TOTAL CROSS SECTIONS AND CONE SLOPE - THEORY AND EXPERIMENT

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The theory of complex angular momenta, which takes into account the contributions of the Regge poles and the rescatterings by these poles, enables us to find the asymptotic scattering amplitude at $s \approx 2m_N E >> m_N^2$ and $|t| = |q^2| \lesssim m_N^2/\ln(E/m_N)$. The higher the energy, the more simple and exact the theoretical formulas, which contain as parameters the Regge coupling constants or the residues $\gamma_a = g_{A_a} g_{B_a}$ and the radii R_a^2 of the form factors, which are the analogs of the charges, magnetic moments, and electromagnetic radii in electrodynamics. In addition, the contributions of the rescatterings (branch points) on the Regge poles are characterized by the constants C_a, which determine the probability of the production of showers of low-mass particles on a given Regge pole. High-mass showers correspond to the so-called enhanced diagrams, the contributions of which are small (the most important case of P-reggeons is discussed below).

The previously presented (1969-1970) description [1, 2] of all the known experimental data on πN, kN, and NN - NN interactions has shown that there exists a single system of parameters that give good agreement with experiment (without enhanced diagrams). The purpose of the present article is to compare the theory, given the same parameters, with the new data of the Institute of High Energy Physics (IHEP) and CERN on the values σ^{tot} and on the slopes $b_0 = b_0(E)$ of the pp-scattering cone.

In Fig. 1, the new experimental data [3] are placed on the previously obtained σ^{tot} curves. We see that our main prediction, that $\sigma^{tot}_{k^+p}$ increases in the region $E \gtrsim 10$ GeV is very accurately confirmed by experiment. The experimental data on $\sigma^{tot}_{k^-p}$ and $\sigma^{tot}_{\pi^\pm p}$ agrees well with the theory, although the points for $\sigma^{tot}_{\pi^-p}$ in the region $E \sim 60$ GeV is somewhat higher than the theoretical curve.

For σ^{tot}_{pp}, the theory yields the curve of Fig. 1 with a gently sloping minimum in the region $E \sim 60$ GeV. This curve increases with increasing E by approximately 1.2 mb for $\Delta E \sim 10^3$ GeV. The IHEP data [3] for σ^{tot}_{pp} agree with the theory with surprising accuracy in the region of the minimum. It remains now to verify that at $E \sim 200$ GeV and $\sim$500 GeV the Batavia accelerator will indeed reveal an increase of σ^{tot}_{pp} to 38.6 and 39 mb, respectively, as would follow[1)] from Fig. 1. It is very important that the future experiments confirm the main conclusion of the theory [4], that all the cross sections σ^{tot} should increase with increasing energy in the region $E \gtrsim 200$ GeV (at $E \gtrsim 500$ GeV in the case of $\sigma^{tot}_{\bar{p}p}$).

The slope of the diffraction cone, $b(E, t) \approx b_0(E) + tb'(E)$ determines the t-dependence of the elastic scattering cross section, $d\sigma/dt = C \exp(bt)$. It is connected with the asymptotic amplitude M(E, t) by the formula

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[1)] The preliminary CERN data $\sigma^{tot}_{pp} \approx 40.3 \pm 2$ mb at $E \sim 10^3$ GeV are not very accurate and agree with Fig. 1 (as do the data obtained by the Grigorov group [7] in cosmic rays).

$$b = t^{-1}\ln \overline{|M(E,t)|^2}/\overline{|M(E,0)|^2}$$

(the bar denotes averaging over the spin states) and can be obtained theoretically for $t \to 0$ by expanding $M(E, t)$ in powers of t.

When account is taken of the contribution of the Regge poles and of the PP branch cuts, nonenhanced [4] and enhanced [5], it is easy to obtain (cf. [6]):

$$b_0(E) = b(E,0) = 2\lambda_P + \Delta b + O(E^{-1/2}),$$

$$b'(E) = [\partial b(E,t)/\partial t]_{t=0} = \lambda'_P - \frac{C_P\gamma_P}{4}\left(\frac{1}{4} + \beta_A + \beta_B\right)\lambda_P - \frac{(\Delta b)^2}{2}. \quad (1)$$

Here $\lambda_P = R_P^2 + \alpha'_P \ln E$, $\lambda'_P = \rho_P^4 + \alpha''_P \ln E$, $E = E_{lab}$ GeV, α'_P, α''_P, R_P^2, ρ_P^4 are the parameters of the P-trajectory and the residues of the P pole, $\alpha_P \simeq 1 + a'_P t + \alpha''_P(t^2/2)$, $\gamma_P(t) = \gamma_P \exp(R_P^2 t + \rho_P^4 t^2/2)$ (the power-law form of the residue $\gamma_P(t) = \gamma_P \exp[-\nu \ln(1 - R^2 t/\nu)]$, where ν is a certain power, corresponds to the value $\rho_P^2 = R_P^2/\sqrt{\nu}$). The quantity

$$\Delta b = \frac{C_P\gamma_P}{4}\left[1 + 2(\beta_A + \beta_B)\ln\frac{\lambda_P}{\lambda_0}\right], \quad \lambda_0 = 0.56\,\alpha'_P, \quad (2)$$

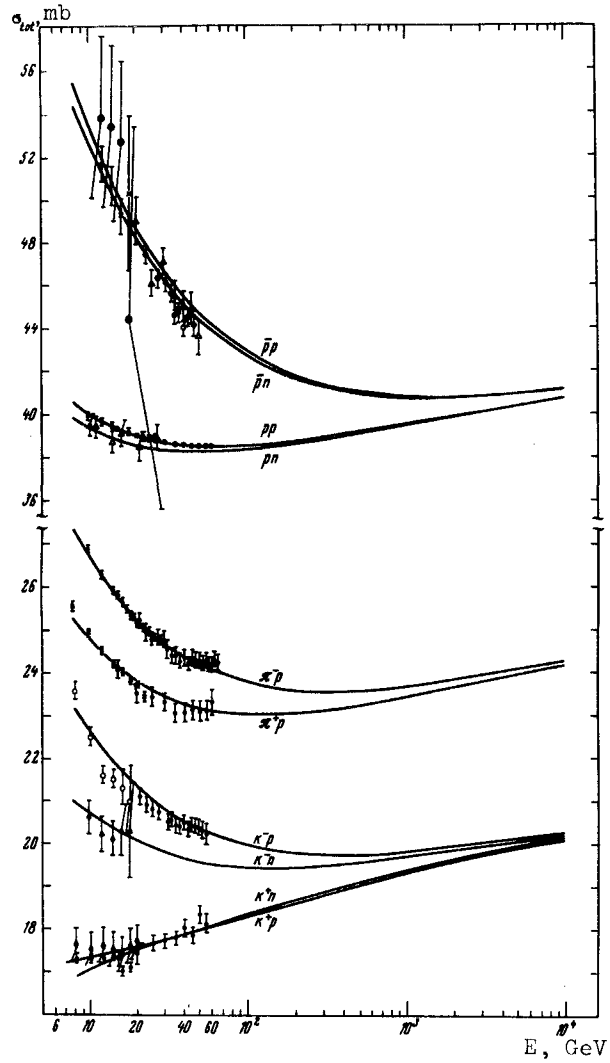

Fig. 1

is the contribution made to b_0 by the PP branch cuts, C_P is the coefficient of shower acceleration, which is close to unity and is known [8] from experimental data, while $\beta_A = r_0/C_P g_A$, $\beta_B = r_0/C_P g_B$, g_A and g_B are the coupling constants of the colliding particles A and B with the P-pole, with $\gamma_P(0)$ and $\gamma_P = g_A g_B$, and r_0 is the constant of the vertex $r = r_0 a'_P t$ of the transformation of the P-reggeon into two ($P \to 2P$). The quantity [6]

$$O(E^{-1/2}) = \Sigma_{a \neq P}\frac{v_a}{E^{1-\alpha_a(0)}} \approx \frac{1}{E^{1/2}}\sum_a v_a,$$

$$v_a = \frac{\pm\gamma_a}{\gamma_P}\left[\lambda_a - \lambda_P + \frac{C_a\gamma_a\lambda_P^2}{(\lambda_a + \lambda_P)^2}\right] \quad (3)$$

denotes the contribution of the poles $a \neq P$, which fades rapidly when $E \to \infty$, and γ_a, C_a, and γ_a are quantities defined in exactly the same manner as γ_P, $C_P = C$,

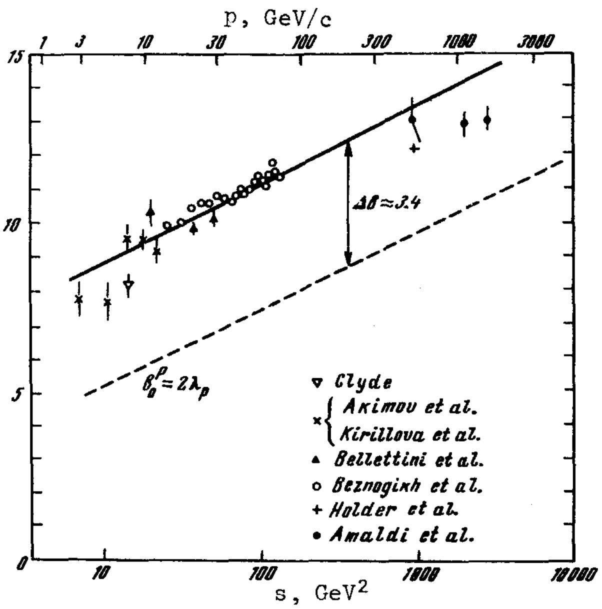

Fig. 2

and λ_P, while the sign of $\pm\gamma_a$ corresponds to the sign of the contribution of the given pole to the Regge asymptotic form. In (3) we have a = P', ω, ρ, or A_2, the most important contributions being those of P' and ω. For $\bar{p}p$ and K^-p scattering these contributions add up in (3), and for pp and K^+p scattering they cancel each other almost completely. Therefore the term $O(E^{-1/2})$ in (1) is very small already in the region E ≳ 10 GeV. In (1) and (2) we neglected the terms $\sim\rho_A\rho_B$ and disregarded the contribution of the four-reggeon enhanced diagram[2]. These terms are small (or unknown), and it can be shown that they do not change the conclusions presented below.

Let us consider pp scattering, for which $\beta_A = \beta_B = \beta$ is unknown, $C_P\gamma_P/4 \approx 1.7$, and $R_P^2 = 1.7$ [1, 2] (all the quantities are given in $(GeV/c)^{-2}$ or $(GeV/c)^{-4}$). As seen from Fig. 2, the IHEP data [9] and the CERN date [10] confirm the main theoretical conclusion that follows from (1), that $b_0(E)$ increases linearly with increasing ln E. In addition, experiment shows that when |t| increases the slope b(E, t) decreases, i.e., that b'(E) is positive (and of the order of ∿10). Therefore the CERN points [8] lie on Fig. 2 somewhat below the theoretical line (the distance between them decreases with decreasing |t|). The quantity $2\lambda_P$, shown dashed in Fig. 2, lies lower than the experimental data by an amount $(\Delta b)_{exp} \approx 3.4$. Therefore, for $b_0(E)$ of (1) to agree with Fig. 2 it is necessary either (a) to choose $\beta \approx 0.1$ or (b) to assume that $\beta = 0$ but R_P^2 is larger than the previously obtained [1, 2] value 1.7 by approximately 0.6 - 0.7 (this change of R_P^2 can occur when the nonlinearity $\lambda_P' t^2/2$ in the exponent of the P-pole contribution is taken into account).

The most reasonable estimate of λ_P' (at $\rho_P^2 \approx R_P^2 = 1.7$ and $\alpha'' \approx 0.3$) yields at E ∿ 10^3 GeV values $\lambda_P' \approx 7 - 10$. Therefore in case (a) at $\beta \approx 0.1$ and $\Delta b \approx 3.4$ we obtain the excessively low value $b' \approx \lambda_P' - 7 \approx 0$, and in case (b), when $\beta = 0$ and $\Delta b = 1.7$ we obtain $b' \approx \lambda_P' - 2$, i.e., the more reasonable value $b' \approx 6$.

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[2]) When these are not neglected, it is necessary to add in (2) the term $\Delta' b = 4\ell_0^2[\ln(\lambda_P/\lambda_0) - 1/2]$ and

$$b' \approx \lambda_P^0 - \frac{C_P\gamma_P}{4}\left(\frac{1}{4} + \beta_A + \beta_B + 2\beta_A\beta_B \ln\frac{\lambda_P}{e\lambda_0}\right)\lambda_P - \frac{3}{2}\ell_0^2\left(\ln\frac{\lambda_P}{\lambda_0} - \frac{5}{2}\right),$$

where ℓ_0 is the amplitude of the P → 4P decay. In addition, formulas (1) and (2) can be made more exact, by means of the substitutions $\gamma_P \to \gamma_P(1 - u/\lambda_P)^{-1}$ and $\ell_0^2 \to \ell_0^2(1 - u/\lambda_P)^{-1}$, where $u = C\gamma_P/4 + 4\ell_0^2/3$.

Experimental observation of a linear growth of b'(E) with increasing ln E would be direct evidence of curvature of the P trajectory, i.e., of $\alpha''_P > 0$.

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YIELD COEFFICIENT OF CYCLOTRON RADIATION FROM A "THERMONUCLEAR" PLASMA

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An important role will be played in the energy balance of thermonuclear reactors by cyclotron radiation, which can be calculated only by numerical methods, owing to the complexity of the formulas for the absorption coefficient. The results are usually represented in graphical form. It may be very useful however, to have very simple approximation formulas for these graphs.

An analysis of three examples [1 - 3] of numerical calculations shows that the radiation yield coefficient can be approximated, with 50% accuracy, by a "universal" formula suitable for a plasma layer, cylinder, and torus:

$$\Phi \simeq 60 \frac{t^{3/2}}{\sqrt{p_a}} \sqrt{1-r} \sqrt{1+\chi_T}. \qquad (1)$$

Here $t = T/mc^2$ is the temperature in units of $mc^2 = 511$ keV, $p_a = a\omega^2/c\omega_B$ is the dimensionless "opacity parameter," in which a is the radius of the cylinder, the thickness of the flat layer, or the minor radius of the torus, $\omega_0^2 = 4\pi n_e e^2/m$ is the square of the plasma frequency, $\omega_B \simeq eB/mc$ is the cyclotron frequency of the electron, c is the speed of light, r is the reflection coefficient of the mirrors located at the plasma boundary, and $\chi_T = a/R\sqrt{t}$ is the parameter of the