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CPT INVARIANCE OF CP-NONINVARIANT THEORY OF K^0 AND $\bar{K}^0$ MESONS AND PERMISSIBLE MASS DISTRIBUTIONS OF THE K_S AND K_L MESONS

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Submitted 15 March 1972
ZhETF Pis. Red. 15, No. 9, 548 - 552 (5 May 1972)

1. The usual phenomenological theory of K^0 and $\bar{K}^0$ mesons (see, e.g., [1, 2]) used in the discussion of the CP-invariance problem is based on the most general principles of quantum theory (superposition principle, unitarity) and on the essential additional assumption that the K_S and K_L mesons decay exponentially. This assumption, which is equivalent to the well-known Weiskopf-Wigner (W-W) approximation [3], states that the mass distributions of the K_S and K_L mesons have a single-pole character. Attention was called already earlier [4] to the large sensitivity of the results of the phenomenological theory (e.g., the unitarity relation, CPT tests, T-invariance) to the validity of this assumption. We shall show below that within the framework of the CT-noninvariant theory the CPT-invariance requirement forbids single-pole distributions of the K_S and K_L meson masses. Within the framework of the CP-invariance theory, on the other hand, a single-pole distribution of these masses is permissible.

2. We assume that the theory is CPT-invariant but CP-noninvariant, i.e.,

$$[CPT, H] = 0, \quad [CP, H] \neq 0, \tag{1}$$

where H is the total energy operator[1])

$$H = H_{St} + H_{\gamma} + H_{Wk} = H_0 + H_{Wk}. \tag{2}$$

Let $|K^0\rangle$ and $|\bar{K}^0\rangle$ be the normalized eigenvectors of the strangeness operator S and of the operator H_0 [1]:

$$\begin{gathered} S|K^0\rangle = 1|K^0\rangle, \quad S|\bar{K}^0\rangle = -1|\bar{K}^0\rangle, \quad H_0|K^0\rangle = m_{K^0}|K^0\rangle, \\ H_0|\bar{K}^0\rangle = m_{\bar{K}^0}|\bar{K}^0\rangle, \quad [S, H_0] = 0, \quad \langle K^0|K^0\rangle = 1 = \langle \bar{K}^0|\bar{K}^0\rangle, \\ \langle \bar{K}^0|K^0\rangle = 0. \end{gathered} \tag{3}$$

1) The notation is that of [1].

By virtue of the proposed CPT invariance ($[CPT, H_0] = 0$), we have $m_{K^0} = m_{\bar{K}^0}$. Under these assumptions [1] we get $|\bar{K}^0> = CPT|K^0>$. Let $\{|\phi(m)>\}$ be a complete system of orthonormal eigenvectors of the operator H:

$$H|\phi(m)> = m|\phi(m)>, \quad <\phi(m)|\phi(m')> = \delta(m - m'). \tag{4}$$

We expand the vectors $|K^0>$ and $|\bar{K}^0>$ in terms of this complete system:

$$|K^0> = \int c_{K^0}(m)|\phi(m)> dm, \quad |\bar{K}^0> = \int c_{\bar{K}^0}(m)|\phi(m)> dm. \tag{5}$$

Since $|\bar{K}^0> = CPT|K^0>$, it follows from (5) on the basis of the proposed CPT-invariance that

$$\omega_{K^0}(m) \equiv |c_{K^0}(m)|^2 = |c_{\bar{K}^0}(m)|^2 \equiv \omega_{\bar{K}^0}(m) \tag{6}$$

i.e., the mass distributions (in the sense of (5)) of the K^0 and $\bar{K}^0$ mesons are identical.

3. We define the vectors of the K_S and K_L mesons [1, 2]:

$$\begin{cases} |K_S> \equiv p|K^0> + q|\bar{K}^0>, \; |K_L> \equiv p|K^0> - q|\bar{K}^0>, \; |p|^2 + |q|^2 = 1 \\ <K_S|K_S> = 1, \; <K_L|K_L> = 1, \; <K_S|K_L> = <K_L|K_S> = |p|^2 - |q|^2 \end{cases} \tag{7}$$

From this definition and (5) we obtain

$$\begin{cases} c_S(m) = <\phi(m)|K_S> = p c_{K^0}(m) + q c_{\bar{K}^0}(m) \\ c_L(m) = <\phi(m)|K_L> = p c_{K^0}(m) - q c_{\bar{K}^0}(m) \end{cases}, \tag{8}$$

so that $\omega_S(m) = |c_S(m)|^2$ and $\omega_L(m) = |c_L(m)|^2$ are the mass distributions of the K_S and K_L mesons and determine, on the basis of the Fock-Krylov theorem [5], the decay amplitudes:

$$\begin{cases} <K_S|K_S(t)> = <K_S|\exp[-iHt]K_S> = p_S(t) = \int \exp[-imt]\,\omega_S(m)\,dm \\ <K_L|K_L(t)> = <K_L|\exp[-iHt]K_L> = p_L(t) = \int \exp[-imt]\,\omega_L(m)\,dm \end{cases} \tag{9}$$

From (8) we obtain

$$c_{K^0}(m) = (2p)^{-1}[c_S(m) + c_L(m)], \quad c_{\bar{K}^0}(m) = (2q)^{-1}[c_S(m) - c_L(m)]. \tag{10}$$

Substituting in (6), we obtain as a result of the CPT-invariance, within the framework of the CP-noninvariant theory,

$$<K_S|K_L>[|c_S(m)|^2 + |c_L(m)|^2] = [c_S^*(m)c_L(m) + c_S(m)c_L^*(m)], \tag{11}$$

which connects the mass distributions of the K_S and K_L mesons. The mass distributions $|c_S(m)|^2$ and $|c_L(m)|^2$, which satisfy (11), will be called admissible.

We note that the CPT-invariance condition is automatically satisfied within the framework of the CP-invariant theory, and imposes no limitations on the admissible distributions of K_1^0 and K_2^0 meson masses. Indeed, in this case $[CP, H] = 0$ and consequently

$$\begin{cases} H\,|\,\phi^{(1)}(m)> \, = m|\,\phi^{(1)}(m)>, \quad H\,|\,\phi^{(2)}(m)> \, = m|\,\phi^{(2)}(m)> \\ CP|\,\phi^{(1)}(m)> \, = \ |\,\phi^{(1)}(m)>, \quad CP|\,\phi^{(2)}(m)> \, = - \ |\,\phi^{(2)}(m)>, \\ \qquad\qquad < \phi^{(1)}(m)|\,\phi^{(2)}(m)> \, = 0 \end{cases} \tag{12}$$

and

$$\begin{cases} |\,K^{\circ}> \, = \int c^{(1)}_{K^{\circ}}(m)\,|\,\phi^{(1)}(m)> dm + \int c^{(2)}_{K^{\circ}}(m)\,|\,\phi^{(2)}(m)> dm \\ |\,\bar{K}^{\circ}> \, = \int c^{(1)}_{\bar{K}^{\circ}}(m)\,|\,\phi^{(1)}(m)> dm + \int c^{(2)}_{\bar{K}^{\circ}}(m)\,|\,\phi^{(2)}(m)> dm \end{cases} \tag{13}$$

On the other hand, by virtue of the CPT-invariance we obtain, just as in (6),

$$|c^{(1)}_{K^{\circ}}(m)|^2 = |c^{(1)}_{\bar{K}^{\circ}}(m)|^2, \quad |c^{(2)}_{K^{\circ}}(m)|^2 = |c^{(2)}_{\bar{K}^{\circ}}(m)|^2. \tag{14}$$

Since the eigenvectors $|K_1^0>$ and $|K_2^0>$ of the CP operator are defined as

$$\begin{cases} |K_1^{\circ}> = (1/\sqrt{2})(|K^{\circ}> + |\bar{K}^{\circ}>),\ |K_2^{\circ}> = (1/\sqrt{2})(|K^{\circ}> - |\bar{K}^{\circ}>),\ <K_1^{\circ}|K_2^{\circ}> = 0 \\ |K_1^{\circ}> = \int c_1^{(1)}(m)\,|\,\phi^{(1)}(m)> dm, \quad |K_2^{\circ}> = \int c_2^{(2)}(m)\,|\,\phi^{(2)}(m)> dm \end{cases} \tag{15}$$

we obtain, with allowance for (13)

$$\begin{cases} c^{(1)}_{K^{\circ}}(m) = (1/\sqrt{2})\,c_1^{(1)}(m), \quad c^{(2)}_{K^{\circ}}(m) = (1/\sqrt{2})\,c_2^{(2)}(m) \\ c^{(1)}_{\bar{K}^{\circ}}(m) = (1/\sqrt{2})\,c_1^{(1)}(m), \quad c^{(2)}_{\bar{K}^{\circ}}(m) = (-1/\sqrt{2})\,c_2^{(2)}(m) \end{cases} \tag{16}$$

and the CPT-invariance conditions (14) are satisfied for arbitrary mass distributions $\omega_1(m) = |c_1^{(1)}(m)|^2$ and $\omega_2(m) = |c_2^{(2)}(m)|^2$ of the K_1^0 and K_2^0 mesons. In particular, both single-pole distributions of the K_1^0 and K_2^0 masses (with W-W approximation) and two-pole distributions (the model of the induced poles [6]) are admissible.

4. We now formulate the main statement of this paper. Within the framework of a theory that is CPT-invariant but CP-noninvariant, single-pole distributions of the K_S and K_L meson masses are not admissible. More accurately, if

$$c_S(m) = \phi_S(m)/m - m_S + i\Gamma_S, \quad c_L(m) = \phi_L(m)/m - m_L + i\Gamma_L \tag{17}$$

$$m_S \neq m_L, \ \Gamma_S \neq \Gamma_L, \ \phi_S(m_S - i\Gamma_S) \neq 0, \ \phi_L(m_L - i\Gamma_L) \neq 0, \tag{18}$$

then the "preparatory" analytic functions $\phi_S(m)$ and $\phi_L(m)$, which have no complex singularities, are not admissible.

We shall prove this statement by induction. The CPT-invariance conditions (11) go over on the basis of (17) into

$$|\phi_L(m)|^2 - |\phi_L(m)|\,|\phi_S(m)| < K_S|K_L>^{-1}\Big\{\exp[i\arg\phi_S(m) - i\arg\phi_L(m)] \times$$
$$\times \frac{m - m_L + i\Gamma_L}{m - m_S + i\Gamma_S} + \exp[-i\arg\phi_S(m) + i\arg\phi_L(m)]\,\frac{m - m_L - i\Gamma_L}{m - m_S - i\Gamma_S}\Big\} + |\phi_S(m)|^2\,\frac{(m-m_L)^2 + \Gamma_L^2}{(m-m_S)^2 + \Gamma_S^2} = 0. \tag{19}$$

Taking (18) into account, we readily obtain from (11)

$$- \langle K_S | K_L \rangle | \phi_S(m_S - i\Gamma_S)|^2 / 2i\Gamma_S = |\phi_S(m_S - i\Gamma_S)| \, |\phi_L(m_S - i\Gamma_S)| \times$$
$$\times (m_S - m_L - i\Gamma_S + i\Gamma_L) \exp[i \arg \phi_S(m_S - i\Gamma_S) - i \arg \phi_L(m_S - i\Gamma_S)]. \tag{20}$$

Let us solve (19) with respect to $|\phi_L(m)|$:

$$|\phi_L(m)| = \frac{1}{2} \langle K_S | K_L \rangle^{-1} |\phi_S(m)| \{ \exp[i \arg \phi_S(m) - i \arg \phi_L(m)] \times$$
$$\times \frac{m - m_L + i\Gamma_L}{m - m_S + i\Gamma_S} + \exp[-i \arg \phi_S(m) + i \arg \phi_L(m)] \frac{m - m_L - i\Gamma_L}{m - m_S - i\Gamma_S} \} \pm$$
$$\pm \sqrt{\frac{1}{4} \langle K_S | K_L \rangle^{-2} |\phi_S(m)|^2 \{ \ldots \}^2 - |\phi_S(m)|^2 \frac{(m - m_L)^2 + \Gamma_L^2}{(m - m_S)^2 + \Gamma_S^2}} . \tag{21}$$

From this, taking (20) into account, we find that $\phi_L(m)$ has a singularity at $m = m_S - i\Gamma_S$. This contradiction completes the proof.

5. The inadmissibility of the single-pole K_S and K_L mass distributions means that the Weiskopf-Wigner approximation is incorrect within the framework of a CPT-invariant but CP-noninvariant theory. This limitation on the validity of the Weiskopf-Wigner method does not coincide with the known limitation imposed on this method by the non-exponential character of the decay laws [7, 8], due to the threshold behavior of $\omega_S(m)$ and $\omega_L(m)$, by virtue of their semi-finite nature (the spectrality principle). It is easy to note that the condition of semi-finiteness of $\omega_S(m)$ and $\omega_L(m)$ satisfies the CPT-invariance condition (11).

The non-admissibility of single-pole distributions of the K_S and K_L masses changes significantly the situation with the CP-invariance problem [1, 2], particularly the paradox with the $K_L \to 2\mu$ decay (see, e.g., [9]). The most critical test of the entire problem would be an investigation of the reaction

$$e^- + e^+ \to \phi \to K_S + K_L \tag{22}$$

as noted by the author earlier [10]. In particular, the non-single-pole character of the K_S and K_L mass distributions would produce in (22) interference in the channel of the decay into two pions, whereas this interference is strictly equal to zero in the ordinary phenomenological theory with violation of CP-invariance and with a single-pole distribution of the K_S and K_L masses.

A detailed investigtion of the CP-invariance problem in connection with the arrived-at conclusion that single-pole K_S and K_L mass distributions are inadmissible is planned for future articles.

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DISCRETE LEVELS IN RANDOM FIELD OF SOUND WAVES AND A NEW MECHANISM OF NON-LINEARITY OF THE SOUND AMPLIFICATION COEFFICIENT

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Submitted 18 March 1972
ZhETF Pis. Red. 15, No. 9, 553 - 555 (5 May 1972)

A number of authors have shown experimentally and theoretically that the forbidden band of a disordered semiconductor is "jammed" with discrete levels (see, e.g., the review article [1]). An investigation of the behavior of the carriers in a random field shows [2] that this circumstance is due not to some concrete singularities of the structure of glasses or liquids, but to the very existence of spatial fluctuations of the potential - regardless of their physical origin. In this sense, one should include among the disordered materials also those (perhaps even ideally crystalline ones) with sufficiently large Maxwellian relaxation times [3], and also substances in which a sufficiently intense low-frequency acoustic field with random phases of the component harmonics has been produced. Indeed, if the essential frequencies are low compared with the reciprocal values of the free path time and of the Maxwellian relaxation time, then the energy U of the interaction between the carriers and the acoustic field can be regarded as a static quantity; the random character of variation of U in space is ensured by the randomness of the phases. Such a formulation of the problem is meaningful under conditions where sound is amplified by a stream of electrons, when by virtue of the very structure of the absorption coefficient (see, e.g., the review [4]) only waves in a limited range of frequencies and wave vectors are effectively amplified[1]. We shall henceforth have in mind precisely such a group of waves with a central wave number q_0.

Obviously, the discrete fluctuation levels which arise in the random phase under consideration can play the role of ordinary traps (the influence of the latter on the sound amplification coefficient is considered in [5, 6]). This will be the situation if the corresponding time of electron capture turns out to be small compared with the time that the given group of waves stays in the crystal (this condition imposes a limitation only in the case of motion of acoustic phonons; it is necessary here also that the domain, as is usually the case, be of macroscopic dimensions over which self-averaging of the considered random quantities can occur). The difference compared with the ordinary traps consists, however, in the fact that in this case (a) the traps are produced by the noise itself, and their number is determined by the strength of the sound P

[1] We are referring, of course, to incoherent waves, i.e., to the amplification of noise.