The intensity of the deflected wave increases with increasing field intensity E_0 of the incident wave. In this case, however, the nonlinear effects due to the field E_0 without participation of the particle also increase. Nonetheless, the use of the influence of the dispersion of the dielectric constant makes it possible to weaken a number of extraneous nonlinear effects. The reason is that in processes in which the self field of the particle participates there is always a field component with a wave vector that satisfies the "synchronism condition," and for nonlinear effects with plane waves this condition is by far not always satisfied. The influence of the background of the Rayleigh scattering will be negligible if the deflected-wave frequency ω does not equal ω_0.

A detailed analysis of the indicated questions will be reported separately.

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EFFECTS CONNECTED WITH THE RESONANT BEHAVIOR OF THE GAP IN A SUPERCONDUCTOR

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The behavior of superconductors in alternating electromagnetic fields has a number of distinguishing features. Thus, for example, if the frequency of the radiation incident on the superconductor exceeds double the size of the gap, then single-quantum absorption of the radiation, with breaking of a Cooper pair, becomes possible. This effect is linear in the field. However, besides the linear terms, the expression for the current contains terms of higher powers in the field. These include terms cubic in the field potential, corresponding to two-quantum effects, and also terms due to oscillations of the modulus of the gap. The latter, as will be shown below, lead to a unique frequency dependence of the characteristics of the superconductor.

To this end, let us find the change of the equilibrium value of the gap under the influence of a weak alternating field. We assume this field to be transverse, and the phase Δ equal to zero. Assume also that everything occurs in a flat film, the thickness of which is small compared with the spatial dimensions of the variation of Δ and $\vec{A}$, so that it is possible to disregard the coordinate dependence of these quantities. Assume further that the number of impurities is sufficiently large to allow us to disregard the character of the electron reflection from the sample boundary.

The correction to the gap

$$\Delta_\omega = \lambda \int_{-\infty}^{\infty} \frac{d\epsilon}{4\pi i} \int \frac{d0_p}{4\pi} f_{\epsilon\epsilon-\omega} \qquad (1)$$

is expressed in terms of the Green's function $F_{\varepsilon\varepsilon-\omega}(\vec{p})$ integrated with respect to $\zeta = \vec{v}\cdot(\vec{p} - \vec{p}_0)$, $f_{\varepsilon\varepsilon-\omega} = \int F_{\varepsilon\varepsilon-\omega}(\vec{p})d\xi$.

For the functions integrated with respect to ξ we have the system of equations derived by Eliashberg [1]

$$\omega g_{\epsilon\epsilon-\omega} = \left\{\frac{e}{c} vAg - g\frac{e}{c} vA - \Delta f + f\Delta\right\}_{\epsilon\epsilon-\omega} + I^{(ph)}_{\epsilon\epsilon-\omega},$$

$$(2\epsilon-\omega) f_{\epsilon\epsilon-\omega} = \left\{\frac{e}{c} vAf + f\frac{e}{c} vA - g\Delta - \Delta g\right\}_{\epsilon\epsilon-\omega} + K^{(ph)}_{\epsilon\epsilon-\omega}, \qquad (2)$$

where averaging over the momentum direction is implied in the right-hand and left-hand sides, and $I^{(ph)}$ and $K^{(ph)}$ are the collision integrals due to interactions with the phonons; the explicit expression for these integrals is too unwieldy.

The solution of (2) in second order in the field without allowance for the energy relaxation, for which the collision integrals are responsible, takes the form

$$\Delta_\omega \frac{4\Delta^2-\omega^2}{2\omega\Delta} \int \frac{d\epsilon}{2\epsilon-\omega}\left(f^{(0)}_\epsilon - f^{(0)}_{\epsilon-\omega}\right) = \int \frac{d\epsilon}{2\epsilon-\omega} \times$$

$$\times \left\{\frac{e}{c} vA\left(f^{(1)} - \frac{2\Delta}{\omega} g^{(1)}\right) + \left(f^{(1)} + \frac{2\Delta}{\omega} g^{(1)}\right)\frac{e}{c} vA\right\}_{\epsilon\epsilon-\omega}, \qquad (3)$$

where $f^{(0)}$ and $f^{(1)}$ are the functions in the zeroth and first order in the field, respectively,

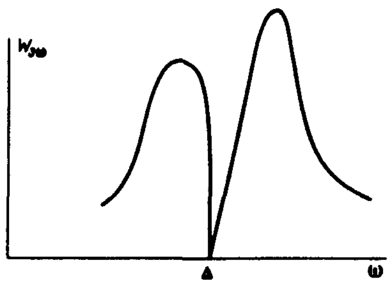
$$f^{(0)}_\epsilon = 2i\pi\Delta \,\mathrm{th}\frac{|\epsilon|}{2T} \cdot \frac{\theta(\epsilon^2-\Delta^2)}{\sqrt{\epsilon^2-\Delta^2}}.$$

From (3) we see that the behavior of the small nonstationary increment to the resonant value of the gap has an essential singularity. At the frequency $\omega = 2\Delta$ it behaves in a resonant manner, i.e., natural oscillations Δ_ω are possible in the absence of an external periodic field. On the other hand, if the frequency of the perturbation in the right side of (3) is close to 2Δ, then oscillations will build up to a value determined in any case by nonlinear effects. However, even in the approximation linear in the field intensity, corresponding to Eq. (3), it is possible for a finite oscillation amplitude to become established, owing to the energy relaxation of the single-particle-oscillation distribution function perturbed by the gap oscillations to an equilibrium value. These processes lead to the appearance of damping of the natural oscillations Δ_ω. This damping is small in magnitude because of the large time connected with the inelastic collisions with phonons. To find this damping, it is necessary to take into account the collision integrals in the right-hand side of (2).

The right-hand side of (3) at $\omega = 2\Delta$ does not contain any zeroes or singularities, and at $|\omega - 2\Delta| \ll \Delta$ we obtain

$$\Delta_\omega = 3K\left(\frac{1}{2}\right)\frac{\sqrt{\Delta|\omega-2\Delta|}}{\omega-2\Delta+i\gamma} D\left(\frac{e}{c}\right)^2 A^2_{\omega/2} \begin{cases} \dfrac{1}{\pi}, & \omega < 2\Delta \\ \dfrac{1}{\ln(3+2\sqrt{2})}, & \omega > 2\Delta \end{cases}, \qquad (4)$$

where $K(\frac{1}{2})$ is a complete elliptic integral of the first kind, and $D = v^2\tau/3$ is the diffusion coefficient.



In the limit of low temperature, $T \ll \Delta$, we have for γ

$$\gamma = 15(1 - 2^{-7/2})\pi^{3/2}\lambda\,\zeta\left(\frac{7}{2}\right)\left(\frac{\Delta}{p_0 s}\right)^2\left(\frac{T}{\Delta}\right)^{7/2}\Delta, \tag{5}$$

where $\zeta(7/2)$ is the Riemann Zeta function and s is the speed of sound. It is clear that in order of magnitude formula (5) yields the value of γ also when $T \sim \Delta$. At low temperatures, besides $(\Delta/p_0 s)^2 \ll 1$, the damping contains an additional smallness $(T/\Delta)^{7/2}$, which enhances the effect.

The presence of impurities does not affect the magnitude of the damping. The smearing of the root singularities in the density of states of the superconductor, due to the anisotropy, to inhomogeneities, etc., likewise does not lead to the appearance of an imaginary part in the denominator of (4), and only shifts the position of its zero. This is easiest to verify using as an example an alloy with paramagnetic impurities, in which the indicated smearing takes place [2]. In the case of high concentration of paramagnetic impurities $\Delta\tau_s \ll 1$ we get in the left-hand side of (3) the expression

$$\Delta_\omega\left[\omega^2 - 4\Delta^2\left(1 - \frac{1}{\Delta^2 \tau_s^2}\right)\right]\int \frac{d\epsilon}{2\epsilon - \omega}\left(\frac{\mathrm{th}\,\frac{\epsilon}{2T}}{\epsilon^2 + \tau_s^{-2}} + \frac{\mathrm{th}\,\frac{\epsilon - \omega}{2T}}{(\epsilon - \omega)^2 + \tau_s^{-2}}\right) \tag{6}$$

which has no zeroes. At low concentration of the paramagnetic impurities, $\Delta\tau_s \gg 1$, the zero will be determined by the previous expression $\omega^2 - 4\Delta^2 = 0$. Thus, the appearance of the singularity is connected with the presence of a sufficiently sharp maximum in the density of states.

Allowance for the coordinate dependence of $\Delta_\omega(\underset{\sim}{k})$, as shown by calculation, leads to an increase of the frequency of the natural oscillation by an amount proportional to v^2k^2/Δ, but a damping of the same order appears simultaneously. That is to say, the relatively long-lived harmonics of Δ_ω are those which are spatially homogeneous, and the effect will be most pronounced in small samples and thin films, where there is no coordinate dependence of the gap. In such samples, the damping of the natural oscillations Δ_ω at finite temperature is due to the phonon relaxation mechanism (5) and is small.

Returning to the electromagnetic properties of superconductors, we can see that the nonlinear part of the current, which is proportional to Δ_ω, will have a characteristic frequency dependence of the type (4). To obtain a corresponding response in the current we can use the method of averaging over the impurities [3]. As a result of calculation we get for $|\omega - \Delta| \ll \Delta$

$$\mathbf{J}_{3\omega} = \alpha\frac{\sigma}{c}\Delta_{2\omega}\mathbf{A}_\omega\,. \tag{7}$$

The complex quantity α can be expressed in terms of elliptic integrals

$$\alpha = \frac{1}{2i}\int_{-\infty}^{\infty} \frac{\theta(x^2-1)\,dx}{\sqrt{x^2-1}}\left[\frac{1}{2}\left(\frac{1}{\zeta^R_{x+1}} - \frac{1}{\zeta^R_{x+3}}\right)\left(5x+2+\frac{\zeta^R_{x+1}\zeta^R_{x+3}}{x+2}\right) - \frac{4}{\zeta^R_{x+3}}\right],$$

where

$$\zeta^R_x = \begin{cases} \sqrt{x^2-1}, & x > 1 \\ i\sqrt{1-x^2}, & |x| < 1. \\ -\sqrt{x^2-1}, & x < -1 \end{cases}$$

The nonlinear increment to the current, as seen from (7) and (4), has a sharply split maximum at $\omega - 2\Delta \sim \gamma$. In the experiment, this effect can be observed by watching the intensity $W_{3\omega}$ of the harmonic of the electromagnetic field of frequency 3ω, generated by a superconducting sample irradiated by an external field of frequency ω. $W_{3\omega}$ is proportional to the cube of the initial-field intensity, and its frequency dependence is shown schematically in the figure. The ratio of the maximum value of the intensity of the harmonic $W_{3\omega}$ to its value far from resonance amounts to $\Delta/\gamma \sim (\omega_D/T_c)^2$ where ω_D is the Debye frequency.

The harmonics $j_{5\omega}$ and so on also have the indicated singularity, but contain an additional smallness in the field intensity. In the calculations, Δ_ω was assumed small in comparison with the unperturbed value of Δ, which is possible when $(H/H_c)^2 \ll T_c/\omega_D$. In the real experimental situation, $\Delta \sim T$, and to observe the resonance it is necessary to use millimeter waves.

It should be noted that the considered singularities in the behavior of the gap, which were pointed out already in [4], are closely connected with collective excitations in superconductors [5].

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