where κ'_ℓ has, as before, only two components differing from zero, namely $\kappa'_{\ell\ell}$ and $\kappa'_{\ell\ell+1}$, and is perpendicular to the vector ξ,

$$\kappa'_\ell \xi = 0, \quad \kappa'_\ell \kappa'_{\ell+1} = 1 - a_0, \quad \kappa'_i \kappa'_k = 0, \quad k > i \; k \neq i + 1, \quad \xi^2 = -\lambda,$$

$$\kappa'^2_\ell = 2(a_0 - 1). \tag{17}$$

Then (if the conditions (17) are satisfied), the conditions (10) - (12) are satisfied and contribution to the states $|\lambda>$, which factorize the amplitude, will be made by the operators $a^+_{n,i+1}a^+_{n\xi}$ and $b^+_{m,i+1}b^+_{m\xi}$.

In conclusion, the author thanks E.M. Levin for useful discussions and the participants of the theoretical seminar of the Leningrad Institute of Nuclear Physics for interest in the work.

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VIRTUAL γγ SCATTERING AND NEW TYPE OF SCALING

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It has been pointed out in recent papers [1] (see also [2]) that it is possible to measure experimentally the absorptive part of the amplitude of virtual forward γγ scattering (Fig. 1a). The initial reaction is "e + e → e + e + hadrons" in colliding electron beams, and its amplitude includes the diagram of Fig. 1b. Thus, the cross section summed over all the hadronic states includes the absorptive part of the γγ scattering amplitude.

In this paper we present quantitative predictions for the asymptotic form of the amplitude of Fig. 1a in the region of large photon masses.

From the theoretical point of view, the virtual γγ scattering affords new interesting possibilities, since all four particles are off the mass shell. Unlike the case of inelastic ep scattering, it is now already necessary to have information on the properties of the products not only of the local operators on the light cone, but also on the bilocal ones.

The amplitude has the following form (for simplicity we consider scalar photons):

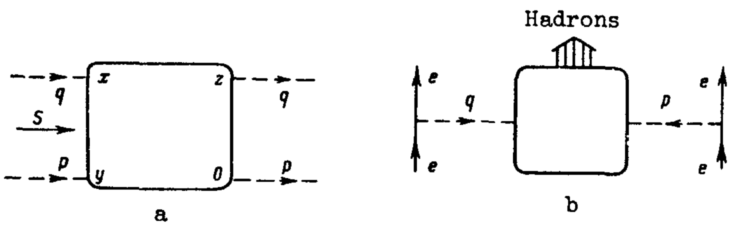

Fig. 1

$$T(s, p^2, q^2) \sim \int dx\, dy\, dz\, e^{-iq(x-z)-ipy} < 0 | T\{ I(x) I(y) I(z) I(0) \} | 0 >. \tag{1}$$

The matrix element in (1) is a function of six variables. We represent it in the form

$$< 0|T\{I(x)I(y)I(z)I(0)\}|0> \equiv \phi_{tot} = \{\phi_4^a[(x-y)^2, (x-z)^2, z^2, y^2] + \phi_4^b + \tilde{\phi}_4^a\} + \phi_{res} \tag{2}$$

(ϕ_4^b is obtained from ϕ_4^a by making the substitution $x \leftrightarrow y$, and $\tilde{\phi}_4^a$ is obtained by the substitution $x \leftrightarrow z$). In the limit when the strong interaction is turned off, the functions ϕ_4 remain, while $\phi_{res} \to 0$. We make the following assumption concerning the behavior of ϕ_{tot} at short distances.

I. The functions ϕ_4 have simple poles in all their arguments, regardless of the sequence in which the limit is taken. For example,

$$\phi_4^a \sim \frac{1}{(x-y)^2 - i\epsilon} \; \frac{1}{(x-z)^2 - i\epsilon} \; \frac{1}{z^2 - i\epsilon} \; \frac{1}{y^2 - i\epsilon}, \tag{3}$$

when all the distances are small.

II. ϕ_{res} has simple poles in not more than two out of the six arguments simultaneously, for example,

$$\phi_{res} \sim \frac{1}{(x-z)^2 - i\epsilon} \; \frac{1}{y^2 - i\epsilon} \phi[x^2, z^2, (x-y)^2, (z-y)^2], \tag{4}$$

where ϕ is already regular at small values of its arguments. (Assumptions I and II can be formulated also in terms of the expansion of the products of local and bilocal operators near the light cone.)

The degrees of the singularities in (3) and (4) correspond to the canonical dimensionality for the scalar current. We note that (3) and (4) are satisfied in ϕ^3 theory (accurate to the square of the logarithm).

Assumptions I and II suffice to obtain the asymptotic form of the amplitude in the region of large photon masses:

$$\text{I.} \quad p^2 \sim q^2 \to -\infty \; p^2 q^2 \mathrm{Ab}_s T(s, p^2, q^2) \to \phi_a\left(\frac{sm^2}{p^2 q^2}\right) + \phi_b\left(\frac{s}{p^2}, \frac{p^2}{q^2}\right) + \phi_c\left(\frac{s}{m^2}, \frac{p^2}{q^2}\right),$$

$$\phi_a(x) = \phi_{(4a)}(x) + \hat{\phi}_{res}(x), \; \hat{\phi}_{res}(x)|_{x<<1} \to 0, \; \phi_{(4a)}(x)|_{x<<1} \to \text{const},$$

$$\phi_b(x, y)|_{x<<1} \to 0, \quad \phi_b(x, y)|_{x>>1} \to 0, \; \phi_c(x, y)|_{x>>1} \to 0. \tag{5}$$

Here p^2 and q^2 are the photon masses, $s = (p + q)^2$, m is the characteristic mass ~1 GeV which determines the region where the asymptotic form is assumed. Let us note the characteristic features of (5): (a) when $|p^2| \sim |q^2| << s << |p^2 q^2|/m^2$ and $m^2 << s << |p^2| \sim |q^2|$ the dependence on all three variables disappears, i.e., $p^2 q^2 \mathrm{Ab}_s T(s, p^2, q^2) \sim$ const; (b) $p^2 q^2 \mathrm{Ab}_s T$ does not tend to zero towards the threshold (resonant) region $s \sim m^2$; (c) at all $sm^2 \gtrsim |p^2 q^2|$ there remains a dependence on only one parameter $sm^2/p^2 q^2$, which agrees

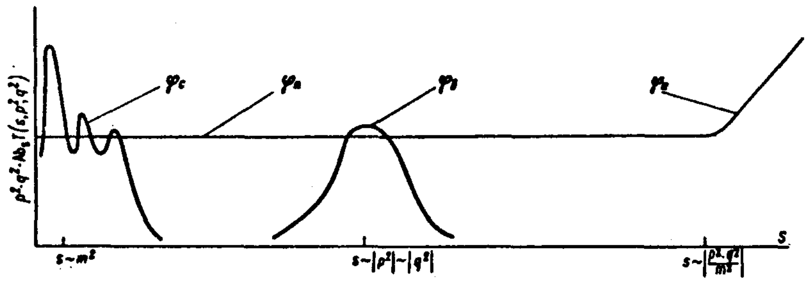

Fig. 2

with the Regge factorization of the residues [3, 4] and with the predictions of the parton model for $\gamma\gamma$ scattering [5]. (We note that in all three references [3 - 5], the indicated region of applicability of the result, $s >> |p^2| \sim |q^2|$, in incorrect. Actually, under the assumptions used in [3 - 5], the behavior in the region $|p^2| \sim |q^2| << s << |p^2q^2|/m^2$ cannot be determined, and the result is thus meaningful only at $sm^2 \gtrsim |p^2q^2|$.) Figure 2 shows one of the variants of the qualitative behavior of the amplitude.

$$\text{II.}\quad p^2 = \text{const} < 0,\ q^2 \to -\infty.\quad p^2q^2\mathrm{Ab}_s T(s, p^2, q^2) \to \frac{p^2}{m^2}\phi\left(\frac{s}{q^2}, \frac{p^2}{m^2}\right),$$
$$\phi(x, y)|_{x<<1} \gtrsim 0(x),\quad y\phi(x, y)|_{y>>1} \to \phi_a\left(\frac{x}{y}\right) = \phi_a\left(\frac{sm^2}{p^2q^2}\right). \qquad (6)$$

(In ϕ^3 theory, the function ϕ_b from (5) represents the contribution of the diagram of Fig. 3b), the function ϕ_a in (5) and (6) represents Figs. 3a and 3d, the function ϕ_c in (5) represents Fig. 3c, while the function ϕ in (6) represents Fig. 3e.)

We note that at $m^2 << s << |p^2q^2|/m^2$, the main contribution to $\mathrm{Ab}_s T$ is made by the region where all the distances are small. Thus, the contribution of the functions ϕ_4^a and ϕ_4^b (in ϕ^3-theory - the diagrams of Figs. 3a and 3b) predominate. Strong interaction in this region is effectively excluded.

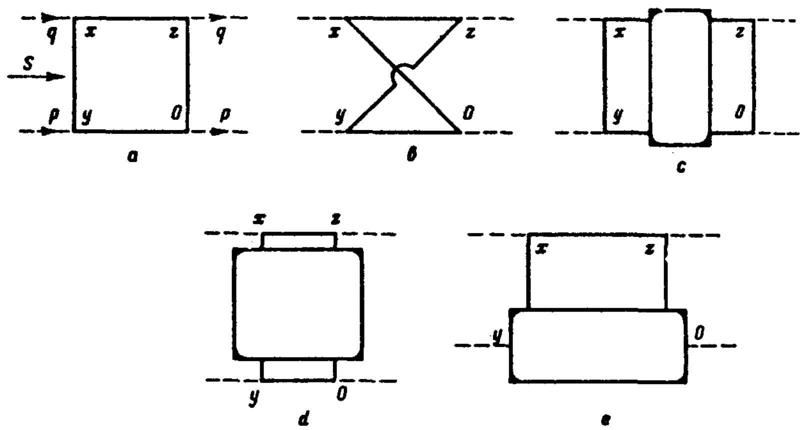

Fig. 3

A common scale factor in the left-hand sides of (5) and (6) is due to the use of scalar photons. For photons with spin unity, roughly speaking, $Ab_s\bar{f}(s, p^2, q^2) \leftrightarrow p^2q^2 Ab_s T(s, p^2, q^2)$, where $\bar{f}$ is the amplitude averaged over the polarizations. We note, however, that the simple approach described in this paper corresponds effectively to partons with integer spin, and this apparently explains the absence of a dependence on p^2/q^2 in the region $m^2 \ll s \ll |p^2| \ll |q^2|$. The use of partons with half-integer spin (see Figs. 3a and 3b. internal line - spinor field, external - photon with unity spin) leads to a dependence on p^2/q^2.

The behavior of 1b agrees with the calculation of the single-resonance contribution to Ab_sT. Since the matrix element for Ab_sT factors out in this case, it suffices for the calculation to use only an expansion of the product of two currents near the light cone [6]. In addition, since the use of only the expansion of the product of two currents suffices for the calculation of the asymptotic of a two-photon form factor of any resonance, the notion of the Regge trajectory as an aggregate of resonances [7] makes it possible to obtain the asymptotic of the Regge residue. The residue behaves in such a way, that the contributions of the Regge trajectories remain in the limit of large mass photons. In other words, the Regge trajectories make a contribution to the functions ϕ_a in (5) and ϕ in (6), and by the same token determine their asymptotic behavior:

$$\phi_a(x)|_{x \gg 1} \to \Sigma \beta_a x^a, \quad \phi(x, y)|_{x \gg 1} \to \Sigma \gamma_a(y) x^a. \tag{7}$$

An analogous result can be obtained also for two-Reggeon branch points.

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INTEGRAL EQUATION FOR THE PAIRED CORRELATION FUNCTION OF DENSE GASES AND A PLASMA

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The main task of the equilibrium statistical theory of dense gases and a plasma is the determination of the paired correlation function [1 - 3]. One of the possible methods of solving this problem reduces to a solution of approximate integral equations for the correlation function $g_2 = f_2 - 1$, where f_2 is the paired distribution function.