describe the interaction of the system with a thermostat, and therefore do not conserve the energy, and lead also to irreversibility.

An hypothesis that the equations of quantum mechanics have a more complicated form than (1) or (5) was advanced by V.M. Galitskii already a few years ago. In particular, he considered a linear equation for the density matrix elements of the $K^0\bar{K}^0$ system. I am grateful to him for an interesting discussion.

In a recent paper [3], Eberhard also discusses the possibility of describing a system with the aid of the evolution of a density matrix that does not satisfy an equation of the type (5). The scheme proposed by him, however, does not contain any equation whatever for the density matrix, and this scheme is not equivalent to the approach discussed here. In addition, the question of energy conservation still remains open.

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MULTIPERIPHERAL MODEL OF μ-MESON INTERACTION

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There are presently grounds for assuming that the main contribution to the hadron interaction cross section at high energies is made by processes of the "ladder" type [1], Fig. 1. The characteristic kinematics of such processes is such that all the produced particles have small transverse momenta, and their longitudinal momenta are ordered, so that the momentum of each next particle is several times larger than the momentum of the preceeding one. The paired energies of the neighboring hadrons are small and it is possible to calculate the particle scattering cross section at high energies if one knows the interaction amplitudes at low energies.

Unfortunately, if we construct a ladder in which the exchange is effected by particles with zero spin (pions), then, at the experimental values of the coupling constant, such a model yields a cross section that decreases like $S^{-0.7}$ [2], and if the coupling constant is increased to such an extent that the cross section σ_{tot} becomes constant, then the ππ scattering cross section at low energies becomes larger than the unitary limit, and σ_{tot} turns out to be too large

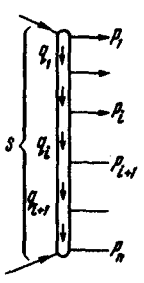

Fig. 1

$$\sigma \sim \frac{16\pi^3}{Nm^2} \sim 300 \text{ mb} \quad (\text{at } m = m_\rho) \tag{1}$$

where m is the mass of the emitted particle and N is the dimensionality of the isotopic multiplet.

We shall show in this paper that the situation improves markedly if we take into account a reasonable dependence of the amplitude of production of "n" particles on the paired energies $S_{ik} = (p_i + p_{i+1} + \ldots + p_k)^2$. Indeed, let there exist only one type of particle (say the ρ meson) and let the matrix element be

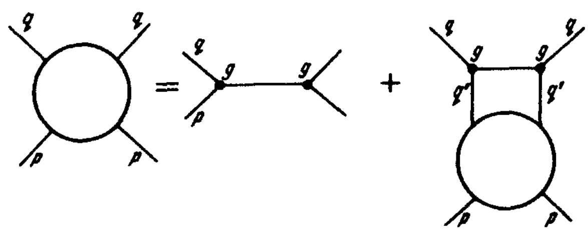

Fig. 2

$$M_n = g^2 \prod_{i=1}^{n-1} \frac{g^2}{q_i^2 - m^2} \left(\frac{S_{in}}{S_{i+1,n}} \right)^{1/2} . \quad (2)$$

(For the last (n - 1)-st step of Fig. 1, we write simple $(S_{n-1,n})^{1/2}$, i.e., S_{nn} is assumed equal to 1 GeV^2.) Such a formula (2) corresponds to exchange of a "standing" Reggeized ρ meson, i.e., its trajectory is $\alpha(t) \equiv \alpha(0) = 1/2$. The diagrams of Fig. 1 with the matrix element (2) can be easily summed with the aid of an equation of the Amati, Fubini, and Stanghellini (AFS) type [4], see Fig. 2. We then obtain for the imaginary part of the forward scattering amplitude $A(S, q^2)$ for zero isotopic spin of the t-channel

$$A(S, q^2) = \pi g^2 \delta(s - m^2) + \frac{g^2}{16\pi^2} \int^{S_{max}} \frac{dS'}{S} \left(\frac{S}{S'} \right) \frac{d|q'^2|\, A(S', q'^2)}{(q'^2 - m^2)^2} , \quad (3)$$

where

$$S'_{max} = \frac{S}{2|q^2|} \left((m^2 - q^2 - q'^2) - \sqrt{(m^2 - q^2 - q'^2)^2 - 4q^2 q'^2} \right) \quad (4)$$

$$S' = (q' + p)^2; \quad S = (q + p)^2 .$$

Equation (3) differs from the original AFS equation by the factor S/S' in the right-hand side. The behavior of the amplitude at high energies is determined by the homogeneous equation (5)

$$A(S, q^2) = \frac{g^2}{16\pi^2} \int^{S'_{max}} \frac{dS'}{S} \left(\frac{S}{S'} \right) \frac{d|q'^2|\, A(S', q'^2)}{(q'^2 - m^2)^2} , \quad (5)$$

where the role of the effective coupling constant $\tilde{g}^2$ is played by the quantity $g^2(S/S')$. To estimate the value of S/S', we put $q^2 \sim q'^2 \sim -m^2$. It is then seen from (4) that $S/S'_{max} \sim 3$; since S' varies between m^2 and S'_{max}, we get for the mean value $\langle S/S' \rangle \sim 5$. We thus have a fivefold gain ($\tilde{g}^2 \sim 5g^2$) and we can obtain a constant total cross section at a relatively small constant g^2. The total cross section, on the other hand, is determined by the inhomogeneous equation (3) and is proportional to g^2, and not to $\tilde{g}^2 = g^2(S/S')$. In our parametrization (2) we should therefore obtain a constant cross section of small magnitude (since g^2 is small). Equation (5) was solved numerically. We sought a solution as $S \to \infty$ in the form $A(S, q^2) = S\sigma(q^2)$, corresponding to a constant cross section, determined the smallest eigenvalue g^2, and then obtained $\sigma(q^2)$ by the same method as in [4].

Putting $m^2 = m_\rho^2$, we obtain $g^2 = 16\pi \times 6.2$ GeV^2, $\sigma(0) = 55$ mb, and an average number of ρ mesons $N_\rho = 0.5 \ln S$. The perpendicular momenta of the pions produced in the decay of ρ will be of the order of $m_\rho/2$ in this model, and the average number of pions is $N_\pi = 1 \times \ln S$. At $\alpha(t) \equiv 0$, (i.e., a ρ meson with zero spin), Eq. (3) would go over into the usual AFS equation [4], and we would have $g^2 = 16\pi \times 29\ \text{GeV}^2$, $\sigma(0) = 230$ mb, and $N_\pi = 1.6 \ln S$. If we write for the ρρ four-point diagram a formula of the Veneziano type [5]

$$B(S, t) = \frac{\Gamma(1-\alpha(S))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(S)-\alpha(t))},$$

then the obtained value $g^2 = 16\pi \times 6.2\ \text{GeV}^2$ corresponds to a rather large ρρ scattering cross section at low energies (∿40 mb at $S \sim 3\ \text{GeV}^2$), but not exceeding anywhere the unitary limit. The pole diagram showing the inhomogeneous term of Eq. (3), Fig. 2, yields at our value of g^2 in the physical region ($S > 4m_\rho$) a cross section σ_p smaller than the unitary limit of the S wave $\sigma_S(\sigma_p/\sigma_S)_{max} = 0.65$. At $S = 3.3\ \text{GeV}^2$ we have $\sigma_p = 13$ mb and $\sigma_S = 20$ mb.

We have thus shwon that, even in such a simplified model, a reasonable dependence of the amplitude on the paired energies S_{ik} leads, at not too large a coupling constant, to a constant cross section of reasonable value, and the decay of the reasonances into pions makes it possible to obtain in this case a multiplicity N_π and perpendicular momenta close to the experimental ones [6, 7]. In a more realistic situation, q^2 can differ appreciably from m^2_ρ. To demonstrate the dependence of the solution on m^2, we present the results at $m^2 = 0.1\ \text{GeV}^2$

$$g^2 = 16\pi \cdot 1.1\ \text{GeV}^2;\ \sigma(0) = 50\ \text{mb};\ N_\pi = 1 \ln S.$$

A detailed description of the model, taking into account the dependence of the matrix element on the paired energies, and also the presence of different particles (π, ρ, ω, A_2, f) [3] and their quantum numbers will be published in the future. For example, inclusion of the f meson decreases g^2 and the total cross section to almost one-half ($g^2 = 16\pi \times 2.9\ \text{GeV}^2$, $\sigma = 32$ mb, and $N_\pi = 0.94 \ln S$).

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