STATISTICAL APPROACH TO THE CALCULATION OF RESONANT WIDTH AND MULTIPLICITY IN DUAL MODELS

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The large degeneracy of the resonant spectrum in dual models (DRM) leads naturally to a statistical description of their physical characteristics [1]. However, the presently available papers deal in essence with only one such quantity, the average spin.

In the present paper we make the next step in this direction, and study within the framework of the statistical approach the total and partial widths of the resonances, and also the multiplicity of production of stable mesons (arbitrarily, pions) in the DRM.

Assuming, in accord with experiment, a cascade decay of the resonances [2] and the DRM unitarization scheme proposed in [3], we obtained the following expression for the total width of the resonance $|\{\ell\}>$

$$\Gamma = \frac{\gamma^2\alpha'}{8\pi^2}\int\frac{d^3p}{2p_0}\sum_{\{\ell'\}}<\{\ell\}|V(p)\delta(H-\alpha(M'^2))|\{\ell'\}><\{\ell'\}|V(-p)|\{\ell\}>, \quad (1)$$

where γ is the interaction constant, $\alpha(x) = \alpha(0) + \alpha'x$ is the Regge trajectory,

$$V(p) = \exp\left[-\sqrt{2\alpha'}\,p\sum_{n=1}^{\infty}\frac{a^{(n)+}}{\sqrt{n}}\right]\exp\left[\sqrt{2\alpha'}\,p\sum_{n=1}^{\infty}\frac{a^{(n)}}{\sqrt{n}}\right]$$

is the usual Veneziano amplitude, $H = -\sum_{\mu=0}^{3}g_{\mu\mu}\sum_{n=1}^{\infty}na_\mu^{(n)+}a_\mu^{(n)}$; $g_{\mu\mu} = (1, -1, -1, -1)$; M' is the mass of the secondary resonance and is determined from the kinematics of the process

$$M'^2 = M^2 + m^2 - 2M\sqrt{m^2 + p^2} \quad (2)$$

M is the mass of the decaying resonance, and m is the pion mass.

With increasing M, the number of possible decay channels increases exponentially and the calculation of (1) is an extremely complicated process. To solve this problem, we use the methods of statistical mechanics.

Our first step is to replace the δ function in (1) by the operator of the canonical distribution

$$\delta(H - \alpha(M'^2)) \to C\frac{1}{T'(p)}e^{-\frac{H}{T'(p)}}, \quad (3)$$

where C is a certain constant.

Such a substitution was already used for other purposes in [4], where it was shown that to retain the Regge properties in the DRM the temperature introduced in (3) must be proportional to the square of the mass of the resonance: $T'(p) = k\alpha(M'^2)$.

We now rewrite (1) in the form

$$\Gamma = \frac{\gamma^2 \alpha' C}{8\pi^2} \int \frac{d^3p}{2p_0} \frac{1}{T'(p)} \langle\{\ell\}|\, V(p)\, e^{-\frac{H}{T'(p)}} V(-p)\, |\{\ell\}\rangle . \qquad (4)$$

The quantity $\bar{\Gamma}$ of interest to us is obtained by averaging over the microcanonical ensemble the external states $|\{\ell\}\rangle$ with energy equal to the eigenvalue $H\alpha(M^2) = N$, in the form [5]

$$\bar{\Gamma} = \frac{\gamma^2 \alpha' C}{8\pi^2} \int \frac{d^3p}{2p_0} \frac{1}{T} \frac{1}{\oint dz z^{-N-1} \mathrm{Sp}\, z^H} \oint dz z^{-N-1} \mathrm{Sp}[z^H V(p) e^{-\frac{H}{T'}} V(-p)]. \qquad (5)$$

An analysis of (5) shows that in the region $T'(p) < T$, where T is the temperature determined from the condition that the average energy of the canonical ensembles of the external states coincide with the energy $N = \alpha(M^2)$ of the microcanonical ensemble (at large N we have $T \approx (3N/2\pi^2)^{1/2}$ [1]), the integrand in (5) decreases exponentially with increasing T. On the other hand, in the region $T'(p) > T$ we can use the saddle-point method (which is equivalent to going over to the canonical ensemble) and obtain in the limit of large M:

$$\bar{\Gamma} = \frac{\gamma^2 \alpha' C}{8\pi^2} \int \frac{d^3p}{2p_0} \frac{1}{T'} e^{\frac{\alpha(M^2)}{T'}} \left(1 - e^{-\frac{1}{T'}}\right)^{2\alpha' m^2} \exp\left[- 2\alpha' m^2 \times \right.$$

$$\left. \sum_{n=1}^{\infty} \frac{e^{-\frac{n}{T}}\left(1 - e^{-\frac{n}{T'}}\right)\left(e^{\frac{n}{T'}} - 1\right)}{n\left(1 - e^{-\frac{n}{T}}\right)} \right] \approx \frac{\gamma^2 C}{8\pi} \int_T^{k(\alpha' M^2 - 2\alpha' Mm)} \frac{dT'}{kM} \times$$

$$\times \sqrt{\left(\frac{\alpha' M^2 - \frac{T'^2}{k}}{2\alpha' M}\right)^2 - m^2} \, (T')^{-1-2\alpha' m^2} e^{-\frac{\alpha(M^2)}{T'}} \left(\frac{\sin \frac{\pi T}{T'}}{\frac{\pi T}{T'}} \right)^{2\alpha' m^2} \approx$$

$$\approx \mathrm{const} \cdot \gamma^2 (\alpha(M^2))^{-2\alpha' m^2} . \qquad (6)$$

The sum in the argument of the exponential in (6) is calculated by going over to integration.

Thus, for negative $\alpha(0)$ (it is precisely for such $\alpha(0) = -\alpha' m^2$ that the dual models have been presently constructed) the total average width decreases like $(M^2)^{2\alpha(0)}$ with increasing M.

Since there is no universally accepted interpretation of ghost states, it is impossible to draw final conclusions concerning the true physical meaning of the decrease of the width $\bar{\Gamma}$ (6) (the "ghosts" should have a negative width in the DRM). It is only clear that such a behavior is evidence in favor of the self-consistency of the DRM unitarization scheme [3, 6], which is based on the assumption that the resonance widths are small.

The integrand in (6) makes it possible to draw conclusions concerning the behavior of the partial widths in the DRM. In particular, its maximum (the most probable value of T') is located at the point $T' \simeq (1/2)\alpha(M^2)$ (if $\alpha(0) \ll 1$).

We consider now the question of the average multiplicity. If the mean-squared mass of the secondary resonance is $M'^2 = rM^2$, then the average number (n) of the pions in the cascade decay of the resonance with $M^2 = s$ is determined by the condition

$$s_0 = r^n s \tag{7}$$

where s_0 is a certain small value of the square of the energy, below which the statistical analysis ceases to be valid. From (7) we obtain

$$n = -\frac{1}{\ln r} \ln s - \ln s_0 . \tag{8}$$

Using (6), we can obtain

$$\bar{M}'^2 = \frac{\bar{T}'}{\alpha'} = \frac{1}{\alpha'} \frac{1}{\int_T^{T'_{max}} dT' f(T')} \int_T^{T'_{max}} dT' T' f(T') ,$$

where f(t') is the integrand in (6).

In the limit as $M \to \infty$, assuming the definition of T' given in [4], which yields $k \simeq 1$ when $\alpha(0) \ll 1$, we obtain $\bar{M}'^2 = M^2/2$, i.e., $(-1/\ln r) \simeq (1/\ln 2) \simeq 1.4$, which is close to the experimental data.

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INTERPOLATION FORMULA FOR THE SPECTRA OF PARTICLES PRODUCED AT HIGH ENERGY

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If we observe any one of the particles produced in an inelastic reaction, then we can obtain information concerning the quantity $d\sigma_C = \sum_X d\sigma(A + B \to C + X)$, which is called the total inclusive cross section for the production of the particle C with momentum $P_C = (\vec{P}_{C\parallel} P_{C\perp})$. By X we denote the system of particles produced together with the given particles, and summation over all the possible states of this system is carried out in the experiment.