ABSORPTION OF FIRST SOUND IN SUPERFLUID HELIUM NEAR T = 0

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Submitted 19 June 1972
ZhETF Pis. Red. 16, No. 3, 177 - 179 (5 August 1972)

The absorption of sound in superfluid helium at low temperatures was investigated in [1] by solving the kinetic equation for the phonons (the influence of the rotons can be neglected at temperatures below 0.6°K). The results obtained in this manner should coincide at low frequencies with the results of the solution of the hydrodynamic equations [2]. In a recent paper, Saslow [3] states that there is a discrepancy between the sound absorption coefficients calculated by these two methods. We shall show that there is an error in [3], since Saslow used for the sound-absorption coefficient an expression obtained from the hydrodynamic equations while neglecting the thermal expansion coefficient $(\partial\rho/\partial T)_p$ of He^4. As indicated in [1], this is not permissible at low temperatures.

At T < 0.6°K, when only phonons are excited in the helium, all the kinetic coefficients are equal to zero, with the exception of the first viscosity coefficient η. We write down the hydrodynamic equations

$$
\begin{aligned}
&\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x} j = 0,\\
&\frac{\partial j}{\partial t} + \frac{\partial p}{\partial x} = \frac{4}{3}\eta\frac{\partial^2 v_n}{\partial x^2},\\
&\frac{\partial v_s}{\partial t} + \frac{\partial\mu}{\partial x} = 0,\\
&\frac{\partial}{\partial T}(\sigma\rho) + \sigma\rho\frac{\partial v_n}{\partial x} = 0.
\end{aligned}
\qquad (1)
$$

The notation is the same as in [1]. In the acoustic wave, the velocities v_n and v_s, and the varying parts of the thermodynamic quantities ρ' and σ' (which we choose to be independent variables) vary like $\exp[-i\omega(t - x/v)]$ (x is the wave propagation direction and ω is the frequency of the sound). After eliminating the variables v_n and v_s, the linearized system (1) takes the form

$$
\begin{aligned}
&\left[\left(\frac{\partial p}{\partial\rho}\right)_\sigma - V^2 - \frac{4}{3}\eta\frac{i\omega}{\rho}\right]\rho' + \left[\left(\frac{\partial p}{\partial\sigma}\right)_\rho - \frac{4}{3}\eta\frac{i\omega}{\sigma}\right]\sigma = 0,\\
&\left[-\sigma\left(\frac{\partial T}{\partial\rho}\right)_\sigma + \frac{4}{3}\eta\frac{i\omega}{\rho^2}\right]\frac{\rho_s\sigma}{\rho_n}\rho' + \left[V^2 - \frac{\rho_s\sigma^2}{\rho_n}\frac{\partial T}{\partial\sigma} - \frac{4}{3}\eta\frac{i\omega}{\rho}\frac{\rho_s}{\rho_n}\right]\sigma' = 0.
\end{aligned}
\qquad (2)
$$

We present below expressions for all the derivatives in (2). These expressions are valid in the temperature region where all the dynamic quantities are determined by the phonons.

The free energy $F = F_0 + F_1$ (where F_0 is the free energy at T = 0) and the pressure p are, respectively,

$$F_1 = -\frac{\pi^2}{90}\frac{(kT)^4}{\rho c^3} = -\frac{\Gamma}{\rho},$$

$$p = p_o + p_1 = p_o + \Gamma(1 + 3u) \qquad \left(u = \frac{\rho}{c}\frac{\partial c}{\partial \rho}\right). \tag{3}$$

The entropy and the ratio ρ_n/ρ are given by

$$\sigma = -\frac{\partial F}{\partial T} = \frac{4\Gamma}{\rho T},$$

$$\rho_n/\rho = \frac{4}{3}\frac{E}{\rho c^2} = \frac{4\Gamma}{\rho c^2} \tag{4}$$

This yields

$$\left(\frac{\partial p}{\partial \rho}\right)_\sigma = \left(\frac{\partial p_o}{\partial \rho}\right)_\sigma + \left(\frac{\partial p_1}{\partial \rho}\right)_\sigma = c^2 + \frac{1}{4}\frac{\rho_n}{\rho}c^2\left(\frac{4}{3} + 8u + 3w\right)$$

$$\left(w = \frac{\rho^2}{c}\frac{\partial^2 c}{\partial \rho^2}\right),$$

$$\left(\frac{\partial T}{\partial \rho}\right)_\sigma = \frac{1}{\rho^2}\left(\frac{\partial p}{\partial \sigma}\right)_\rho = \frac{T}{3\rho}(1 + 3u), \tag{5}$$

$$\left(\frac{\partial T}{\partial \sigma}\right)_\rho \frac{\rho_s}{\rho_n}\sigma^2 = \frac{c^2}{3}\left[1 + \frac{1}{3}\frac{\rho_n}{\rho}(1 + 3u)^2\right].$$

The condition for the compatability of the system (2) gives a dispersion equation, from which, taking (5) into account, we obtain the complex sound velocity V

$$\frac{V_1^2}{c^2} = 1 + \frac{1}{4}\frac{\rho_n}{\rho}\left(\frac{4}{3} + 8u + 3w\right) + \frac{1}{6}\frac{\rho_n}{\rho}(1 + 3u)^2 -$$

$$- 3(1 + u)^2\frac{i\omega\eta}{\rho c^2}. \tag{6}$$

The real part of (6) determines the sound velocity

$$\frac{V_1}{c} = 1 + \frac{\rho_n}{\rho}\left[\frac{1}{8}\left(\frac{4}{3} + 8u + 3w\right) + \frac{1}{12}(1 + 3u)^2\right] =$$

$$\equiv 1 + \frac{\rho_n}{\rho}\left[-\frac{3}{2}\left(u^2 - \frac{1}{4}w\right) + \frac{1}{4}(1 + 3u)^2\right]. \tag{7}$$

The imaginary part of (6), on the other hand, gives the sound absorption coefficient

$$\alpha_1 = \operatorname{Im}\frac{\omega}{V_1} = \frac{3}{2}\frac{\omega^2}{\rho c^3}(u + 1)^2\eta. \tag{8}$$

This expression differs from the well-known expression used by Saslow [3] and obtained neglecting $(\partial\rho/\partial T)_p$ by an additional factor $9(u + 1)^2/4$.

We substitute in (8) the expression given in [1] for η:

$$\eta = \frac{1}{5} c^2 \rho_n \tau_{ph} . \tag{9}$$

We thus obtain

$$a_1 = \frac{3}{10} (u + 1)^2 \frac{\omega^2 \tau_{ph}}{c} . \tag{10}$$

Formulas (7) and (10) coincide exactly with those obtained in [1] by solving the kinetic equation.

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[3] W.M. Saslow, Phys. Rev. A5, 1491 (1972).

QUALITATIVE FEATURES OF THE LEPTON SPECTRA IN THE PROCESS OF W-BOSON PRODUCTION IN A NEUTRINO BEAM

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Submitted 19 June 1972
ZhETF Pis. Red. 16, No. 3, 180 - 183 (5 August 1972)

A search for the intermediate W boson with mass $m \leq 4 - 6$ GeV is being planned in neutrino experiments with the accelerator of the Institute of High Energy Physics [1]. The W boson is produced in the electromagnetic field of the nucleon or nucleus in accordance with the scheme

$$\nu_\mu + N \to \mu^- + W^+ + N \tag{1}$$

with subsequent decay $W^+ \to \ell^+ + \nu_\ell$ ($\ell^+ = \mu^+$ or e^+). Recent numerical calculations of the differential spectra of μ^- and ℓ^+ leptons in the reaction (1) [2] have revealed that they differ qualitatively. The physical reason for this difference was not discussed in [2].

The main contribution of the process (1) comes from the region near the minimal momentum transfers q^2. This can be readily seen by analyzing the approximate formula for the doubly-differential cross section of the process (1), obtained by the Weizsacker-Williams method [4]:

$$\frac{\partial^2\sigma}{\partial s\, \partial q^2} = \frac{\alpha z^2}{\pi} \frac{(q^2 - q^2_{min})}{q^4} F^2(q^2) \sigma_{\gamma\nu}(s) s^{-1}, \tag{2}$$

where $\sqrt{s}$ is the effective mass of the $(\mu^- W^+)$ system (s-system), and $\sigma_{\gamma\nu}(s)$ is the cross section of the process $\gamma + \nu \to \mu^- + W^+$. The cross section (2) has a maximum at $q^2 \simeq 2q^2_{min} \simeq s^2/2E^2_\nu$, with width $\sim 2q^2_{min}$, and the form factor is $F(q^2) \simeq 1$. In order for the amplitude of this maximum not to be cut off by the form factor, it is necessary to satisfy the inequality $3q^2_{min} \leq q^2_{el}$, ($q^2_{eff} \simeq$