$$\frac{\partial \ln H}{\partial r} \approx -4\pi\left\{\frac{3}{2}<r>+14\frac{<r^2>}{<r>}\right\}\frac{Bh^2}{cT^2}\frac{L}{4\pi r^2}, \quad (6)$$

where L is the luminosity. Substituting τ, we get

$$\frac{\partial \ln H}{\partial r} \approx -\frac{3\pi\cdot 10^4}{2\lambda}\frac{\hbar^2}{m^{5/2}c^4T^{1/2}}\frac{L}{4\pi r^2},$$

where λ is the Coulomb logarithm. Assuming that $T \approx T_0(r/r_1)^{-\alpha}$, where T_0 is the temperature of the internal convection region, in ergs, and $\alpha > 2$, we find that the magnetic field increases in the interior of the star in accordance with the law

$$H \sim \exp\left\{-\left(\frac{r}{\delta}\right)^{\frac{\alpha}{2}-1}\right\},$$

where the characteristic length δ is given by

$$\delta \approx \left(\frac{\alpha}{2}-1\right)\left[\frac{2\cdot 10^{-4}\lambda}{3\pi}\frac{m^{5/2}c^4T_0^{1/2}}{\hbar^2}\frac{4\pi r_1}{L}\right]^{\frac{2}{\alpha-2}} r_1 .$$

Assuming that near r_1 we have $\alpha \approx 4$ and $T_0 \approx 5\times 10^{-11}$ erg (values typical of the sun), we obtain the estimate $\delta \approx 10^{-1}R$, where R is the radius of the sun. For stars hotter than the sun, the characteristic length is even smaller.

This enhancement of the magnetic field is produced by currents that flow along meridians and return through the star's rotation axis. These currents produce their own magnetic field, having also a toroidal structure and coinciding in direction with the magnetic field produced by the convection. Its maximum lies approximately near the center of the meridian-axis current contour, as a result of which the main field is also enhanced in the interior of the star up to this line, beyond which the enhancement decreases.

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FLUCTUATION SPECTRUM IN SUPERCONDUCTING POINT JUNCTIONS

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1. The question of fluctuations in superconducting point junctions in which the Josephson effect takes place is of great interest, since in most applications, the limiting characteristics are determined precisely by the fluctuations. It is important here to determine the fluctuation spectrum, particularly the values of the spectral density at low frequencies and the Josephson-generation

line shape, all the more since it is just these quantities that are easiest to measure experimentally [1].

A feature of point junctions (and of arbitrary S-c-S Josephson structures) is the relative smallness of the intrinsic capacitance. This circumstance makes it impossible[1] to apply to them directly the well-known theory of fluctuations in tunnel junctions [2], in which the changes in the voltage across the junction are assumed to be small in the absence of fluctuations. For an autonomous junction we can, to the contrary, assume also the total current I to be constant.

The purpose of the present paper is to find the spectral density of the fluctuations of the voltage V(t) across a point junction (or a bridge of small dimensions), for which the model developed by Aslamazov and Larkin (AL) is valid [3].

2. If all the dimensions of the superconducting electrons forming the contact are large compared with the coherence length, then the intrinsic fluctuations of the ordering parameter in the electrodes are exceedingly small at $T < T_c$. Therefore the source of the fluctuations is the noise of the normal resistance R of the junction [2, 4]. The Langevin equation for the phase difference ϕ on the junction is written as follows [2 - 4]:

$$\dot{\phi} + \sin\phi = i + \tilde{i}, \quad \overline{\tilde{i}} = 0, \tag{1}$$

where i and $\tilde{i}$ are the displacement and fluctuation currents, normalized to the critical current I_0, and the differentiation is carried out with respect to the time $\tau = \omega_0 t$, where $\omega_0 = (2e/\hbar)I_0R$ is the characteristic frequency of the junction.

3. The determination of the spectral density of the voltage $S_v(\omega)$ for equation (1) is in the general case more complicated than the determination of the mean values [4], even if $\tilde{i}$ is white noise. However, if $\tilde{i}$ is sufficiently small, ordinary correlation theory can be applied to Eq. (1). In the first approximation in $\tilde{i}$ we have

$$\dot{\tilde{\phi}} + \tilde{\phi}\cos\phi^{(0)} = \tilde{i}, \tag{2}$$

where $\tilde{\phi} = \phi - \phi^{(0)}$ and $\phi^{(0)}$ is the solution of Eq. (1) at $\tilde{i} = 0$ [3]. At $|i| < 1$ (the stationary Josephson effect) we have $\cos\phi^{(0)} = (1 - i^2)^{1/2}$ and changing over to the Fourier transforms i_Ω and v_Ω of the current $\tilde{i}$ and of the voltage $\tilde{v} = \dot{\tilde{\phi}} = V/I_0R$ $(\Omega = \omega/\omega_0)$ we obtain immediately

$$v_\Omega = i_\Omega[1 - i(1 - i^2)^{1/2}/\Omega]^{-1}, \tag{3}$$

whence

$$S_v(\Omega) = [1 + (1 - i^2)/\Omega^2]^{-2} S_i(\Omega). \tag{4}$$

With our normalization for the normal resistance R ($I_0 = 0$) we would have $S_v(\Omega) \equiv S_i(\Omega)$. Therefore formula (4) describes the suppression of the noise in the junction on going over to the superconducting state, $S_v(0) = 0$. This suppression does not come into play at frequencies $\omega \gg \omega_0$.

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[1] If the point junction is not connected in a special low-resistance electrodynamic system.

At $i > 1$ (process of Josephson generation), by solving (2) relative to $\tilde{v}$, we obtain

$$\tilde{v} = \tilde{i} + \ddot{\phi}^{(0)} \int (\tilde{i}/\dot{\phi}^{(0)})\,d\tau. \tag{5}$$

Changing over to Fourier transforms, we easily obtain

$$v_\Omega = \sum_k z_k i_{\Omega - kv}, \tag{6}$$

where z_k are the coefficients of transformation of the fluctuation frequency as a result of their mixing with the Josephson oscillations (normalized frequency), $v = \bar{V}/I_0R = (i^2 - 1)^{1/2}$

$$|z_k| = \left|\delta_{k,0} + ik\frac{(i-v)^{|k|}}{\Omega - kv} - \frac{1}{2}\left\{\frac{(k-1)(i-v)^{|k-1|}}{\Omega - (k-1)v} + \frac{(k+1)(i-v)^{|k+1|}}{\Omega - (k+1)v}\right\}\right|. \tag{7}$$

From (6) we obtain for the spectral density of the voltage

$$S_v(\Omega) = \sum_k |z_k|^2 S_i(\Omega - kv). \tag{8}$$

4. From (7) and (8) we have for low frequencies $\Omega \ll v$

$$S_v(0) = [i^2 S_i(0) + (1/2) S_i(v)] v^{-2}. \tag{9}$$

For frequencies near nv ($|\Omega - nv| \ll v$) we have

$$S_v(\Omega) = n^2 (i - v)^{2n} [i^2 S_i(0) + (1/2) S_i(v)] (\Omega - nv)^{-2}. \tag{10}$$

This expression gives the form of the lower part of the Lorentz curve of the n-th harmonic of the Josephson generation. From this expression we can find the width $2\Gamma_n$ of this line under the condition that $\Gamma_n/\omega_0 \ll v$.[2] To this end, writing down the general formula for a Lorentz line

$$S_v(\Omega) = (2\pi)^{-1} P_n (\Gamma_n/\omega_0)[(\Omega - nv)^2 + (\Gamma_n/\omega_0)^2]^{-1}, \tag{11}$$

we equate the total "radiation power" at the n-th harmonic

$$P_n = 2\int_{nv-\epsilon}^{nv+\epsilon} S_v(\Omega)\,d\Omega, \quad \Gamma_n/\omega_0 \ll \epsilon \ll v, \tag{12}$$

to its value $P_n = 2v^2(i - v)^{2n}$ in the absence of fluctuations [3]. Then, comparing formulas (10) and (11) at $|\Omega - nv| \gg \Gamma_n$, we obtain:

$$\Gamma_n = n^2 \pi \omega_0 [i^2 S_i(0) + (1/2) S_i(v)] v^{-2} = n^2 \Gamma_1. \tag{13}$$

Inasmuch as in the first approximation in $\tilde{i}$ the fluctuations do not change the value of v, the differential dc resistance of the junction is $R_d \equiv d\bar{V}/dI = Ri/v$. We can therefore write

$$\Gamma_1/\pi\omega_0 = S_v(0) = [R_d^2/R^2] S_i(0) + (1/2v^2) S_i(v). \tag{14}$$

[2] It is this condition which determines the limiting fluctuation intensity at which the conclusions of the present paper are valid in the region $\Omega \simeq nv$.

5. In the case of low frequencies and voltages ($\hbar\omega$, eV << T), the noise of the resistance R can be regarded as thermal

$$S_I(\omega) \equiv \omega_0 R^{-2}(2e/\hbar)^{-2} S_i(\Omega) = (\pi R)^{-1} T. \tag{15}$$

When v >> 1, formulas (14) - (15) yield the usual expression [2]

$$\Gamma_1 = (2e/\hbar)^2 RT, \tag{16}$$

but for low generation frequencies ($v \to 0$) the voltage fluctuations and the generation line width increase sharply[3])

$$\Gamma_1 = \pi\omega_0 S_v(0) = (3/2)(2e/\hbar)^2(R_d^2/R)T. \tag{17}$$

6. If, as assumed in the AL model [3], the electron mean free path ℓ in the region of the junction is much smaller than its characteristic dimension a, then the spectral current density is given by the usual equilibrium formula

$$S_I(\omega) = (\pi R)^{-1}(\hbar\omega/2)\,\mathrm{cth}\,(\hbar\omega/2T) \tag{18}$$

even when eV > T. Indeed, the shot noise [2] begins to make a noticeable contribution only when the energy $eE\ell$ connected with a single scattering act is comparable with the temperature T ($E \simeq V/a$). Formula (18) can therefore be used for all values $eV \lesssim T(a/\ell) >> T$. It is seen from (14) and (18) that for high frequencies ($\bar{v}$ >> 1) the line width will be determined by expression (16) even in the "quantum" case $e\bar{V} > T$.

7. A comparison of the present results with the experimental data known to us [1] does not permit a final conclusion concerning their relation. It would be desirable to carry out analogous experiments under conditions when I_0R and T/e differ sufficiently ($I_0R \simeq T/e$ in most experiments), and the junctions are not shunted by the bias source and by the measuring system.

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3) In tunnel junctions, when self-detection is taken into account, we also have Γ_1, $S(0) \sim R_d^2$ [2], but the dependence on the current is essentially different.