2) The indicated non-equilibrium character of the distribution function can exist only because of the gap. This means in turn that the considered state is metastable and can be obtained by first illuminating an equilibrium superconductor, and then raising the temperature.

3) The calculation does not take into account the finite lifetime of the Cooper pairs. This account should decrease the value of the ordering parameter Δ, and it is possible that the superconductivity at $T > T_c$ will be gapless.

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[1] L.D. Landau and E.M. Lifshitz, Statisticheskaya fizika (Statistical Physics), Nauka, 1964, p. 300.
[2] A.G. Aronov and V.L. Gurevich, Fiz. Tverd. Tela 14, 1129 (1972) [Sov. Phys.-Solid State 14, No. 3 (1972)].
[3] G.M. Eliashberg, ZhETF Pis. Red. 11, 186 (1970) [JETP Lett. 11, 114 (1970)]; Zh. Eksp. Teor. Fiz. 61, 1254 (1971) [Sov. Phys.-JETP 34, 668 (1972)].

CONTRIBUTION TO ELECTRODYNAMIC THEORY OF VAN DER WAALS FORCES BETWEEN MACROSCOPIC BODIES

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Van der Waals forces between macroscopic bodies (e.g., between two plates - regions 1 and 2) can be calculated by an electrodynamic method using the fluctuation-dissipation theorem. The corresponding theory was initially developed (see [1, 2], Sec. 92) as applied to two half-spaces with parallel boundaries, separated by an empty gap (region 3). A generalization of the calculations to the case of a gap filled with a medium was realized, however, only by using the complicated formalism of the temperature Green's functions. At the same time, the results obtained in [3] for filling of a gap with an arbitrary medium were recently obtained [4, 5] by an incomparably simpler procedure. Namely, the free energy F or the internal energy U of the system (media 1, 2, and 3) are represented in the form of sums of contributions of harmonic oscillators with surface-oscillation (wave) frequencies ω_α, corresponding to the problem in question. For example, it is assumed that

$$U(\ell) = \sum_\alpha \phi(\omega_\alpha, T), \quad \phi(\omega_\alpha, T) = \frac{\hbar\omega_\alpha}{2} \operatorname{cth} \frac{\hbar\omega_\alpha}{2kT}, \tag{1}$$

where ℓ is the width of the gap 3, on which the frequencies ω_α depend, and the index α combines both the discrete variables and the wave vector k in the plane of the gap. Knowing the frequencies $\omega_\alpha(\ell)$, we obtain $U(\ell)$ or $F(\ell)$, and by differentiating with respect to ℓ we subsequently obtain the force $f(\ell)'$, which coincides with that obtained in [1 - 3]. In principle it is easy to generalize the method to the case of more complicated configurations; in the particular case of ideally conducting plates in vacuum, this method reduces to a procedure already long in use [7] (see also [8, 9]). Both in this limiting case, and in the more general case, when all the media 1, 2, and 3 are non-absorbing, the meaning of the expression (1) is obvious, and it is obtained formally by standard quantization of the field in the medium (see, e.g., [10]). The theory of

Van der Waals forces between bodies has been developed, however, for absorbing media, and allowance for absorption, generally speaking, is absolutely essential [1 - 3]. On the other hand, in the presence of absorption, the natural frequencies ω_α are complex, and expression (1) does not have explicitly the meaning of the internal energy. The latter is clear both formally and from a comparison with a RCL circuit, for which in the presence of a resistance $R \neq 0$ the average (internal) energy is far from equal to the circuit energy at $R = 0$ (see [11]). Therefore, the results of the calculations of [4 - 6] coincide with those obtained in [1 - 3] to a certain degree only formally, inasmuch as the permittivities ε_1, ε_2, and ε_3 of media 1, 2, and 3 enter in [1 - 3] in a general manner, and they can be regarded, in particular, as real; in addition, in the final expressions of [1 - 6] all the permittivities $\varepsilon_j(\omega)$ ($j = 1, 2, 3$) are taken on the imaginary frequency axis $\omega = i\xi$, where the permittivity is real. At the same time, the approach of [4 - 6] (see also [12 - 14]) is undoubtedly effective, and it is natural to attribute its success to the possibility of consistently extending it to the case of absorbing media. This is indeed the situation.

In fact, the problem reduces to finding the internal energy of an electromagnetic field $U(\ell)$ in an absorbing medium. In the general case, such a thermodynamic quantity does not exist at all, since it is impossible, by virtue of the energy conservation law, to separate uniquely the dissipative term (see, e.g., [2], Sec. 61 and [15], Sec. 3). However, in the state of thermal equilibrium, which we have here in mind [1 - 6], there is no dissipation at all (we refer here to statistical mean values, henceforth designated by angle brackets $\langle\ \rangle$), and the internal energy has a definite value also for an absorbing medium. This is particularly clearly seen, using as an example an RCL circuit described by the equation

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = K, \tag{2}$$

where $I = \dot{q}$ is the current and K is a fluctuating (random) emf.

In thermal equilibrium, obviously, $R\langle I^2\rangle = \langle KI\rangle$, and the internal energy is (for details see [11])

$$U = \left\langle \frac{q^2}{2C} \right\rangle + \left\langle \frac{LI^2}{2} \right\rangle = \frac{1}{\pi}\int_0^\infty \frac{RC\phi(\omega, T)\,d\omega}{(LC\omega^2 - 1)^2 + R^2C^2\omega^2} + \frac{1}{\pi}\int_0^\infty \frac{RLC^2\phi(\omega, T)\omega^2\,d\omega}{(LC\omega^2 - 1)^2 + R^2C^2\omega^2}; \tag{3}$$

The electromagnetic field in an arbitrary system can be regarded as situated in a certain auxiliary resonator surrounding the system, and as expanded in the natural modes of this resonator; the frequencies of such modes are $\omega_\alpha(\omega)$, where the real frequency ω enters as a parameter (see Secs. 100 - 102 of [16] and below). Application of such a procedure to an RCL circuit shows that in this case the frequency is

$$\omega_\alpha(\omega) \equiv \omega_1(\omega) = \sqrt{\frac{1}{LC} - i\frac{R}{L}\omega}, \tag{4}$$

where ω_1 is a solution of the free equation (2), in which we put $R(\omega_1) =$

$(\omega/\omega_1)R$, which indeed corresponds to introducing a corresponding auxiliary circuit (for details see [16]). With the aid of the frequency $\omega_1(\omega)$, expression (3) can be rewritten in the form

$$\begin{cases} W_E = \left\langle \dfrac{q^2}{2C} \right\rangle = -\dfrac{i}{2\pi}\displaystyle\int_{-\infty}^{+\infty} \dfrac{\phi(\omega,T)\omega\, d\omega}{\omega_1^2(\omega)-\omega^2} + \dfrac{i}{2\pi}\displaystyle\int_{-\infty}^{\infty} \dfrac{\phi(\omega,T)\dfrac{d\omega_1^2(\omega)}{d\omega}}{\omega_1^2(\omega)-\omega^2}\, d\omega \\ W_H = \left\langle \dfrac{L I^2}{2} \right\rangle = -\dfrac{i}{2\pi}\displaystyle\int_{-\infty}^{+\infty} \dfrac{\phi(\omega,T)\omega\, d\omega}{\omega_1^2(\omega)-\omega^2}, \quad U = W_E + W_H \end{cases} \qquad (5)$$

For a system of quite general form (with account taken of absorption, frequency, and spatial dispersion, anisotropy, smooth inhomogeneities, and the presence of boundaries), the natural frequencies $\omega_\alpha(\omega)$ enter in the field equations for the eigenfunctions (fields) $\vec{E}_{\omega_\alpha}$ and $\vec{H}_{\omega_\alpha}$ in the auxiliary resonator enclosing the system:

$$\begin{cases} \mathrm{rot}\, \mathbf{H}_{\omega_\alpha(\omega)}(\omega, r) = -\dfrac{i\omega_\alpha(\omega)}{c}\displaystyle\int \hat{\epsilon}(\omega, r, r')\mathbf{E}_{\omega_\alpha(\omega)}(\omega, r')\, dr', \\ \mathrm{rot}\, \mathbf{E}_{\omega_\alpha(\omega)}(\omega, r) = \dfrac{i\omega_\alpha(\omega)}{c}\mathbf{H}_{\omega_\alpha(\omega)}(\omega, r) \end{cases} \qquad (6)$$

where $\hat{\varepsilon}$ is a linear operator that degenerates into a scalar only for an isotropic medium without spatial dispersion. In the state of thermal equilibrium, as is clear from (5), the internal energy of the system is

$$U = -\frac{i}{\pi}\sum_\alpha \int_{-\infty}^{\infty} \frac{\phi(\omega,T)\omega\, d\omega}{\omega_\alpha^2(\omega)-\omega^2} + \frac{i}{2\pi}\sum_\alpha \int_{-\infty}^{\infty} \frac{\phi(\omega,T)}{\omega_\alpha^2(\omega)-\omega^2}\, \frac{d\omega_\alpha^2(\omega)}{d\omega}\, d\omega. \qquad (7)$$

Putting $\omega_\alpha(\omega) = \mathrm{Re}\, \omega_\alpha(\omega) - i\xi$, and taking the limit $\xi \to +0$, we arrive to expression (1) in the case of a transparent medium. This conclusion is obvious also from the well-known result for an RCL circuit (we have in mind expression (3) as $R \to 0$; see [11]).

From the internal energy U it is easy to change over to the free energy

$$F = -kT\int \rho(\beta)\, d\beta \sum_\alpha \left\{ \frac{i}{\pi}\int_{-\infty}^{\infty} \frac{\left(\omega - \dfrac{1}{2}\dfrac{d\omega_{\alpha\beta}^2(\omega)}{d\omega}\right)}{\omega_{\alpha\beta}^2(\omega)-\omega^2} \ln\left[2\,\mathrm{sh}\frac{\hbar\omega}{2kT}\right] d\omega + \int_{-\infty}^{\infty} \frac{\left(\omega - \dfrac{1}{2}\dfrac{d\omega_{\alpha\beta}^2(\omega)}{d\omega}\right)}{\omega_{\alpha\beta}^2(\omega)-\omega^2}\, d\omega \right\}. \qquad (8)$$

Here β stands for those indices α which are continuous, with a density of state $\rho(\beta)$. The dispersion equation that follows from (6) and determines the frequency $\omega_{\alpha\beta}(\omega)$ is written in the form

$$D_\beta(\omega', \omega) = 0. \qquad (9)$$

The index β is regarded here as a fixed parameter (the argument of the function D), and the values $\omega' = \omega_{\alpha\beta}(\omega)$ are the roots of (9).

For the real (but not auxiliary) problem, as is clear from (6), $\omega_{\alpha\beta} = \omega$ and the dispersion equation is $D_\beta(\omega, \omega) = D_\beta(\omega) = 0$. By a procedure related to that described in [5], using the well-known theorem concerning the properties of a logarithmic residue, we transform (8) into

$$F = kT \int \left\{ \sum_{n=0}^{\infty}{}' \ln D_\beta(\omega_n) \right\} \rho(\beta)\, d\beta, \quad \omega_n = i \frac{2\pi nkT}{\hbar}, \tag{10}$$

where the prime at the summation sign denotes that the term with $n = 0$ is taken with weight 1/2; it can be shown (see also [5, 14]) that expression (10) coincides with that obtained by E. Lifshitz for arbitrary complex $\varepsilon_1(\omega)$, $\varepsilon_2(\omega)$, and $\varepsilon_3(\omega)$ [1 - 3].

Thus, it seems to us that there is full justification for the use of the method of expansion in natural modes[1)] in the calculation of Van der Waals forces in media of a quite general class (we note that the question of macroscopic Van der Waals forces has been attracting much attention; see [1 - 9, 12 - 14, 17 - 24]). The obtained expressions for the energy of the electromagnetic field in an equilibrium absorbing medium is also of wide significance.

[1] E.M. Lifshitz, Zh. Eksp. Teor. Fiz. 29, 94 (1955) [Sov. Phys.-JETP 2, 73 (1956)].
[2] L.D. Landau and E.M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), GTTI, 1957 [Pergamon, 1959].
[3] I.E. Dzyaloshinskii, E.M. Lifshitz, and L.L. Pitaevskii, Usp. Fiz. Nauk 73, 381 (1961) [Sov. Phys.-Usp. 4, 153 (1961)]; A.A. Abrikosov, L.P. Gor'kov, and I.E. Dzyaloshinskii, Metody kvantovoi teorii polya v statisticheskoi fizike (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1965].
[4] N.G. van Kampen, B.R.A. Nijboer, and K. Schram, Phys. Lett. 26A, 307 (1968).
[5] B.W. Ninham, V.A. Parsegian, and G.H. Weiss, J. Statistical Physics. 2, 323 (1970).
[6] E. Garlach, Phys. Rev. B4, 393 (1971).
[7] H.B.G. Casimir, Proc. Kon. Ned. Akad. Wetenschap. 51, 793 (1948).
[8] T.H. Boyer, Ann. of Phys. 56, 474 (1970).
[9] W. Lukosz, Physica 56, 109 (1971).
[10] V.L. Ginzburg, Zh. Eksp. Teor. Fiz. 10, 589 (1940); Journal of Physics USSR 2, 441 (1940).
[11] V.L. Ginzburg, Usp. Fiz. Nauk 46, 348 (1952).
[12] B.W. Ninham and V.A. Parsegian, J. Chem. Phys. 52, 4578 (1970); 53, 3398 (1970).
[13] D.B. Chang, R.L. Cooper, J.E. Drummond, and A.C. Young, Phys. Lett. 37A, 311 (1971).
[14] B. Davies, Phys. Lett. 37A, 391 (1971).
[15] V.M. Agranovich and V.L. Ginzburg, Kristallooptika s uchetom prostranstvennoi dispersii i teorii eksitonov (Crystal Optics with Account of Spatial Dispersion and Exciton Theory), Nauka, 1965.
[16] L.A. Vainshtein, Elektromagnitnye volny (Electromagnetic Waves), Soviet Radio, 1957.
[17] A.D. McLachlan, Proc. Roy. Soc. 274, 80 (1963).

[1)] The relative simplicity and effectiveness of this method is due to the fact that its use requires knowledge of only the frequency $\omega_{\alpha\beta}(\omega)$ or even only of the initial dispersion equation $D_\beta(\omega)$.

[18] L.S. Brown and G.J. Maclay, Phys. Rev. 184, 1272 (1969).
[19] P.G. deGennes, C.R. Acad. Sc. Paris 271, B469 (1970).
[20] M.L. Levin and S.M. Rytov, Teoriya ravnovesnykh teplovykh fluktuatsii v elektrodinamike (Theory of Equilibrium Thermal Fluctuations in Electrodynamics), Nauka, 1967.
[21] M.J. Renne, Physica 53, 193 (1971); 56, 125 (1971).
[22] R.H. Winterton, Contemp. Phys. 11, 559 (1970); Usp. Fiz. Nauk 105, 307 (1971)
[23] E.I. Kats, Zh. Eksp. Teor. Fiz. 60, 1172 (1971) [Sov. Phys.-JETP 33, 634 (1971)].
[24] D. Langbein, J. Phys. Chem. Solids 32, 1657 (1971).

ENERGY SPECTRUM OF NEUTRONS OF TERNARY FISSION OF Cf^{252} NUCLEI

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So far there is only one reported measurement of the energy spectrum of the neutrons from ternary fission of U^{235} by thermal neutrons [1], which has shown that the average energy of the ternary-fission neutrons $\overline{E_t}$ is smaller than $\overline{E_b}$ by 8%. In the literature there are also data on the average number of neutrons $\overline{\nu_t}$ per act of ternary fission of Cf^{252} nuclei [2, 3], and also on the dependence of the agerage number of neutrons on the mass numbers of the fragments and their kinetic energy. According to the data of Adamov et al. [2], $\overline{\nu_t} = 2.83 \pm 0.67$, while the measurements of Nardi and Fraenkel [3] gave for this quantity values $\overline{\nu_t} = 3.11 \pm 0.06$.

We note that in the case of binary fission of Cf^{252} nuclei, the average number of neutrons is $\overline{\nu_b} = 3.787$ [4].

The energy spectra of the neutrons from binary fission of nuclei by thermal neutrons and from spontaneous fission of Cf^{252} nuclei have been investigated in sufficient detail.

All are well described by a Maxwellian distribution $N(E_n) \sim E_n^{-1/2} \exp(-E_n/T)$, where $N(E_n)$ is the number of neutrons with kinetic energy E_n in the laboratory frame and T is a parameter connected with the average neutron energy by the relation $\overline{E_n} = 3T/2$.

If it is assumed that in both cases the neutrons are emitted from fully accelerated fragments, then the very fact that the average number of neutrons is smaller in ternary fission indicates a smaller excitation energy of the fragments produced in this process, in comparison with the binary-fission fragments. Therefore the spectrum of the ternary-fission neutrons should be "softer" than for binary fission. The parameter T in the case of binary spontaneous fission of Cf^{252} nuclei, according to the data of [5, 6], is $T = 1.40 \pm 0.05$ [7, 8]. For the energy spectrum of the ternary-fission neutrons one should expect a smaller value of T.

We studied the energy spectrum of the neutrons from ternary fission of Cf^{252} nuclei for the purpose of verifying this assumption and obtaining additional information concerning the excitation energies of the fragments produced in this process.

The spectra of the Cf^{252} fission neutrons were measured by the time-of-flight method with a base of 0.5 m and a resolution ±2.5 nsec, determined by