A common scale factor in the left-hand sides of (5) and (6) is due to the use of scalar photons. For photons with spin unity, roughly speaking, $Ab_s\bar{f}(s, p^2, q^2) \leftrightarrow p^2q^2Ab_sT(s, p^2, q^2)$, where $\bar{f}$ is the amplitude averaged over the polarizations. We note, however, that the simple approach described in this paper corresponds effectively to partons with integer spin, and this apparently explains the absence of a dependence on p^2/q^2 in the region $m^2 \ll s \ll |p^2| \ll |q^2|$. The use of partons with half-integer spin (see Figs. 3a and 3b. internal line - spinor field, external - photon with unity spin) leads to a dependence on p^2/q^2.

The behavior of 1b agrees with the calculation of the single-resonance contribution to Ab_sT. Since the matrix element for Ab_sT factors out in this case, it suffices for the calculation to use only an expansion of the product of two currents near the light cone [6]. In addition, since the use of only the expansion of the product of two currents suffices for the calculation of the asymptotic of a two-photon form factor of any resonance, the notion of the Regge trajectory as an aggregate of resonances [7] makes it possible to obtain the asymptotic of the Regge residue. The residue behaves in such a way, that the contributions of the Regge trajectories remain in the limit of large mass photons. In other words, the Regge trajectories make a contribution to the functions ϕ_a in (5) and ϕ in (6), and by the same token determine their asymptotic behavior:

$$\phi_a(x)|_{x \gg 1} \to \Sigma \beta_a x^a, \quad \phi(x, y)|_{x \gg 1} \to \Sigma \gamma_a(y) x^a. \tag{7}$$

An analogous result can be obtained also for two-Reggeon branch points.

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INTEGRAL EQUATION FOR THE PAIRED CORRELATION FUNCTION OF DENSE GASES AND A PLASMA

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The main task of the equilibrium statistical theory of dense gases and a plasma is the determination of the paired correlation function [1 - 3]. One of the possible methods of solving this problem reduces to a solution of approximate integral equations for the correlation function $g_2 = f_2 - 1$, where f_2 is the paired distribution function.

At the present time, three such equations are used, corresponding to different approximations [4, 5]. These are the equations of Kirkwood, Bogolyubov, and Born-Green, in which a superposition approximation, the Percus-Yevick equation, and the so-called equation of super-entangled chains, is used for the distribution function. Complex variants of the last two equations [4, 5] are also used.

For dense gases and liquids the Percus-Yevick equation is apparently most convenient. For a plasma, the best approximation is given by the equation of the super-entangled chains.

The method of justifying these approximations, based on a functional expansion in terms of a small inhomogeneity, is unfortunately too formal and obscures the physical nature of the assumptions made during the derivation of the equation.

In the present paper we propose other integral equations for the correlation functions of dense gases and a plasma, the physical meaning of which is more lucid.

We represent the equilibrium equation for the paired distribution function in the form

$$f_2(r_1, r_2) = e^{-(\phi_{12}/kT)} F(r_1, r_2); \frac{1}{V^2} \int f_2\, dr_1 dr_2 = 1.$$

We have introduced here the function

$$F = \frac{V^2}{Z} \int \exp\left[- \frac{\sum_{3 \leqslant j \leqslant N} (\phi_{1j} + \phi_{2j}) + \sum_{3 \leqslant i < j \leqslant N} \phi_{ij}}{kT} \right] dr_3 \ldots dr_N$$

Z is the statistical integral and V is the volume.

$F \to 1$ when the distances $|r_1 - r_j|$ and $|r_2 - r_j| \to \infty$ at all values of j, i.e., then the correlation of the particles 1 and 2 with the surrounding particles tends to zero.

We represent the function $F - 1$ in the form of a series in the correlation functions with increasing powers of the density. The series should have a structure such that the expression for f_2 would coincide with the corresponding virial expansion in the density when the function g_2 is replaced by its value in the zeroth approximation in the density ($g_2^0 = \exp(-\phi_{12}/kT) - 1$).

In the first approximation in the density we have

$$f_2(1, 2) \equiv g_2(1, 2) + 1 = e^{-(\phi_{12}/kT)} (1 + n \int g_2(1, 3) g_2(2, 3)\, dr_3). \tag{1}$$

When g_2 is replaced in (1) by g_2^0, the expression (1) for f_2 gives the first two terms of the expansion in the density, and consequently determines the first three terms in the virial expansion of the thermodynamic functions.

We write, for comparison, in the assumed notation, the Percus-Yevick equation

$$f_2(1, 2) = e^{-(\phi_{12}/kT)} \Big(1 + n \int e^{(\phi_{13}/kT)} f_2(1, 3) g_2^0(1, 3) g_2(2, 3)\, dr_3\Big). \tag{2}$$

Equation (2) goes over into (1) following the substitution $g_2^0(1, 3) \to g^2$,

$$f_2(1,3) \to f_2^{\circ} = \exp\left(-\frac{\phi_{13}}{kT}\right).$$

Equation (1) has a structure similar to (2) but is more symmetrical. The interpretation of (1) is simpler. It takes into account the correlations of the surrounding particles with the separated particles 1 and 2 in the self-consistent approximation in the correlation functions.

When writing down the corresponding equations for the plasma in the equation for g_{ab}, it is necessary to take into account the contribution of the dynamic polarization. It is shown in [6] that the averaged contribution of the dynamic polarization can be taken into account by introducing an effective potential $\tilde{\phi}_{ab}$ into the equation for g_{ab}. In the equilibrium state we have

$$\tilde{\phi}_{ab}(r) = \frac{e_a e_b}{r} e^{-r/r_d},$$

i.e., they coincide with the Debye potential.

The equation for g_{ab} then takes the form

$$f_{ab}(r_a, r_b) = e^{-(\tilde{\phi}_{ab}/kT)}\left(1 + \sum_c n_c \int g_{ac}(r_a, r_c) g_{bc}(r_b\, r_c)\, dr_c\right) \tag{3}$$

a, b, and c are the indices of the plasma components.

In the zeroth approximation in the density and at $\tilde{\phi}_{ab}/kT \ll 1$, expression (3) leads to a correct result also in the first approximation in the plasma parameter.

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