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BOUND STATES OF ELECTRON, HOLE, AND PHONON IN A STRONG MAGNETIC FIELD

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Recently, in connection with a number of optical experiments [1, 2], interest has increased in the bound states of quasiparticles with long-wave optical phonons [3 - 5]. We show in the present paper that bound states of three particles (electron, hole, and phonon) with zero total momentum exist in a strong magnetic field H. When these states are formed the electron and hole are near the bottoms of the corresponding lower Landau bands; therefore the excitation energy of such a state lies above the threshold for the production of an electron-hole pair and a phonon:

$$\epsilon = \Delta E + \frac{1}{2}(\omega_{c1} + \omega_{c2}) + \omega_0 - W,$$

where ΔE is the width of the forbidden band, ω_{c1} and ω_{c2} are the cyclotron frequencies of the electron and of the hole, ω_0 is the phonon frequency, and W is the binding energy. The presence of such states should lead to a fine structure of the intrinsic-absorption phonon replicas.

The existence of bound states of three particles was proved under the following assumptions: 1) only an electron interacts with the phonons, and this interaction is weak (coupling constant $\alpha \ll 1$); 2) the Coulomb energy is less than the magnetic one in the sense of $L \equiv \ln(\omega_c/R) \gg 1$, where ω_c is the cyclotron frequency and R is the Rydberg energy with reduced mass m. If there are no other small parameters, i.e., $\omega_{c1} \sim \omega_{c2} \sim \omega_0$, and the electron-hole binding energy $W_c \sim RL^2$ [6] is of the same order as the electron-phonon binding energy $W_p \sim \alpha^2\omega_0$ [5], then the indicated inequalities ensure the existence of three-particle bound states, with $W \sim W_c \sim W_p$. Here $W \ll \omega_0$, so that the energy of the bound state is superimposed on the continuum of the dissociated states of the electron-hole pair; therefore the state can decay with vanishing of a

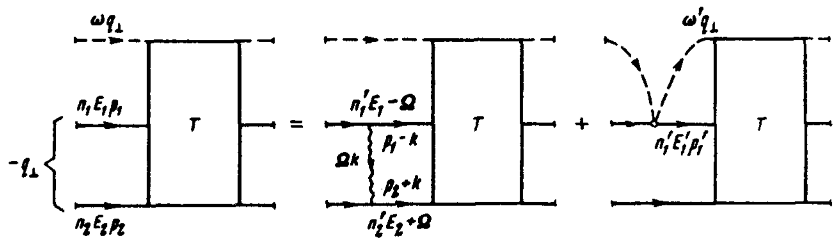

Fig. 1

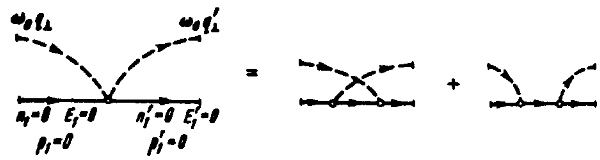

Fig. 2

phonon. It is important, however, that the corresponding width Γ is smaller than the binding energy: $\Gamma/W \sim (W/\omega_0)^{3/2}$, which indeed justifies the consideration of such bound states.

Since the interactions are weak, the dangerous cross sections in the diagrams of the three-particle T-matrix are those in which a real decay into three particles is possible. The essential diagrams are summed by the equation in Fig. 1, where the dashed line represents a phonon, the lower solid line a hole, the upper solid line an electron, the wavy line Coulomb interaction, and the point polaron interaction (Fig. 2). The inhomogeneous terms of the equation have been discarded, since they play no role in the determination of the spectrum. For the same reason, the quantum numbers of the input end of the T-matrix are not shown. n, E, and p with subscripts 1 and 2 denote the Landau quantum number, the energy parameter, and the longitudinal momentum of the electron and the hole. Instead of p_{1x} and p_{2x} we use in each cross section the total transverse momentum $-q_\perp$ of the pair [7]; this is explicitly indicated for the input ends. The integration with respect to Ω, E_1', and ω' can be carried out, after which an equation is obtained for the T-matrix, in which the phonon and hole ends are on the mass shell. When integrating in the dangerous cross sections, it is necessary to retain only the near-threshold regions of the variables [8], meaning that all the Landau quantum numbers n vanish and all the longitudinal momenta in the interaction matrix elements can be assumed equal to zero. As a result we obtain the equation

$$T(p_1p_2;\, q_\perp) = \int \frac{dk}{2\pi} V(-q_\perp) G(\epsilon - \omega_0,\, p_1 - k,\, p_2 + k) T(p_1 - k,\, p_2 + k;\, q_\perp) + \int \frac{dq_\perp'}{(2\pi)^2} \int \frac{dp_1'}{2\pi} U(q_\perp,\, q_\perp') G(\epsilon - \omega_0,\, p_1',\, p_2) T(p_1' p_2;\, q_\perp'). \tag{1}$$

Here $V(-q_\perp)$ is the zeroth Fourier component of the Coulomb potential averaged over the transverse motion, at a transverse pair momentum $-q_\perp$. The kernel $U(q_\perp, q_\perp')$ of the polaron interaction is shown in Fig. 2. Furthermore.

$$G(\epsilon,\, p_1,\, p_2) = \left(\epsilon - \frac{p_1^2}{2m_1} - \frac{p_2^2}{2m_2} + i0\right)^{-1}, \tag{2}$$

where m_1 and m_2 are the effective masses and $\varepsilon = E_1 + E_2$ is reckoned from the threshold of production of a free pair without a phonon.

Further progress is possible because V is independent of $q_\perp$ if L >> 1. Therefore the dependence of T on $q_\perp$ is determined by the eigenfunctions $\chi(q_\perp)$ of the polaron kernel, and the solution can be sought in the form

$$T(p_1p_2,\, q_\perp) = T_{\parallel}(p_1p_2)\, \chi(q_\perp). \tag{3}$$

If we put $GT_{\parallel} = \psi$ and change over to the coordinate z-representation with respect to p_1 and p_2, then we obtain the Schrodinger equation

$$\left[\frac{1}{2m_1}\frac{\partial^2}{\partial z_1^2}+\frac{1}{2m_2}\frac{\partial^2}{\partial z_2^2}+\epsilon-\omega_0-U\delta(z_1)-V\delta(z_1-z_2)\right]\psi(z_1z_2)=0, \quad (4)$$

where U is the polaron-kernel eigenvalue corresponding to χ. The immobile center with which the electron interacts is a phonon with infinite mass.

If V = 0, then bound states of the electron and phonon exist; this means that there exist χ such that U < 0 [9]. Thus, each bound state of the electron and phonon corresponds to bound states of three particles, which are determined by solving (4). At least one such state always exists.

The energies obtained in the solution of (4) do not contain damping. This is due to the fact that the decay of the bound state with vanishing of a phonon occurs in nuclear-hole-pair states that are far from the pair-production threshold; yet such states are not taken into account in (1). If they are taken into account by perturbation theory [10], then we obtain the estimate given above.

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THEORY OF BOUND STATES OF PHONONS WITH IMPURITY CENTERS AND EXCITONS

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Kogan and Suris [1] have shown that when the phonon frequency ω_0 is at resonance with one of the electronic frequencies of the local centers, local oscillations of a new type are produced. They were soon observed experimentally [2] and called dielectric modes. They were observed also in centers in which there was no resonance [3, 4].

A general theory of dielectric modes for $\alpha << 1$, where α is the electron-phonon constant, is proposed here. It is shown that an infinite number of such modes always exists if phonon dispersion is neglected. In the absence of resonances they represent bound states of a phonon near an impurity center.

For states whose energy is close to the phonon emission threshold, we can confine ourselves in the mass operator M to the diagrams of Fig. 1, in which all the single-phonon cross sections are dangerous [5]; in connection with the problem of bound states, they were already considered in [6, 7]. Here s and t are the indices of the Coulomb states, and 0 is the ground state. Writing down