DESCRIPTION OF TEMPORAL EVOLUTION OF THE $K^0\bar{K}^0$ SYSTEM ON THE BASIS OF THE RELAXATION EQUATION FOR THE DENSITY MATRIX

M.S. Marinov
Institute of Theoretical and Experimental Physics
Submitted 17 April 1972
ZhETF Pis. Red. 15, No. 11, 677 - 680 (5 June 1972)

The time evolution of a beam of neutral K mesons is usually described [1] by the Schrodinger equation for a two-component wave function ψ_α($\psi^*_\alpha\psi_\alpha$ = N is the total number of particles)

$$i\frac{d\psi}{dt} = H\psi \equiv (M - i\Gamma/2)\psi. \tag{1}$$

Here $H^{\alpha'}_{\alpha}$ is a second-order matrix, which can be expanded in Pauli matrices

$$H = h_0 + h_\ell\sigma_\ell, \quad \ell = 1, 2, 3, \tag{2}$$

h_λ are complex numbers. We choose a representation in which Y = σ_3 (hypercharge). CP = σ_1 (combined inversion). If CP parity is conserved, then h_2 = h_3 = 0, and in the case of CPT symmetry h_3 = 0. All the observed quantities are expressed in terms of the density matrix

$$\rho^{a'}_a = \psi_a \otimes \psi^{*}_{a'}, \quad \mathrm{Sp}\,\rho = N, \tag{3}$$

which can be represented in the form

$$\rho = \frac{1}{2}N(1 + b_\ell\sigma_\ell). \tag{4}$$

For pure states (3) we have $\sum b^2_\ell$ = 1. By virtue of Eq. (1), the density matrix satisfies the equation

$$i\frac{d\rho}{dt} = H\rho - \rho H^+, \tag{5}$$

the solution of which is $\rho(t) = \exp(-iHt)\rho(0)\exp(iH^+t)$.

Let us assume that weak interactions lead to an equation more general than (5):

$$i\frac{d\rho}{dt} = \mathcal{H}\rho, \tag{6}$$

where $\mathcal{H}$ is a linear operator acting in the space of the matrices ρ($\mathcal{H}$ is a matrix with two pair of indices, a tetrad). Equation (6) is linear in ρ and therefore does not contradict the superposition principle. The solution of (6) can be written in matrix form

$$\rho(t) = W(t)\rho(0), \tag{7}$$

where W is not a direct product of two matrices. Application of Eq. (6) to the matrix ρ instead of (5) can cause the pure state to become mixed in the course of time. This is formally expressed in the fact that the quantity $\sum b^2_\ell$ is not conserved. Naturally, the operator $\mathcal{H}$ must be such as not to violate the main properties of ρ as a density matrix, viz.,

A. Hermitian property, $\rho^+ = \rho$.

B. Non-negativity, Tr $\rho > 0$, det $\rho \geq 0$.

C. "Unitarity," $dN/dt \leq 0$.

Since we are dealing with decay, it suffices to consider times $t > 0$. The conditions A to C are identically satisfied for Eq. (5) (condition C is satisfied if $\Gamma = (1/i)(H^+ - H)$ is a positive matrix).

For an analysis of Eq. (6) it is convenient to consider, besides the matrix ρ, a real 4-vector ρ_λ with components

$$\rho_0 = \frac{1}{2} N, \quad \rho_\ell = \frac{1}{2} N b_\ell, \tag{8}$$

For pure states $\rho_0^2 - \rho_\ell^2 = 0$, i.e., the vector ρ_λ lies on the upper "light cone." The mixed states correspond to "time-like" vectors lying inside the cone: $\rho_0^2 - \rho_\ell^2 > 0$. The time evolution of the system is described by real linear transformations, under which the vector ρ_λ does not go outside the cone, and the component ρ_0 decreases (for an unstable system). Formulas (6) and (7) take the form

$$\frac{d\rho_\lambda}{dt} = - G_{\lambda\mu}\rho_\mu, \quad \rho_\lambda(t) = W_{\lambda\mu}(t)\rho_\mu(0), \quad W = \exp(-Gt). \tag{9}$$

We put

$$G_{\lambda\mu} = g\delta_{\lambda\mu} - \widetilde{G}_{\lambda\mu}; \quad g = \frac{1}{4}(G_{00} + G_{\ell\ell}). \tag{10}$$

Conditions B and C lead to limitations on the quantities $G_{\lambda\mu}$. It follows from condition C that $G_{0\mu}$ is a time-like vector. As to non-negativity, the necessary conditions are quite complicated in form. The sufficient conditions, however, are easy to formulate. If the matrix

$$F = \eta\widetilde{G} + \widetilde{G}^T\eta \; (\eta = \mathrm{diag}\,(1, -1, -1, -1)) \tag{11}$$

is non-negative, then $\exp(4gt)$ det ρ does not decrease, and det ρ does not vanish. The usual equation (5) corresponds to $F = 0$. The transformation of the vector ρ_λ with time is then a direct product of the Lorentz transformation by uniform compression along all axes, $\exp(-gt)$, and the vector ρ_λ does not leave the cone.

It is very important that the positiveness condition can be formulated independently of the usual (Hamiltonian) terms. The point is that when a beam of K^0 mesons passes through matter, there are added to the "vacuum" Hamiltonian terms describing the interaction of the K mesons with the nuclei and depending, in particular on the K-meson velocity and on the density of the medium.

The solution of (9) becomes much simpler in the presence of CP symmetry. Let us consider this case and use it as an example to trace qualitatively the difference between the solutions of Eqs. (5) and (6). CP-inversion corresponds to the transformation $(\rho_0, \rho_1, \rho_2, \rho_3) \to (\rho_0, \rho_1, -\rho_2, -\rho_3)$. Equation (9) conserves CP-parity if the component pairs (ρ_0, ρ_1) and (ρ_2, ρ_3) are transformed independently, i.e., the matrix G is a direct sum of two second-order matrices.

The solution of (9) reduces then to a calculation of exponentials of real second-order matrices. The number of the K_1 and K_2 mesons varies with time in accordance with the following law:

$$N_1(t) = e^{-\Gamma_1 t} N_1(0) + \frac{1}{2} e^{-\Gamma_2 t} (1 - e^{-(\Gamma_1 - \Gamma_2)t}) \sin\delta_1 \times$$

$$\times [\mathrm{tg}\frac{1}{2}\delta_1 N_1(0) + e^{-\beta} N_2(0)],$$

$$N_2(t) = e^{-\Gamma_2 t}\left\{N_2(0) - \frac{1}{2}(1 - e^{-(\Gamma_1 - \Gamma_2)t}) \sin\delta_1 \times \right. \tag{12}$$

$$\left. \times \left[e^{\beta} N_1(0) - \mathrm{tg}\frac{1}{2}\delta_1 N_2(0)\right]\right\},$$

where Γ_1 and Γ_2 are the widths of the K_1 and K_2 mesons, $\delta_1 \geq 0$ characterizes the deviation of the theory from the canonical one, and β is an arbitrary parameter. We present also the time dependence of the numbers $N_\pm$ of the K and $\bar{K}$ mesons, assuming for simplicity that $N_-(0) = 0$:

$$N_\pm(t) = \frac{1}{4} N(0)\left\{e^{-\Gamma_1 t}(1 - \sin\delta_1 \mathrm{ch}\beta) + e^{-\Gamma_2 t}(1 + \sin\delta_1 \mathrm{ch}\beta) \pm \right.$$

$$\left. \pm 2\exp\left[-\frac{1}{2}(\Gamma_1 + \Gamma_2)t - 2\lambda t\right](\cos\mu t - \sin\delta_2 \sin\gamma \sin\mu t)\right\}. \tag{13}$$

Here μ is the mass difference, and $\delta_2 \geq 0$, $\lambda \geq 0$, and γ are three more parameters. The theory thus contains five additional parameters.

The number of parameters increases even more if CP-violation is taken into account. The formulas become much more complicated, but if the CP-violation is assumed to be small and perturbation theory is used, then the result can be readily written in explicit form.

The most significant distinguishing feature of (12) is the presence of two exponentials in the decay of the K_1 mesons. If $\Gamma_1 t \gg 1$, after most K_1 mesons have already decayed, there still remains a certain number of CP-even mesons that decay with a time characteristic of K_2 mesons. However, without violating CP parity it is impossible to explain the experimentally observed $K^0 \to 2\pi$ decays at considerable distances from the source. Such a possibility is excluded, as is also the "independent particle model," by the known experiments on vacuum regeneration, in which it has been shown that there is interference in the $K^0 \to 2\pi$ decays.

According to the experimental data the parameters δ_1, δ_2, and λ are small and barely exceed 10^{-3}. However, to establish their scales correctly, additional data are needed. Of particular importance is the performance of a "complete experiment" in which one could determine all the four elements of the density matrix at a given instant of time.

Equation (6) for the $K\bar{K}$ system should be the consequence of an analogous equation for the entire (K^0 + decay products) system. The question is whether it is possible, without violating the condition of non-negativity and the conservation of the probability (Tr ρ = const) and of the energy, to construct an equation that differs from the canonical one, still remains open. We know that the well-known relaxation equations (cf., e.g., [2]) for the density matrix describe the interaction of the system with a thermostat, and therefore do not conserve the energy, and lead also to irreversibility.

An hypothesis that the equations of quantum mechanics have a more complicated form than (1) or (5) was advanced by V.M. Galitskii already a few years ago. In particular, he considered a linear equation for the density matrix elements of the $K^0\bar{K}^0$ system. I am grateful to him for an interesting discussion.

In a recent paper [3], Eberhard also discusses the possibility of describing a system with the aid of the evolution of a density matrix that does not satisfy an equation of the type (5). The scheme proposed by him, however, does not contain any equation whatever for the density matrix, and this scheme is not equivalent to the approach discussed here. In addition, the question of energy conservation still remains open.

The author is grateful to A.D. Dolgov, V.I. Zakharov, I.Yu. Kobzarev, L.B. Okun', V.I. Roginskii, and I.S. Shapiro for interest in the work and for a discussion of its results.

[1] T. Lee and C.S. Wu, Weak Interactions (Russ. transl.) Mir, 1968.
[2] A.A. Belavin, B.Ya. Zel'dovich, A.M. Perelomov, and V.S. Popov, Zh. Eksp. Teor. Fiz. 56, 264 (1969) [Sov. Phys.-JETP 29, 145 (1969)].
[3] P. Eberhard, Preprint CERN 72-1, 1972.

MULTIPERIPHERAL MODEL OF μ-MESON INTERACTION

E.M. Levin and M.G. Ryskin
Leningrad Institute of Nuclear Physics
Submitted 14 April 1972
ZhETF Pis. Red. 15, No. 11, 681 - 684 (5 June 1972)

There are presently grounds for assuming that the main contribution to the hadron interaction cross section at high energies is made by processes of the "ladder" type [1], Fig. 1. The characteristic kinematics of such processes is such that all the produced particles have small transverse momenta, and their longitudinal momenta are ordered, so that the momentum of each next particle is several times larger than the momentum of the preceeding one. The paired energies of the neighboring hadrons are small and it is possible to calculate the particle scattering cross section at high energies if one knows the interaction amplitudes at low energies.

Unfortunately, if we construct a ladder in which the exchange is effected by particles with zero spin (pions), then, at the experimental values of the coupling constant, such a model yields a cross section that decreases like $S^{-0.7}$ [2], and if the coupling constant is increased to such an extent that the cross section σ_{tot} becomes constant, then the $\pi\pi$ scattering cross section at low energies becomes larger than the unitary limit, and σ_{tot} turns out to be too large

$$\sigma \sim \frac{16\pi^3}{Nm^2} \sim 300 \text{ mb} \quad (\text{at } m = m_\rho) \tag{1}$$

where m is the mass of the emitted particle and N is the dimensionality of the isotopic multiplet.

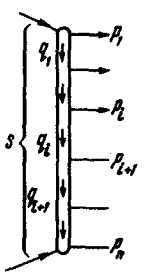

Fig. 1