smaller the length of the instability development. Therefore in helium, where the breakdown field is smaller than in air by a factor 4 - 5 [5], the beam penetration length is larger. We note that the dielectric strength of the gas may be decreased by its heating by the powerful beam, which leads to an increase in the propagation length.

Formula (7) agrees also with measurements performed earlier by Link [1]. In these experiments, in air at atmospheric pressure, L is approximately 4 times larger than in our case, owing to the increase of γ and of the energy in the beam.

In conclusion, we note once more that a substantial fraction of the energy can be converted into heat in the phenomenon in question. It can therefore be used for a collective heating of a superdense plasma.

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SEPARABLE EXPANSION OF THREE-PARTICLE AMPLITUDES

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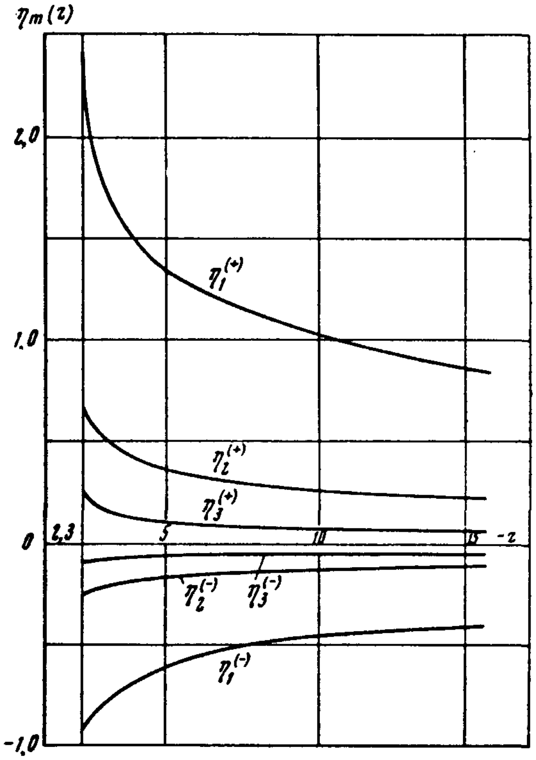
In the present article, using the simplest separable interaction as an example, we consider certain properties of the Hilbert-Schmidt (HS) expansion [1 - 3] for amplitudes in the three-body problem [4]. An analysis of an expansion of this type is very important both from the point of view of clarifying the analytic structure of the three-particle amplitudes, and in connection with the possibility of using the HS expansion to solve integral equations for four particles [5, 6].

Our result reduces to the following. For identical particles we can introduce two three-particle amplitudes X(z) and Y(z) [7] (z is the energy parameter), the first of which is connected with the amplitude of physical processes (elastic n-d scattering, disintegration of a deuteron) in a system of three particles. Since the equations for the amplitudes X(z) and Y(z) separate, it follows that Y(z) does not correspond to any physical quantity and is not used in practice in three-particle calculations. The amplitudes X(z) and Y(z) are in the general case $(N \times N)$ matrices, where N is the number of terms in the separable expansion of the two-particle T-matrix. Symbolically, the equation for X(z) can be written in the form

$$X(z) = V(z) - V(z)\Delta(z)X(z), \qquad (1)$$

where V(z) is the effective three-particle potential, and $\Delta(z)$ is a diagonal matrix describing the propagation of a free particle and of a pair of interacting ing particles. The equation for Y(z) differs from (1) in the substitution $V \to V/2$ and in the reversal of the sign of the second term. It follows from (1) that the HS expansion of the amplitude X(z) coincides formally with the HS expansion of the two-particle amplitude. Each term of this expansion contains



a factor $[1 - \eta_m(z)]^{-1}$, where $\eta_m(z)$ are the eigenvalues (EV) of the kernel $V(z)\Delta(z)$ of Eq. (1). This factor determines the positions of the poles z_m of the amplitude $X(z)$: $\eta_m(z_m) = 1$. Negative real poles lying in the region below the two-particle threshold correspond to bound states of the system of three particles. The HS expansion for the amplitude $Y(z)$ differs in that $[1 - \eta_m(z)]^{-1}$ is replaced by $[2 + \eta_m(z)]^{-1}$. As a result the $Y(z)$ have poles $\bar{z}_m$ whose positions are specified by the equation $\eta_m(\bar{z}_m) = -2$. As to the amplitudes $W_{\alpha\beta}(z)$, which determine the kernels of the integral equations for four particles, they represent linear combinations of the amplitudes $X(z)$ and $Y(z)$ and contain all the singularities of these amplitudes. In particular, the amplitudes $W_{\alpha\beta}(z)$ can have on the negative real axis poles that do not correspond to real bound states. From this it follows, in turn, that the kernels of the integral equations for four particles contain only a symmetrized combination of the three-particle amplitudes, since the amplitude $Y(z)$ leads in the four-particle problem to unphysical cuts not connected with real thresholds. To avoid misunderstandings, we emphasize that the foregoing pertains only to a system of identical particles.

We used in the calculations the simplest separable Yamaguchi model [8] with parameters

$$\beta_t = 1.450\ \mathrm{F}^{-1},\ \beta_s = 1.165\ \mathrm{F}^{-1},\ \lambda_t = 0.4156\ \mathrm{F}^{-3},\ \lambda_s = 0.149\ \mathrm{F}^{-3},$$
$$\alpha = \sqrt{m\epsilon_d} = 0.232\ \mathrm{F}^{-1},\ 1/m = 41.47\ \mathrm{MeV\text{-}F}^2\ . \qquad (2)$$

We have observed that in the energy region below the two-particle threshold there are two series of eigenvalues, one of which contains the positive eigenvalues $\eta_m^{(+)}(z)$ and the other negative eigenvalues $\eta_m^{(-)}(z)$. The negative ones become most clearly pronounced for the doublet state of the three-particle system (see the figure). This result indicates that the effective three-particle potential for the separable Yamaguchi interaction is not a positive-definite operator, i.e., it contains both attraction and repulsion, in spite of the fact that in the two-particle system the separable potential contains only attraction.

In the two-particle problem, the good convergence of the HS expansion is due to the fact that the first EV is larger in absolute magnitude than all the others[1]. The rate of decrease in the three-particle problem is approximately

[1] It follows from the HS expansion that only one term predominates near the pole of the amplitude. However, it is easy to verify that for shallow potentials, for which all the EV are small, the HS expansion converges well, as before, and the first HS pole makes the principal contribution to each term of the perturbation-theory series.

the same as in the two-particle problem. In the case the state with quantum numbers S = T = 1/2, however, a unique situation arises for the values of z lying near the two-particle threshold, owing to the proximity of the doublet scattering length to zero [9][2]. In the language of the HS expansion, the proximity of the quantity ${}^2a$ to zero denotes, first, that at the threshold we have $\eta_1 > 1$ and $\eta_2 < 1$, and therefore the first term of the HS expansion makes a positive contribution to the scattering length, and the second a negative one. A similar situation occurs also for triplet NN scattering, but in this case the deuteron pole is close to the cut, and the contribution of the first term to the scattering length turns out to be dominant. For the doublet state of the three-particle system, the pole is sufficiently far from the cut, and therefore the contributions of the first two terms in the HS expansion are comparable in magnitude and opposite in sign; as a result, their total contribution to the scattering length is close to zero. In particular, in the potential model under consideration we have $\eta_1 = 2.44$ and $\eta_2 = 0.68$, and the contribution of the first two terms to the scattering length is equal to 5.00 - 5.36 = -0.36 F. In this situation, the main contribution to the scattering length is made by the EV corresponding to remote poles (their summary contribution is <1 F); as a result, the convergence of the HS expansion becomes much worse. We emphasize that the described cancellation effect does not depend on the choice of the concrete potential model.

Convergence of HS expansion for the scattering length

m	1	2	3	4	5	6
$a_m^{*)}$	6.80	23.55	21.38	20.84	20.64	20.56
${}^2a_m^{(+)}$	5.00	-0.36	-1.07	-1.30	-1.39	–
${}^2a_m^{(-)}$	0.33	0.44	0.49	–	–	–
${}^4a_m^{*)}$	4.01	5.38	5.88	6.09	6.18	6.22

*In [8] are given the values a = 20.40 F and ${}^4a$ = 6.28 F, obtained by solving the inhomogeneous integral equation.

The table lists several of the first terms of the HS expansion of the n-d scattering length for the spinless case (with triplet NN potential), and also for the doublet and quartet states. The contribution of the first m terms to the HS expansion is denoted by a_m. For the doublet state, we give separately the contributions of the positive and negative EV. All the quantities in the table are given in Fermi units.

Taking the calculated terms into account, we get ${}^2a \simeq -1.39 + 0.49 = -0.9$ F. This value differs somewhat from the published value [10] ${}^2a = -1.00$ F. The difference can apparently be attributed to the noticeable sensitivity of ${}^2a$ to the choice of the constants λ_t and α. We have shown, for example, that a variation of λ_t by an amount $\sim\pm10^{-3}$ leads to a change of ${}^2a$ by approximately ∓0.05 F. Furthermore, the discarded terms make an additional negative contribution. A more detailed analysis of the properties of the HS expansion, including local potentials, and also concrete calculations in the four-particle system, will be given in a subsequent paper.

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CPT INVARIANCE OF CP-NONINVARIANT THEORY OF K^0 AND $\bar{K}^0$ MESONS AND PERMISSIBLE MASS DISTRIBUTIONS OF THE K_S AND K_L MESONS

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1. The usual phenomenological theory of K^0 and $\bar{K}^0$ mesons (see, e.g., [1, 2]) used in the discussion of the CP-invariance problem is based on the most general principles of quantum theory (superposition principle, unitarity) and on the essential additional assumption that the K_S and K_L mesons decay exponentially. This assumption, which is equivalent to the well-known Weiskopf-Wigner (W-W) approximation [3], states that the mass distributions of the K_S and K_L mesons have a single-pole character. Attention was called already earlier [4] to the large sensitivity of the results of the phenomenological theory (e.g., the unitarity relation, CPT tests, T-invariance) to the validity of this assumption. We shall show below that within the framework of the CT-noninvariant theory the CPT-invariance requirement forbids single-pole distributions of the K_S and K_L meson masses. Within the framework of the CP-invariance theory, on the other hand, a single-pole distribution of these masses is permissible.

2. We assume that the theory is CPT-invariant but CP-noninvariant, i.e.,

$$[CPT, H] = 0, \quad [CP, H] \neq 0, \tag{1}$$

where H is the total energy operator[1])

$$H = H_{St} + H_{\gamma} + H_{Wk} = H_0 + H_{Wk}. \tag{2}$$

Let $|K^0\rangle$ and $|\bar{K}^0\rangle$ be the normalized eigenvectors of the strangeness operator S and of the operator H_0 [1]:

$$\begin{gathered} S|K^0\rangle = 1|K^0\rangle, \quad S|\bar{K}^0\rangle = -1|\bar{K}^0\rangle, \quad H_0|K^0\rangle = m_{K^0}|K^0\rangle, \\ H_0|\bar{K}^0\rangle = m_{\bar{K}^0}|\bar{K}^0\rangle, \quad [S, H_0] = 0, \quad \langle K^0|K^0\rangle = 1 = \langle\bar{K}^0|\bar{K}^0\rangle, \\ \langle\bar{K}^0|K^0\rangle = 0. \end{gathered} \tag{3}$$

1) The notation is that of [1].