GENERALIZATION OF THE NEVEU-SCHWARZ MODEL TO INCLUDE AN ARBITRARY INTERSECTION OF THE $\rho$ TRAJECTORY

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The construction of an amplitude with N external pions, which includes physical trajectories, is one of the most important problems of the dual approach. Earlier attempts [1, 2] of solving this problem, while including a $\pi$-mesic and $\rho$-mesic trajectory without states with imaginary mass (tachyons), had substantial shortcomings. Principal among them was the absence of factorization at the level of the daughter trajectories.

Neveu and Schwarz [3] proposed a dual amplitude for the $\pi$-mesic N-point diagrams with $\alpha_\rho(0) = 1$ and $\alpha_\pi(0) = 1/2$ without a tachyon on the $\rho$ trajectory and with gauge conditions that excluded ghosts from the spectrum of the states.

The Neveu-Schwarz amplitude is factorized at all states of the spectrum. Halpern and Thorn [4] modified this model for the case of arbitrary $\alpha_\pi(0)$.

The purpose of the present paper was to generalize the Neveu-Schwarz amplitude to arbitrary $\alpha_\rho(0)$. In the constructed model, there is no tachyon on the $\rho$ trajectory, and the contribution of all the states factors out independently of the number of external pions. However, the "super-gauge" conditions no longer lead here to elimination of ghosts from the spectrum. To construct the model we introduce, inaddition to the usual four components of the momentum of the external meson $k_i$, also new components $\kappa_{i\mu}$, $\mu = 5, 6, 7, \ldots$, such that the square of the generalized multi-dimensional vector $\tilde{k}_i \equiv (k_i, \kappa_i)$ remains equal to -1, and the sum of all the vectors $\tilde{k}_i$ remains equal to zero:

$$\frac{\tilde{k}_i^2}{2} = -\frac{1}{2} \qquad \frac{\kappa_i^2}{2} = -\frac{1+\mu^2}{2} \quad \alpha_\pi(0) + \frac{\mu^2}{2} = 0, \tag{1}$$

$$\sum_{i=1}^{N} \tilde{k}_i = 0 \qquad \sum_{i=1}^{N} \kappa_i = 0. \tag{2}$$

If we introduce alongside the new components $\kappa_i$ also the corresponding components of the operators $b_{m\mu}$ and $a_{n\mu}$, then we can see that the new Neveu-Schwarz operators, which depend on $\tilde{k}_i$ and $g_m(\tilde{k}_i)$ will satisfy the previous commutation relations (3) and (4)

$$\{ g_m(\tilde{q}),\ g_n(\tilde{q}) \} = 2L_{m+n}(\tilde{q}), \tag{3}$$

$$[ g_m(\tilde{q}),\ L_n(\tilde{q}) ] = \left(\frac{1}{2}n - m\right) g_{m+n}(\tilde{q}). \tag{4}$$

Writing down the operator vertices in accordance with the Neveu-Schwarz recipe

$$V(\tilde{k}_i) = [g_m, V_o(\tilde{k}_i)] = (\tilde{k}_i H) V_o(\tilde{k}_i) \tag{5}$$

where

$$V_o(\tilde{k}_i) = \exp \sum_{n=1}^{\infty} \frac{(\tilde{k}_i a_n^+)}{n} \exp \sum_{n=1}^{\infty} \frac{(\tilde{k}_i a_n)}{n}, \quad H_\mu = \sum_{m=-\infty}^{\infty} b_{m\mu},$$

$$m = \frac{1}{2}, -\frac{1}{2}, \frac{3}{2} \ldots \quad b_{-m} \equiv b_m^+ , \; [a_{n\mu}, a_{m\nu}^+] = n\delta_{mn} g_{\mu\nu},$$

$$\{ b_{\ell\mu}, b_{m\nu}^+ \} = \delta_{\ell m} g_{\mu\nu}$$

we construct a new expression for the amplitude with N external pions, replacing the momenta $k_i$ by $\tilde{k}_i$ in the initial Neveu-Schwarz amplitude. The values of the masses of the states on the ρ-meson and π-meson trajectories will be determined from relations (6) and (8), and the intersection of the trajectories will be determined from formulas (7) and (9):

$$1 + \frac{\tilde{q}^2}{2} = 1 + \frac{q^2}{2} + \frac{\kappa^2}{2} = n \qquad \kappa = \sum_{j=1}^{i} \kappa_j , \tag{6}$$

$$1 + \frac{\kappa^2}{2} = a_\rho(0) \equiv a_o \tag{7}$$

i is an even number, n = 0, 1, 2, ...,

$$\frac{1}{2} + \frac{\tilde{q}^2}{2} = \frac{1}{2} + \frac{q^2}{2} + \frac{\kappa^2}{2} = n, \quad \kappa = \sum_{\ell=1}^{j} \kappa_\ell \tag{8}$$

$$\frac{1}{2} + \frac{\kappa^2}{2} = a_\pi(0) = -\frac{\mu^2}{2} \tag{9}$$

j is an odd number, n = 0, 1, 2, ...

To satisfy relations (7) and (9) for arbitrary $\alpha_0$ and $\alpha_\pi^{(0)}$, it is necessary and sufficient to make the scalar products of the vectors $\kappa_\ell$ as follows:

$$\kappa_\ell \kappa_{\ell+1} = a_o + \mu^2 , \tag{10}$$

$$\kappa_\ell \kappa_{\ell+2} = \kappa_\ell \kappa_{\ell+4} = \kappa_\ell \kappa_{\ell+6} = \cdots = \kappa_\ell \kappa_{\ell+2k} = -\lambda, \tag{11}$$

$$\kappa_\ell \kappa_{\ell+3} = \kappa_\ell \kappa_{\ell+5} = \cdots = \kappa_\ell \kappa_{\ell+2k+1} = \lambda , \tag{12}$$

$$\lambda = 2a_o + \mu^2 - 1.$$

The condition λ = 0, i.e.,

$$2a_o + \mu^2 - 1 = 0 \tag{13}$$

is the Adler-Lovelace condition. In this case, the construction of the vectors $\kappa_i$ greatly simplifies and one can determine only two of their nonvanishing components $\kappa_{i\mu}$, with i = 1, 2, ..., N − 1, namely, $\kappa_{ii}$ and $\kappa_{ii+1}$. The vector $\kappa_N$ can be obtained in accordance with the "conservation law" (2) as follows:

$$\kappa_N = -\sum_{i=1}^{N-1} \kappa_i .$$

It is easy to see that factorization is observed in this case, i.e., just as in the ordinary dual model, the number of states with a given spin and mass is perfectly defined. Namely, since (3), (4), and (5) are satisfied, the same algebraic manipulations as in [3] will make it possible to write the amplitude of the N-point diagram in the form (14)

$$A = \langle 0 | V(\tilde{k}_2) \frac{1}{L_o - \frac{1}{2}} V(\tilde{k}_3) \ldots \frac{1}{L_o - \frac{1}{2}} V(\tilde{k}_{N-1}) | 0 \rangle . \tag{14}$$

The form (14) excludes automatically the tachyon from the $\rho$ trajectory, since in it the state with the lowest mass is a pion state with mass $\mu$.

Let us consider the factorization of the amplitude A in the channel $q^2 = (k_1 + k_2 + \ldots + k_i)^2$. To this end, we represent expression (14) as usual in the form of a sum over the states $|\lambda\rangle$:

$$A = \sum_n \sum_\lambda \langle 0 | V_2 \frac{1}{L_o - \frac{1}{2}} V_3 \ldots V_i | \lambda \rangle \frac{1}{n - \alpha_{q^2}} \langle \lambda | V_{i+1} \frac{1}{L_o - \frac{1}{2}} \ldots V_{N-1} | 0 \rangle$$

$|\lambda\rangle$ is a state in the occupation-number space

$$|\lambda\rangle \sim (a^+_{1\mu_1})^{r_1} (a^+_{1\mu_2})^{r_2} \ldots (a^+_{2\mu_1})^{s_1} (a^+_{2\mu_2})^{s_2} \ldots (b^+_{\mu_1/2})^{n_1} \ldots |0\rangle .$$

By virtue of (13), the scalar products $\kappa_\ell \kappa_m$ with $\ell = 2, 3, \ldots, i$ and $m = i + 1, \ldots, N - 1$ vanish, with the exception of $\kappa_i \kappa_{i+1}$. Therefore $|\lambda\rangle$ will contain besides the usual four components $a^+_{n\mu}$ and $b^+_{m\mu}$ only one i + 1-st component of the operators $a^+_{n\mu}$ and $b^+_{m\mu}$. The number of states with given mass and spin can be obtained with allowance for these operators. The concrete number of the additional component does not play any role at all (the conditions (10) - (12) are covariant), and after factorization the numbers of the components of the vectors of the right-hand (i + 1)-point diagram and the left-hand (N - i + 1)-point diagram can be chosen in one and the same manner.

Thus, the spectrum is determined by four components of the operators $a_n$ and $b_m$ and by an additional component of these operators $a_{n,i+1}$ and $b_{m,i+1}$.

In this model, the absence of ghosts at the first daughter trajectories of the $\rho$-mesic and $\pi$-mesic trajectories can be demonstrated in standard fashion. As always, this property is a direct consequence of the $SU_{11}$ invariance of the amplitude. The super-gauge conditions, which are connected with the operators $g_m m \geq 3/2$ and $L_n n \geq 2$, do not lead here to a lowering of the spectrum of the states, and consequently to a dropping out of all the ghosts. These conditions express each time the above-described states $|\lambda\rangle_{i+1}$ in terms of new states which include not one but all the additional components of the operators $a^+_{n\mu}$ and $b^+_{m\mu}$. Therefore no connections appear between the states $|\lambda\rangle_{i+1}$ themselves.

Repeating the reasoning of Brower and Thorn [5], we can then show that asymptotically there will be ghosts on sufficiently deep daughter trajectories. We note in conclusion that a forgoing of the Adler-Lovelace condition in this model would lead to a broadening of the spectrum of the states also because of one more component that appears upon factorization. Any vector $\kappa_\ell$ can then be written in the form of a sum

$$\kappa_\ell = \kappa'_\ell + (-1)^\ell \xi , \tag{16}$$

where $\kappa'_\ell$ has, as before, only two components differing from zero, namely $\kappa'_{\ell\ell}$ and $\kappa'_{\ell\ell+1}$, and is perpendicular to the vector $\xi$,

$$\kappa'_\ell \xi = 0, \quad \kappa'_\ell \kappa'_{\ell+1} = 1 - \alpha_0, \quad \kappa'_i \kappa'_k = 0, \quad k > i \ k \neq i + 1, \quad \xi^2 = -\lambda,$$

$$\kappa'^2_\ell = 2(\alpha_0 - 1). \tag{17}$$

Then (if the conditions (17) are satisfied), the conditions (10) - (12) are satisfied and contribution to the states $|\lambda\rangle$, which factorize the amplitude, will be made by the operators $a^+_{n,i+1} a^+_{n\xi}$ and $b^+_{m,i+1} b^+_{m\xi}$.

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## VIRTUAL $\gamma\gamma$ SCATTERING AND NEW TYPE OF SCALING

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It has been pointed out in recent papers [1] (see also [2]) that it is possible to measure experimentally the absorptive part of the amplitude of virtual forward $\gamma\gamma$ scattering (Fig. 1a). The initial reaction is "e + e → e + e + hadrons" in colliding electron beams, and its amplitude includes the diagram of Fig. 1b. Thus, the cross section summed over all the hadronic states includes the absorptive part of the $\gamma\gamma$ scattering amplitude.

In this paper we present quantitative predictions for the asymptotic form of the amplitude of Fig. 1a in the region of large photon masses.

From the theoretical point of view, the virtual $\gamma\gamma$ scattering affords new interesting possibilities, since all four particles are off the mass shell. Unlike the case of inelastic ep scattering, it is now already necessary to have information on the properties of the products not only of the local operators on the light cone, but also on the bilocal ones.

The amplitude has the following form (for simplicity we consider scalar photons):

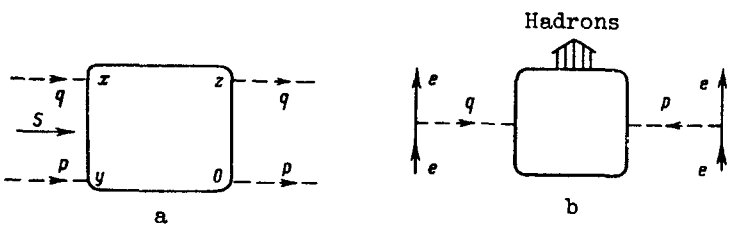


Fig. 1