$$\left[\frac{1}{2m_1}\frac{\partial^2}{\partial z_1^2}+\frac{1}{2m_2}\frac{\partial^2}{\partial z_2^2}+\epsilon-\omega_0-U\delta(z_1)-V\delta(z_1-z_2)\right]\psi(z_1z_2)=0, \tag{4}$$

where U is the polaron-kernel eigenvalue corresponding to χ. The immobile center with which the electron interacts is a phonon with infinite mass.

If V = 0, then bound states of the electron and phonon exist; this means that there exist χ such that U < 0 [9]. Thus, each bound state of the electron and phonon corresponds to bound states of three particles, which are determined by solving (4). At least one such state always exists.

The energies obtained in the solution of (4) do not contain damping. This is due to the fact that the decay of the bound state with vanishing of a phonon occurs in nuclear-hole-pair states that are far from the pair-production threshold; yet such states are not taken into account in (1). If they are taken into account by perturbation theory [10], then we obtain the estimate given above.

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THEORY OF BOUND STATES OF PHONONS WITH IMPURITY CENTERS AND EXCITONS

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Kogan and Suris [1] have shown that when the phonon frequency ω_0 is at resonance with one of the electronic frequencies of the local centers, local oscillations of a new type are produced. They were soon observed experimentally [2] and called dielectric modes. They were observed also in centers in which there was no resonance [3, 4].

A general theory of dielectric modes for $\alpha \ll 1$, where α is the electron-phonon constant, is proposed here. It is shown that an infinite number of such modes always exists if phonon dispersion is neglected. In the absence of resonances they represent bound states of a phonon near an impurity center.

For states whose energy is close to the phonon emission threshold, we can confine ourselves in the mass operator M to the diagrams of Fig. 1, in which all the single-phonon cross sections are dangerous [5]; in connection with the problem of bound states, they were already considered in [6, 7]. Here s and t are the indices of the Coulomb states, and 0 is the ground state. Writing down

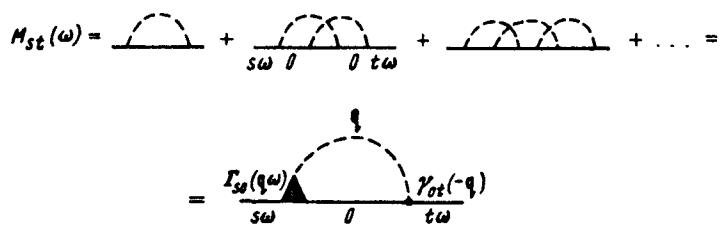

Fig. 1

the corresponding equation for Γ and transforming it in accordance with the figure into an equation for the matrix M, we obtain

$$M^{-1}(\omega) = (\omega - \omega_0 - \epsilon_0) A^{-1} - G^0(\omega - 2\omega_0). \tag{1}$$

Here

$$A_{st} = \sum_q \gamma_{s0}(q)\gamma_{0t}(-q), \quad \gamma_{st}(q) = C_q(s|e^{iqr}|t),$$
$$G^0_{st}(\omega) = (\omega - \epsilon_s)^{-1}\delta_{st}, \tag{2}$$

ϵ_s are the energies of the electronic levels, and C_q are the coefficients in the Hamiltonian of the electron-phonon interaction.

It can be shown that the equation for the poles $G(\omega)$

$$|M^{-1}_{st}(\omega) - G^0_{st}(\omega)| = 0 \tag{3}$$

leads in conjunction with (1) to the secular equation

$$\left| A_{st} - \delta_{st}\lambda(\lambda + \Omega_s - \omega_0)\frac{\omega_0 + \Omega_s}{2\Omega_s} \right| = 0, \quad s, t = 1, 2, \ldots \infty, \tag{4}$$

where $\Omega_s = \epsilon_s - \epsilon_0$, and $\lambda = \epsilon_0 + \omega_0 - \omega$ is the binding energy. At resonance, $\Omega_s \simeq \omega_0$ gives rise to a pair of roots $\lambda \pm \sqrt{A_{ss}}$ [1] and an infinite number of roots with smaller $|\lambda|$.

In the nonresonant case we have

$$\left| A_{st} - \delta_{st}\lambda\frac{\Omega_s^2 - \omega_0^2}{2\Omega_s} \right| = 0, \quad s, t = 1, 2, \ldots \infty \tag{5}$$

The matrix A is positive definite, since it has the structure $A = \gamma\gamma^+$. If all $\Omega_s > \omega_0$, then in accordance with the Sylvester criterion all the eigenvalues λ_k are positive, meaning that all the frequencies of the dielectric modes are lower than ω_0. There exist modes corresponding to all values of the angular momentum; we emphasize that in this case each value of the angular momentum corresponds to an infinite number of frequencies. If certain $\Omega_s < \omega_0$, then an equal number of roots $\lambda_k < 0$ arise. All the frequencies correspond to bound states of a phonon at the impurity center; the single-phonon term predominates in the corresponding wave functions.

Using (2) and (5), we can easily determine the scale of λ_k; for polarization phonons we have

$$\lambda_k = b_k \alpha\omega_0 (\omega_0/R)^{1/2} = b_k \omega_0 \left(\frac{\kappa_0}{\kappa_\infty} - 1\right), \quad b_k \sim 1, \tag{6}$$

κ is the dielectric constant and R is the ionization potential of the center. There is always an infinite number of roots of this order of magnitude. At resonance, as follows from (4), there appears also a pair of large roots $|\lambda| \sim \sqrt{\alpha}\omega_0$; they correspond to hybrid states in which the number of phonons is $\sim 1/2$.

For the calculation of b_k it is convenient to change over to the configuration representation, writing down the Schrodinger equation corresponding to (5):

$$\lambda[(H - \epsilon_0)^2 - \omega_0^2]\psi = 2A(H - \epsilon_0)\psi, \tag{7}$$

where $H(\vec{r})$ is the Hamiltonian of the electron at the center without interaction with the phonons and

$$A(r, r') = \psi_0(r) V(r - r') \psi_0(r'), \quad V(r) = \sum_q C_q^2 e^{iqr}, \tag{8}$$

ψ_0 is the Coulomb ls-function. The kernel A is a unique potential that depends on the electron-phonon interaction via $V(\vec{r})$ and on the Coulomb interaction via ψ_0. $V(\vec{r}) \propto 1/r$ for polarization interaction and $V(r) \propto \delta(r)$ for non-polarization interaction. In the latter case, the integro-differential equation reduces to a differential one.

Using (7), we can find λ by a variational method; with a one-parameter approximation for the first p-level at $R \gg \omega_0$ we obtain a lower bound $b_{2p} \simeq 0.15$, which is three times larger than in the two-level scheme [3, 4].

Thus, electron-phonon interactions give rise to an aggregate of non-resonant levels for the phonon; they have already been observed in [3, 4]. According to (6), at $R \gg \omega_0$ and at fixed κ_0/κ, their binding energy does not depend on R, i.e., the bound states should arise to an equal degree also for deep centers. The acceptor centers in CdS with $R \simeq 4\omega_0$, on which bound states were observed in [4], occupy in this respect an intermediate position. It is of particular interest to separate a sequence of levels with identical symmetry, something not accomplished as yet. Analogous levels should arise also when the center is in an excited electronic state.

An exciton with a large particle mass ratio (e.g., $m_h \gg m_e$) is analogous to an impurity center; bound states of an exciton and phonon should therefore arise. Their formation is hindered by the recoil energy of the exciton, which is of the order of Rm_e/m_h. It's smallness in comparison with the binding energy (6)

$$\frac{m_e}{m_h} \lesssim 0{,}1\left(\frac{\kappa_0}{\kappa_\infty} - 1\right)\frac{\omega_0}{R} \tag{9}$$

is indeed the condition for the formation of bound states. Inasmuch as the numerical factor remains undetermined here, and can be obtained only from a consistent theory, it is difficult to speak of satisfaction of (9) for concrete substances; on the whole, however, this criterion is not too rigorous. Resonant bound states analogous to those of [1] were considered for excitons by a variational method in [8]. The nonresonant bound states considered here could

not be obtained there, since no account was taken in [8] of the two-phonon state, which must be present in the diagrams of the figure.

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CONCERNING THE FEASIBILITY OF A γ LASER BASED ON RADIOACTIVE CRYSTALS

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The feasibility of a γ laser based on Mossbauer radiation without transfer of energy to the nuclei in crystals has been discussed in the literature many times [1 - 7]. This discussion has revealed serious difficulties that stand in the path of realization of a γ laser, and consequently, in so far as the author knows, no serious attempts at its realization have been made so far.

The present article is an extension of [1 - 7] and shows that in principle it is possible to produce a γ laser based on crystals consisting of long-lived nuclear isomers.

When a resonant γ quantum is incident on an excited nucleus, the effective cross section σ of the stimulated emission by the nucleus of a new γ quantum coherent with the incident one is described by the Breit-Wigner formula [1 - 3]

$$\sigma = \frac{\lambda^2}{2\pi} \frac{1+2I_2}{1+2I_1} f \frac{1}{\Gamma\tau} \frac{1}{1+\alpha}, \qquad (1)$$

where λ is the radiation wavelength, I_2 and I_1 are the spins of the upper and lower states of the nucleus, f is the probability of emitting a γ quantum without transferring energy to the nucleus, Γ is the width of the emission length, τ is the lifetime of the excited state, and α is the internal-conversion coefficient.

The gain β of the radiation is equal to

$$\beta = \sigma N, \qquad (2)$$

where N is the difference between the number of excited nuclei and the nuclei in the lower working level.

Let us estimate the factors that determine the gain.