Calculation shows that in this case $P_{circ}(H)$ does not depend on the magnetic field and is equal to the degree of orientation of the holes, if no change in the degree of hole orientation occurs upon binding of the exciton.

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THE "QUASI-EIKONAL" APPROXIMATION

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We propose below for the scattering amplitude a simple approximation, which is quite exact at high energies $s \simeq 2m_N E \gg m_N^2$ (and small $|t| = |q^2| \lesssim m_N^2/\ln(E/m_N)$), with account taken for the contributions of the Regge poles and all the rescatterings [1, 2] from them (Fig. 1).

We write down, in the representation of the impact parameter b, the amplitude[1])

$$M(s, q^2) = \int e^{i\vec{\kappa} b} f(s, b) \frac{d^2 b}{2\pi} = \int_0^\infty I_0(\kappa b) f(s, b) b\, db \qquad (1)$$

in terms of the partial wave f(s, b), in analogy with the eikonal model [3]

$$f(s, b) = [e^{\chi'} - 1 - \chi' + C\Sigma_a \chi_a]/2iC. \qquad (2)$$

Here χ' is the quasi-eikonal

$$\chi = C\chi_P(s, b) + \sum_{a \neq P} C_a \chi_a(s, b),$$

$$\chi_a = 2i\int M^{(1)}(s, \kappa^2)\, e^{-i\vec{\kappa} b} \frac{d^2\kappa}{2\pi}, \qquad (3)$$

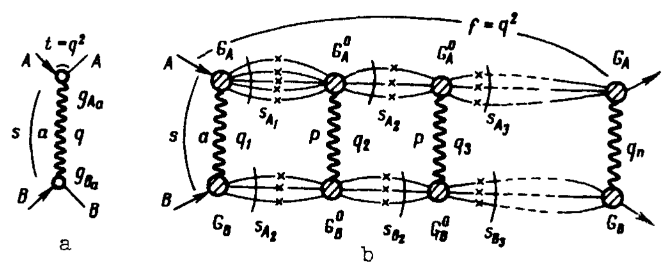


Fig. 1

[1])We are considering spinless particles; the normalization is such that $d\sigma/dt = 4\pi|M(s, t)|^2$, $\sigma^{tot} = 8\pi \mathrm{Im} M(s, 0)$.

and $M^{(1)} = \eta_{\sigma_a}(\alpha_a) g_{A_a}(t) g_{B_a}(t)(s/s_0)^{\alpha_a(t)-1}$ is the contribution of the a-th Regge pole, $s_0 \approx 2m_N^2$, $t = q^2 \approx -\vec{\kappa}^2$, and $\eta_{\sigma_a} = (\sigma_a + \exp[-i\pi\alpha_a])/(-\sin\pi\alpha_a)$ is the signature factor. If

$$\alpha_a \approx \alpha_a^0 + \alpha_a' t$$

and

$$|\eta_{\sigma_a}(\alpha_a)|\, g_{A_a} g_{B_b} \approx |\eta_{\sigma_a}(\alpha_a^0)|\, \gamma_a^0 e^{R_a^2 t},$$

then

$$\chi_a = i\eta_{\sigma_a}(\alpha_a^0)(\gamma_a^0/\lambda_a)(s/s_0)^{\alpha_a^0 - 1}\exp[-b^2/4\lambda_a],$$

$$\lambda_a = R_a^2 + \alpha_a'\left(\ln\frac{s}{s_0} - i\frac{\pi}{2}\right).$$

When $C = C_a = 1$ these formulas go over into the eikonal model [3]. They are, however, much more accurate, since they take into account correctly, owing to the factors C and C_a in [3], the contribution of the rescatterings of Fig. 1b with cascades [2] in the intermediate states (with small masses $\sqrt{s_{A_K}}$ and $\sqrt{s_{B_K}}$).

Indeed, we shall show that when [2, 4]

$$C = 1 + \sigma^{in}/\sigma^{el}, \tag{4}$$

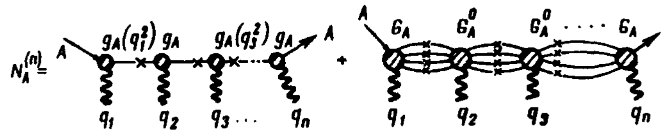
where σ^{el} and σ^{in} are the cross sections of the elastic A + B scattering and of the diffraction formation of cascades, and at a certain choice of the constants C_a, $a \neq P$, formulas (1) - (3) account correctly for the contributions [1, 2] of the diagrams of Fig.1b, corresponding to n-fold P^n and aP^{n-1} rescatterings

$$M^{(n)}(s, q^2) = \nu_n \int N_A^{(n)}(q_k) N_B^{(n)}(q_k)\, \eta_{12\ldots n}(s/s_0)^{\sum_{k=1}^{n}\alpha_k(q_k^2) - n} \times$$

$$\times \frac{d^2\kappa_1}{\pi}\frac{d^2\kappa_2}{\pi}\cdots\frac{d^2\kappa_{n-1}}{\pi}. \tag{5}$$

Here $\eta_{12\ldots n} = \eta_1 i\eta_2 \ldots i\eta_n$, $\eta_k = \eta_{\sigma_k}(\alpha_k)$, $N_A^{(n)}$ and $N_B^{(n)}$ are the vertices for the production of n-reggeons, $g_k \approx (0, 0, \vec{\kappa}_k)$ are the reggeon 4-momenta, $\sum_{k=1}^{n}\vec{\kappa}_k = \vec{\kappa}$, and $t = q^2 \approx -\kappa^2$. For the P^n rescatterings on Fig. 1b we have a = P, i.e., all $\alpha_k \equiv \alpha_P(q_k^2)$ and $\nu_n = 1/n!$, while for the aP^{n-1} rescattering we have $a \neq P$, $\alpha_1 = \alpha_a(q_1^2)$, $\alpha_k = \alpha_P(q_k^2)$, $k > 1$, and $\nu_n = 1/(n-1)!$ (the contribution of the diagram of Fig. 1b does not change when the reggeon lines are rearranged, and we therefore assume that the reggeon of type $a \neq P$ is the farthest one to the left.

The vertices $N_A^{(n)}$ and $N_B^{(n)}$ can be [1, 2] expanded in a series in the complete system of physical intermediate states[2] of the type of Fig. 2, with the largest contribution $(N_A^{(n)})_0 = g_{A_a}(\kappa_1^2) g_{A_P}(\kappa_2^2) \ldots g_{A_P}(\kappa_n^2)$ made by the single-particle state of Fig. 2a. Therefore



a b

Fig . 2

$$N_A^{(n)}(q_k) = g_{A_a}(\kappa_1^2) g_{A_P}(\kappa_2^2) \cdots g_{A_P}(\kappa_n^2) C_A^{(n)}(\kappa_k),$$

where $C_A^{(n)}(\kappa_k)$ is the ratio of the contribution of all the diagrams of Fig. 2 to $(N_A^{(h)})_0$. Hence

$$N_A^{(n)}(\kappa_k) N_B^{(n)}(\kappa_k) = \gamma_a(\kappa_1^2)\gamma_P(\kappa_2^2) \ldots \gamma_P(\kappa_n^2) C_a^{(n)}(\kappa_k), \quad (6)$$

where $\gamma_a = g_{A_a} g_{B_a}$ and $C_a^{(n)} = C_A^{(n)}(\kappa_k) C_B^{(n)}(\kappa_k)$.

From the definitions of $d\sigma^{in}/dt$ and $d\sigma^{e\ell}/dt$ it follows that when a = P and n = 2 we have $C_P^{(2)}(\vec{\kappa}_1, -\vec{\kappa}_1) = 1 + (d\sigma^{in}/d\kappa_1^2)/(d\sigma^{e\ell}/d\kappa_1^2)$ and $C_P^2(0, 0) = C$, where C is a constant [7].

When $s >> m_N^2$ and $\kappa^2 \simeq |t| \lesssim m_N/\ln(s/m_N^2)$, an important role is played in the integral (5) by very small $\kappa_k^2 \sim \kappa^2/n$; we can therefore assume that $C_a^{(n)}(\kappa_k) \simeq C_a^{(n)}(0)$, accurate to terms $\sim \kappa_k^2/m_N^2 \sim 1/\ln(s/m_N^2)$.

The contribution of each link of the diagram of Fig. 1b increases as a result of the formation of a cascade of particles, in comparison with its single-particle value, by an amount $C_P^{(2)}(0, 0) \simeq C$. Therefore, if the structure of all the links of the diagram 1b is the same and the contribution of the particles "circling around" one of the vertices is small(dashed lines in Figs. 2b and 1b), then

$$C_a^{(n)}(\kappa_n) \simeq C_a^{(n)}(0) \simeq C_a C^{n-2}, \quad (7)$$

where $C_a = C$ if a = P. If $a \neq P$, the factor C_a determines the degree of increase of the contribution of the left-hand link on Fig. 1b.

The approximate equality (7) (which is exact if $s \to \infty$ and n = 2) is the basis of our approach.

If we substitute (7) and (6) in (5) and rewrite (5) in the form (1) we find that the contribution of the P^n rescattering, Fig.1b, corresponds in the partial wave to the term $f_P^{(n)}(s, b) = (C\chi_P)/(2iCn!)$, and the contribution of the aP^{n-1} rescattering corresponds to the term $f_a^{(n)}(s, b) = C_a\chi_a(C\chi_P)^{n-1}(2iC(n-1)!)$.

[2] They correspond to particles on the mass shell; the lines of these particles are marked by crosses in Figs. 1 and 2; they correspond to the propagator $-2\pi i\tau(p_0)\delta(p^2 - m^2)$. The large masses of the cascades of these particles, $s_{A_K} \sim s_{B_K} \sim s$, do not matter, for they lead to the so-called enhanced diagrams, the contributions of which are small [5].

A series expansion of $\exp(\chi')$ in (2) shows that terms of just this type appear in (2) and (3). At the same time, the terms of (2) that are linear in χ_a, namely $(1/2i)\sum_a \chi$, yield exactly the contributions of the Regge poles of Fig. 1a. Thus, within the framework of the approximation (7), formulas (1) - (3) reproduce correctly all[3] the rescatterings (5).

Since the outermost cascade production vertices on Figs. 1b and 2b (G_A and G_B) differ from the central ones (G_A^0, G_B^0), we have at a = P a somewhat less accurate approximation than (7), namely,

$$C_P^{(n)} \approx C C_1^{n-2}, \quad C_a^{(n)} \approx C_a C_1^{n-2}, \tag{8}$$

where C_1 is a new constant different from (4). This approximation can also be easily obtained with the quasi-eikonal (2) and (3) by making in (3) the substitutions $C \to C_1$ and $C \to C_a C_1/D$ and in (2) the substitution $C \to C_1^2/C$.

Allowance for the particle spin in πN and kN scattering is trivial [2, 3] and reduces to a replacement of χ_a and χ' in (2) and (3) by the operators $\chi_a' = \chi_{a_0} + \hat{\sigma}_y' \chi_{a_y}$, and $\chi' = \chi_0' + \hat{\sigma}_y' \chi_y'$, where $\hat{\sigma}_y' = \hat{\sigma}[\vec{n}_0 \times \vec{b}/b]$ and $\vec{n}_0 = \vec{p}_A/p_A$, and to a representation of χ_0' and χ_y' in the form of sums (3) with respective coefficients $C_1 = C_{P_0}$ and C_{a_0} or $C_y = C_{P_y}$ and C_{a_y}.

The amplitude $M = M_0 + i\sigma[\vec{n}_0 \times \vec{\kappa}/\kappa]M_y$ is determined in terms of the partial waves

$$f_0(s, b) = \frac{1}{2iC}\{ e^{\chi_0'} \operatorname{ch} \chi_y' - 1 - \Sigma_a (C - C_{a_0}) \chi_{a_0} \},$$

$$f_y(s, b) = \frac{1}{2iC}\{ e^{\chi_0'} \operatorname{sh} \chi_y' + \Sigma_a (C - C_{a_y}) \chi_{a_y} \},$$

in the usual form [2, 3, 6]

$$M_0 = \int_0^\infty f_0 I_0(\kappa b)\, b\, db, \quad M_y = \int_0^\infty i f_y I_1(\kappa b)\, b\, db.$$

Analogous formulas, convenient for numerical calculations, can be obtained for backward πN scattering, charge exchanges, etc. For example, for backward scattering we have

$$\hat{f}(s, b) = \frac{1}{2iC}[C_f \hat{\chi}_f e^{\hat{\chi}'} + (C - C_f)\hat{\chi}_f],$$

where $\hat{\chi}_f(s, p) = \chi_{f_0} + \hat{\sigma}_y' \chi_{f_y}$ is the eikonal (3) corresponding to the contribution $M_f^{(1)}$ of the fermion Regge pole.

We note in conclusion that at high energies the amplitude (2) certainly satisfies the s-unitarity condition Im f $\leq$ 1, and that numerical calculations [6, 7] performed with $C_{a_0} \equiv C_y \equiv C_{a_y} \equiv 1$ gave a good description of all the known experimental data on πN, kN, and NN - $\bar{N}N$ interactions.

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[3] The term $\chi_a \chi_b / 2ic$, which determines rescattering by two poles a, b $\neq$ P, is reproduced in (2) and (3) with the reasonable coefficient $C_{ab} = C_a C_b / C$.

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LOCAL OSCILLATIONS IN AN ANHARMONIC CRYSTAL

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In the problem of local oscillations, anharmonicity is usually considered as the cause of the damping and small shift of the local frequencies. Its influence can lead, however, also to other effects, particularly to the appearance of new local frequencies [1, 2].

It is natural to expect the influence of anharmonicity on the properties of local oscillations to become most strongly pronounced in crystals with large zero-point oscillation amplitudes (quantum crystals), when the usual method of expanding the potential energy in terms of the displacements of the nuclei is not valid. In such systems, the behavior of even an isotropic defect can be significantly different than in a harmonic lattice, for the anharmonicity of the oscillations causes the change of mass to give rise to a change in the effective force constants. We shall show below that under certain assumptions the properties of an isotropic defect in an anharmonic crystal are radically altered.

We consider a one-dimensional crystal containing 2N + 1 atoms, of which 2N have a mass m and one, located at the zeroth site, has a mass m' = Qm. We shall take into account only the nearest-neighbor approximation, which is described by a Morse potential

$$\phi(r) = D[e^{-(r-r_0)/\rho} - 1]. \tag{1}$$

Here D and ρ are constants, r_0 is the distance between sites, and r the instantaneous distance between atoms. We assume also that no external forces act on the system.

To describe the dynamics of this system, we shall use the method of [3 - 6], in which it is not assumed that the anharmonicity is small, and which makes it possible to take into account all the orders of perturbation theory. Although this method was used in the cited papers only for ideal crystals, it is easy to formulate it in a manner that does not presuppose that the lattice is ideal. Disregarding damping processes (pseudoharmonic approximation), this method applied to the potential (1) leads to the following self-consistent system of equations[1)]

$$\omega^2 G_{nn'}(\omega) = \delta_{nn'} + \sum_{n''} D_{nn''} G_{n''n'}(\omega), \quad D_{nn'} = \frac{K_{nn'}}{\sqrt{m_n m_{n'}}}; \tag{2a}$$

[1)] We follow here [4, 5], where the Morse potential is specially considered.