbf{r}, t) \right|^2. \tag{2}$$

3. The change of the polarization $\vec{P}_1(\vec{r}, t)$ is due to the simultaneous action of the plane-wave field ($\vec{H}_0$, $\vec{E}_0$), and the field of the moving particle ($\vec{H}_1$, $\vec{E}_1$) on the substance, and in this sense it constitutes a nonlinear effect. Assuming the fields $\vec{E}_0$ and $\vec{E}_1$ to be small compared with the atomic fields, we can expand the polarization in powers of the summary field $\vec{E}_0 + \vec{E}_1$, after which we write for $\vec{P}_1(\vec{r}, t)$

$$\mathbf{P}_1(\mathbf{r}, t) = \chi\{\mathbf{E}_0 E_1^2 + 2\mathbf{E}_1(\mathbf{E}_0\mathbf{E}_1)\} + \chi\{\mathbf{E}_1\mathbf{E}_0^2 + 2\mathbf{E}_0(\mathbf{E}_1\mathbf{E}_0)\}. \tag{3}$$

The nonlinear susceptibility χ is a material characteristic of a type similar to the dielectric constant ε; dispersion leads to a dependence of χ on the frequencies of all the fields. The nonlinear susceptibility χ was investigated theoretically and experimentally in a number of papers [2, 3], and generally speaking the value of χ for a given substance can be found in an independent experiment. The deviation of the field acting on the atom from the average field leads to anomalously large values of χ in substances with large refractive indices [2, 3].

When $\vec{E}_1 \gg \vec{E}_0$ we can retain only the first term in (3), and it follows from (2) that

$$d\mathcal{E}(\mathbf{n},\omega) = \frac{\omega^2 d\omega d\Omega}{8\pi^3\sqrt{\epsilon(\omega)}}\chi^2 T\{\,|Q(\mathbf{k}-\mathbf{k}_0)|^2\delta(\omega-\omega_0-\mathbf{k}\mathbf{v}+\mathbf{k}_0\mathbf{v}) + + |Q(\mathbf{k}+\mathbf{k}_0)|^2\delta(\omega+\omega_0-\mathbf{k}\mathbf{v}-\mathbf{k}_0\mathbf{v})\}, \tag{4}$$

where T is the total interaction time,

$$Q(\mathbf{p}) = \pi^4\int d^3s\left\{[\mathbf{k}\,\mathbf{E}_0^\circ]\left(\mathbf{E}_1\left(\frac{\mathbf{p}-\mathbf{s}}{2}\right)\mathbf{E}_1\left(\frac{\mathbf{p}+\mathbf{s}}{2}\right)\right)+\left[\mathbf{k}\mathbf{E}_1\left(\frac{\mathbf{p}-\mathbf{s}}{2}\right)\right]\left(\mathbf{E}_0^\circ E_1\left(\frac{\mathbf{p}+\mathbf{s}}{2}\right)\right)+\right.$$

$$\left.+\left[\mathbf{k}\mathbf{E}_1\left(\frac{\mathbf{p}+\mathbf{s}}{2}\right)\right]\left(\mathbf{E}_0^\circ\mathbf{E}_1\left(\frac{\mathbf{p}-\mathbf{s}}{2}\right)\right)\right\},$$

$$\mathbf{E}_0(\mathbf{R},t) = \mathbf{E}_0^\circ\cos(\mathbf{k}_0\mathbf{R}-\omega_0 t);$$

$$E_1(\mathbf{q}) = \frac{ie}{2\pi^2}(\mathbf{v}(\mathbf{q}\mathbf{v})-\mathbf{q}\epsilon^{-1})(q^2-(\mathbf{q}\mathbf{v})^2\epsilon(\mathbf{q}\mathbf{v})^{-1}.$$

When $\vec{E}_1 \ll \vec{E}_0$ we can retain only the second term in (3) and

$$d\mathcal{E}(\mathbf{n},\omega) = \frac{\omega^2 d\omega\, d\Omega}{\sqrt{\epsilon}(\omega)}\chi^2 T 2\pi^5[\delta(\omega-\mathbf{k}\mathbf{v})4|[\mathbf{k}\,\mathbf{E}_1(\mathbf{k})]\,(\mathbf{E}_0^\circ)^2+2[\mathbf{k}\mathbf{E}_0^\circ]\times$$

$$\times(\mathbf{E}_0^\circ\,\mathbf{E}_1(\mathbf{k}))|^2+\{\delta(\omega-2\omega_0-\mathbf{k}\mathbf{v}+2\mathbf{k}_0\mathbf{v})|[\mathbf{k}E_1(k-2k_0)](E_0^\circ)^2+$$

$$+2[kE_0^\circ](E_0^\circ E_1(k-2k_0))|^2+\delta(\omega+2\omega_0-\mathbf{k}\mathbf{v}-2\mathbf{k}_0\mathbf{v})|[\mathbf{k}\,\mathbf{E}_1(\mathbf{k}+2\mathbf{k}_0)]\times$$

$$\times(\mathbf{E}_0^\circ)^2+2[\mathbf{k}\mathbf{E}_0^\circ](\mathbf{E}_0^\circ\,\mathbf{E}_1(\mathbf{k}+2\mathbf{k}_0))|^2\}]. \tag{4'}$$

It is important that in (4) and (4') the angle θ between the direction of deflected wave $\vec{k}$ and the particle velocity $\vec{v}$ is rigidly connected with the frequency of deflected wave ω:

$$\cos\theta = \frac{1}{\mathrm{v}\sqrt{\epsilon(\omega)}}\left(1 \mp \frac{\omega_0-\mathbf{k}_0\mathbf{v}}{\omega}\right) \quad\text{or}\quad \cos\theta = \frac{1}{\mathrm{v}\sqrt{\epsilon(\omega)}}\left(1 \mp 2\frac{\omega_0-\mathbf{k}_0\mathbf{v}}{\omega}\right),$$

which makes it possible to determine the energy of the particle by measuring the angle of inclination of the wave of a given frequency. Unlike the case of Cerenkov radiation, it is possible to choose the values of k_0 and ω such that for any particle velocity the angle of inclination lies in the region favorable for the measurement.

4. The intensity of the deflected wave can be greatly enhanced by using the resonant character of the nonlinear susceptibility. It is possible here to use either the already mentioned resonance $\omega_0 = E_2 - E_1$, or else multiple (two-photon) resonances [3]. It is also possible, in particular, to use the fact that the denominator of (4) contains an expression of the type

$$|(\mathbf{k}\pm 2\mathbf{k}_0)^2-(\omega\pm 2\omega_0)\epsilon(\omega\pm 2\omega_0)|^2,$$

so that the intensity of the deflected wave increases when this expression is small.

The intensity of the deflected wave increases with increasing field intensity E_0 of the incident wave. In this case, however, the nonlinear effects due to the field E_0 without participation of the particle also increase. Nonetheless, the use of the influence of the dispersion of the dielectric constant makes it possible to weaken a number of extraneous nonlinear effects. The reason is that in processes in which the self field of the particle participates there is always a field component with a wave vector that satisfies the "synchronism condition," and for nonlinear effects with plane waves this condition is by far not always satisfied. The influence of the background of the Rayleigh scattering will be negligible if the deflected-wave frequency ω does not equal ω_0.

A detailed analysis of the indicated questions will be reported separately.

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EFFECTS CONNECTED WITH THE RESONANT BEHAVIOR OF THE GAP IN A SUPERCONDUCTOR

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The behavior of superconductors in alternating electromagnetic fields has a number of distinguishing features. Thus, for example, if the frequency of the radiation incident on the superconductor exceeds double the size of the gap, then single-quantum absorption of the radiation, with breaking of a Cooper pair, becomes possible. This effect is linear in the field. However, besides the linear terms, the expression for the current contains terms of higher powers in the field. These include terms cubic in the field potential, corresponding to two-quantum effects, and also terms due to oscillations of the modulus of the gap. The latter, as will be shown below, lead to a unique frequency dependence of the characteristics of the superconductor.

To this end, let us find the change of the equilibrium value of the gap under the influence of a weak alternating field. We assume this field to be transverse, and the phase Δ equal to zero. Assume also that everything occurs in a flat film, the thickness of which is small compared with the spatial dimensions of the variation of Δ and $\vec{A}$, so that it is possible to disregard the coordinate dependence of these quantities. Assume further that the number of impurities is sufficiently large to allow us to disregard the character of the electron reflection from the sample boundary.

The correction to the gap

$$\Delta_\omega = \lambda \int_{-\infty}^{\infty} \frac{d\epsilon}{4\pi i} \int \frac{dO_p}{4\pi} f_{\epsilon\epsilon-\omega} \tag{1}$$

is expressed in terms of the Green's function $F_{\varepsilon\varepsilon-\omega}(\vec{p})$ integrated with respect to $\zeta = \vec{v}\cdot(\vec{p} - \vec{p}_0)$, $f_{\varepsilon\varepsilon-\omega} = \int F_{\varepsilon\varepsilon-\omega}(\vec{p})d\xi$.

For the functions integrated with respect to ξ we have the system of equations derived by Eliashberg [1]