The integrand in (6) makes it possible to draw conclusions concerning the behavior of the partial widths in the DRM. In particular, its maximum (the most probable value of T') is located at the point $T' \simeq (1/2)\alpha(M^2)$ (if $\alpha(0) \ll 1$).

We consider now the question of the average multiplicity. If the mean-squared mass of the secondary resonance is $M'^2 = rM^2$, then the average number (n) of the pions in the cascade decay of the resonance with $M^2 = s$ is determined by the condition

$$s_0 = r^n s \tag{7}$$

where s_0 is a certain small value of the square of the energy, below which the statistical analysis ceases to be valid. From (7) we obtain

$$n = -\frac{1}{\ln r}\ln s - \ln s_0 \,. \tag{8}$$

Using (6), we can obtain

$$\bar{M}'^2 = \frac{\bar{T}'}{\alpha'} = \frac{1}{\alpha'}\frac{1}{\int_T^{T'_{max}} dT' f(T')}\int_T^{T'_{max}} dT' T' f(T') \,,$$

where f(t') is the integrand in (6).

In the limit as $M \to \infty$, assuming the definition of T' given in [4], which yields $k \simeq 1$ when $\alpha(0) \ll 1$, we obtain $\bar{M}'^2 = M^2/2$, i.e., $(-1/\ln r) \simeq (1/\ln 2) \simeq 1.4$, which is close to the experimental data.

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INTERPOLATION FORMULA FOR THE SPECTRA OF PARTICLES PRODUCED AT HIGH ENERGY

V.A. Kolkunov, K.A. Ter-Martirosyan, and Yu.M. Shabel'skii
Institute of Theoretical and Experimental Physics
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If we observe any one of the particles produced in an inelastic reaction, then we can obtain information concerning the quantity $d\sigma_C = \sum_X d\sigma(A + B \to C + X)$, which is called the total inclusive cross section for the production of the particle C with momentum $P_C = (\vec{P}_{C\parallel} P_{C\perp})$. By X we denote the system of particles produced together with the given particles, and summation over all the possible states of this system is carried out in the experiment.

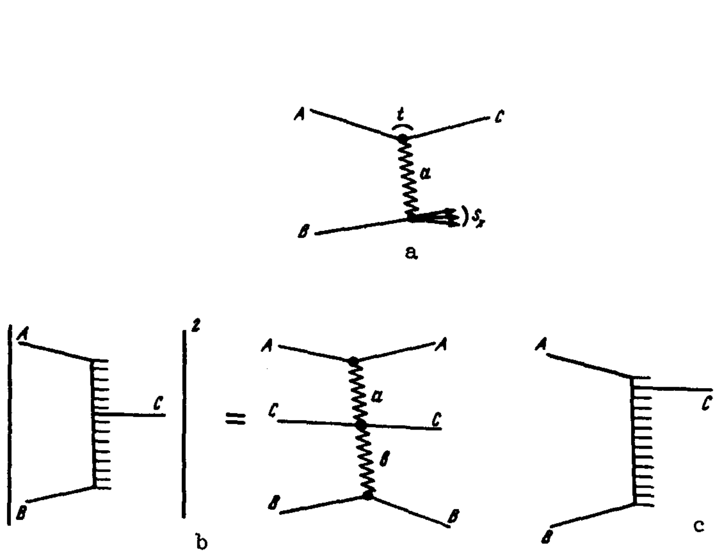

Fig. 1

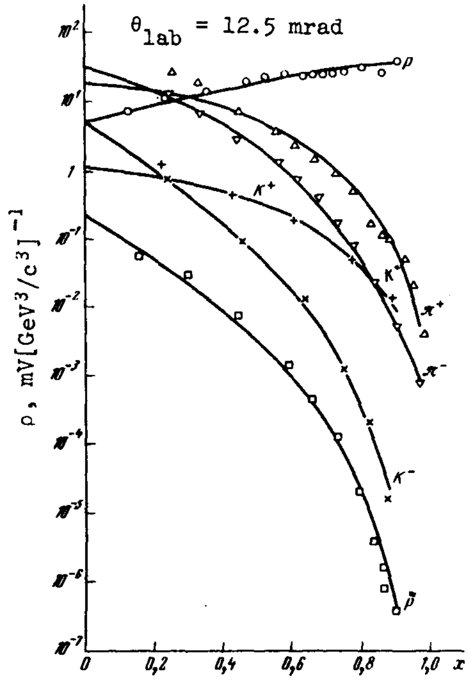

Fig. 2

It follows from the experimental data [1 - 5] that at high energy $E_A = E_B = \sqrt{s}/2$ the inclusive cross section depends not on E_A and $P_{C\parallel}$ separately, but only on their ratio $x = P_{C\parallel}/E_A$, i.e., $d\sigma = \rho d^2 P_C / 2E_C$, where $\rho = \rho(x, P^3_{C\perp})$. In addition, the dependence of the function ρ on x and $P_\perp^2$ is approximately factorized, i.e., $\rho(x, P^2_{1\perp})/\rho(x, p^2_{2\perp})$ depends weakly on x.

The function $\rho(x, P_\perp^2)$ can be found with the aid of the theory of complex angular momenta in the regions $m^2/s \ll 1 - x \ll 1$ and $|x| \ll 1$, corresponding to the diagrams of Figs. 1a and 1b.

In the region $m^2/s \ll 1 - x \ll 1$, in which the particle C is fast in the c.m.s. [6, 7], the diagram of Fig. 1a with reggeon exchange yields

$$\rho(x, P_\perp^2) = |\eta_a g_a(t)|^2 \sigma_{aB}(t) \left(\frac{s}{s_X}\right)^{2\alpha_a(t) - 1}, \tag{1}$$

where

$$s = (P_A + P_B)^2, \quad s_X = (P_A + P_B - P_C)^2, \quad \frac{s}{s_X} \quad \frac{1}{1-x}$$
$$t = -\frac{P_\perp^2}{x} + (1 - x)\left(m_A^2 - \frac{m_C^2}{x}\right) \tag{2}$$

$g_a(t)$ is the Regge residue, which can be parametrized in the form $g_a(t) = g_a^0 \exp(bt)$, b is a parameter, $\eta_a = [1 + \sigma\exp(-i\pi\alpha_a(t)]/[1 - \sin\pi\alpha_a(0)]$ is the signature factor, and $\sigma_{aB}(s_X, t)$ is the total cross section for the scattering of reggeon a by the particle B.

We assume that it is independent of s_X when $s_X \gg m_V^2$. Thus, expression (1) can be written in the form

$$\rho(x, P_\perp^2) = C e^{2R_a^2 t}(1-x)^{1-2\alpha_a(t)}, \qquad (3)$$

where R_a^2 is a certain parameter, and C determines the normalization.

In the region $|x| \ll 1$ (Fig. 1b), where the particle C is slow in the c.m.s. (this is the region of the so-called "pionization"), we can obtain [7, 8]

$$\rho(x, P_\perp^2) = \rho(P_\perp^2) \qquad (4)$$

i.e., it is independent of x.

If both reggeons in Fig. 1b are vacuum, then we have in the soft part of the spectrum

$$\rho_C(P_\perp^2) = \rho_{\bar{C}}(P_\perp^2) \qquad (5)$$

i.e., for the same colliding particles the functions $\rho(P_\perp^2)$ are equal for the case of production of particles and antiparticles. The spectra of certain particles, however, for example of slow protons and antinucleous, can differ appreciably (see, for example, Fig. 2). This is caused by the kinematic suppression of the $P\bar{P}$ pair production in the region of non-ultrahigh energies of the incident particles ($E \lesssim 10^2 - 10^3$ GeV). With increasing input energy, the ratio $\rho_P(P_\perp^2)/\rho_{\bar{P}}(P_\perp^2)$ should tend to unity.[1)]

In the intermediate region $x \lesssim 1$ (Fig. 1c), the function $\rho(x, P_\perp^2)$ can be obtained on the basis of some model representations (for example, with the aid of the diagram of Fig. 1c with one-pion exchange). We propose in this connection the following very simple interpolation formula[2)]

$$\rho(x, P_\perp^2) = C e^{2R_a^2 t x}\left(\frac{1}{1-x} + d\right)^{2\alpha_a(tx)-1}. \qquad (6)$$

with one free parameter d (compared with (3)), which goes over into (3) and (4) as $x \to 0$ and $1 - x \to 0$, respectively.

Particle	a	$\alpha(0)$	$\alpha'(0)$	R_a^2	d
π^+	N	- 0.50	–	2.30	- 0.50
π^-	πN	- 1.50	–	2.30	- 0.40
K^+	Y^*	- 0.40	0.9	1.15	- 0.40
K^-	KN	- 1.70	–	2.30	- 0.60
$\bar{P}$	NN	- 2.00	1.0	1.15	- 0.35
P	P'	0.45	–	1.70	- 0.30

[1)]The authors are grateful to A.B. Kaidalov for a discussion of this question.

[2)]It is possible that this formula should be modified to make $(\partial\rho/\partial x)|_{x=0}=0$.

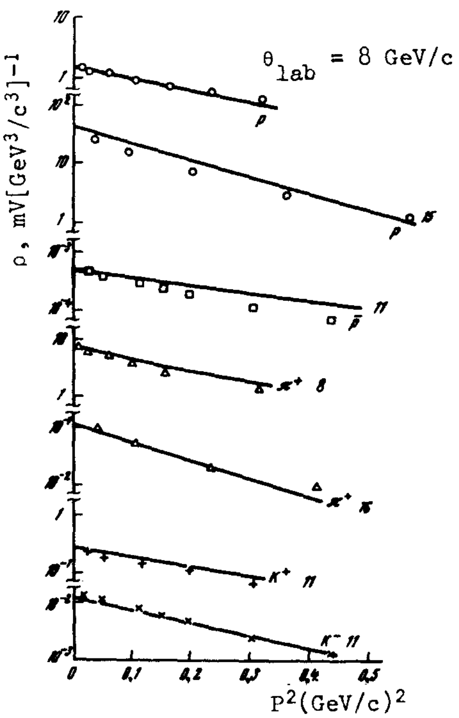

Fig. 3

Formula (6) for inclusive production of $\Pi^{\pm}$, $K^{\pm}$, $\bar{P}$, and P in PP collisions agrees well with the experimental data [1] (see Figs. 2 and 3) at the parameter values listed in the table.

The results of other investigations agree with the data of Allaby et al. [1] when account is taken of the experimental errors.

The fact that the parameter d is negative in all cases leads to a faster growth of the function (6) with increasing x in comparison with the function (3). This corresponds to an effective allowance for the contribution of the diagram of Fig. 1c (which is not taken into account in (3)), the relative fraction of which in comparison with Fig. 1a increases with increasing 1 - x. In the region x > 0.7 - 0.8, all the spectra are satisfactorily described by the limiting formula (3).

In the hard part of the π^- spectrum, a noticeable contribution is made also by the Δ pole. It is accounted for in the spectrum of Fig. 2 with the aid of the formula $p = C[\exp(2R_a^2 tx)]|\tilde{T}|^2$, where

$$\tilde{T} = Y^{\alpha_{\pi N}(0) - \frac{1}{2}} + \delta \eta_\Delta Y^{\alpha_\Delta(0) - \frac{1}{2}},$$

with $Y = 1/(1 - x) + d$, $\alpha_\Delta = 0.15 + 0.9t$, $\eta_\Delta = -i + 0.61$, and the best value of the parameter δ turns out to be $\delta \simeq 5\%$.

Formula (3) was used in [9] to reduce the experimental data, and the resultant parameters differed appreciably from the ones given above. A check has shown, however, that the curves of [9] do not agree with the parameters given there.

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