

case" $c_{\lambda m} = c_{\lambda+1, m+1}$ or $c_{\lambda m} = c_{\lambda+1, m}$.) A study of sufficiently large λ would make it possible to determine the probability that different morphemes are neighbors. The resultant analogy with a language text have been discussed in [3]. In connection with the foregoing, it is of interest to calculate with a computer the melting point of a sequence of pairs specified by means of the text of a sufficiently long book expressed in binary form, and to investigate the inverse problem for this case.

I am deeply grateful to I.M. Lifshitz for very useful discussions, and also to A.I. Larkin and M.D. Frank-Kamenetskii.

- [1] A.A. Vedenov, A.M. Dykhne, and M.D. Frank-Kamenetskii, Usp. Fiz. Nauk 105, 479 (1971) [Sov. Phys.-Usp. 14, 715 (1972)].
- [2] A.A. Vedenov and A.M. Dykhne, Zh. Eksp. Teor. Fiz. 55, 357 (1968) [Sov. Phys.-JETP 28, 187 (1969)].
- [3] M.Ya. Azbel, Ideen des Exakten Wissens, No. 12 (1972)

LOW-FREQUENCY CYCLOTRON RESONANCE IN AN IDEAL METALLIC CRYSTAL

S.M. Cheremisin

Institute of Physics Problems, USSR Academy of Sciences

Submitted 22 June 1972

ZhETF Pis. Red. 16, No. 3, 186 - 190 (5 August 1972)

The progress attained recently in the production of perfect single crystals and in the mastery of the millimeter radio band has made it possible to observe cyclotron resonance in metals under conditions when the free path time is determined not by scattering from the impurities and defects of the crystal, but by the interaction of the resonant electrons with other quasiparticles, primarily with phonons. The most detailed studies were made on bismuth [1] and lead [2, 3].

Sher and Holstein have constructed a rigorous theory [4] that enables us to calculate the cyclotron-resonance line width determined by the electron-phonon interaction. The electromagnetic field frequency was assumed by them to be very high, $\omega = n\omega_c \sim \omega_D$ (ω_c is the cyclotron frequency and ω_D the Debye frequency). The relaxation time in this case is $\tau \sim 1/\omega$ and the influence of the Landau quantization on the scattering can be disregarded even at $T = 0$. Actually, however, the experiments are performed at $\omega \ll \omega_D$ and $\omega\tau \gg 1$, and, as will be shown below, at sufficiently low frequency the relaxation time of the resonant electrons may turn out to be much larger than expected from the theory of [4].

Let us consider a quasiclassical model of electron-phonon interaction at $kT \ll \hbar\omega$. The main process determining the relaxation time will in this case be, as is well known, the spontaneous emission of phonons by the excited resonant electrons. (The residual time of the relaxation determined by the collisions of the electrons with the impurities and defects of the crystal is not taken into account.) The electron energy near the (convex) Fermi surface is given by the formula

$$E = (N + 1/2)\hbar\omega_c + p_z^2/2m_H$$

The energy of an electron having $p_z = 0$ changes by $\Delta E = \Delta N\hbar\omega_c - q_z^2/2m_H$ following the emission of a phonon (q_z is the projection of the phonon momentum on the magnetic-field direction).

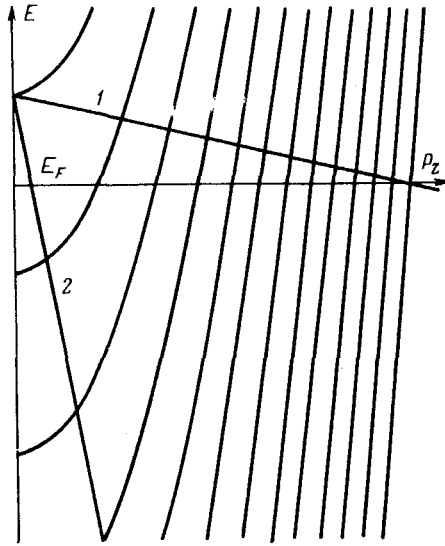


Fig. 1. Scheme for electron-phonon scattering. The final states of the electron lie at the points of intersection of the parabolas $E = (N + 1/2)\hbar\omega_c + p_z^2/2m_H$ and the straight line $\Delta E = q_z s$ (1 or 2).

In the case of scattering along the magnetic field, the values of q allowed by the conservation laws are determined by the intersection of the parabolas $E(p_z, N)$ and of the straight line $\Delta E = q_z s$ (s is the speed of sound).

Let us consider two limiting cases.

I. $\hbar\omega_c/m_H s^2 \gg 1$ (line 1 of Fig. 1).

The electron is scattered by free levels lying above the Fermi level with $\Delta N \gg 1$. The number of states on these levels can be represented, as is usually done in the theory of quantum oscillations, with the aid of a Poisson formula in the form of a sum of a monotonic part, equal to the number of states in the continuous spectrum, and an oscillating part, the relative magnitude of which for fixed \vec{q} decreases like $1/\Delta N$ (in the same manner as the quantum oscillations of the thermodynamic potentials). Thus, if the phonon spectrum has no sharp singularities in the frequency band from 0 to ω , then the quantum oscillations of τ can be neglected and the theory of [4] is fully applicable.

In this case the level width is $\Gamma \sim (|E - E_F|/\hbar\omega_D)^3$; the cyclotron-resonance line

width is determined by the sum of the widths of the upper and lower levels participating in the resonance, and has a minimum when the upper and lower levels are symmetrical with respect to E_F , i.e., $1/\tau \sim (\omega/2\omega_D)^3$ and the main contribution to the resonance is obviously made by the longest-lived electrons. This is precisely the case realized in the experiments [1, 2, 3]: $m_H = (1 - 2)m_e$, $s = (1 - 3) \times 10^5$ cm/sec, $\omega = 10^{11} - 10^{12}$, and $\Delta N = 10 - 100$.

II. In this case $\hbar\omega_c/m_H s^2 \ll 1$ (*) (line 2 in Fig. 1) and it is necessary to consider separately the principal resonance and the resonance with the harmonics of the cyclotron frequency. In the case of resonance with a harmonic $\omega = n\omega_c$, there are $n - 1$ Landau levels between the upper and lower levels taking part in the resonance, and the excited electrons can fall on these levels if they are free, and can fall from them into holes on the lower level if they are occupied. The presence of only one intermediate level suffices to make the relaxation time small, of the same order as in the continuous spectrum. When $n \gg 1$, just as in case I, the Landau quantization can be disregarded and one can use the formulas of [4].

To estimate the width of the principal resonance, it suffices to consider two neighboring Landau levels between which the Fermi level is located, since the relative contribution of the other levels is exponentially small ($kT \ll \hbar\omega$), and only phonons of frequency $\omega = \omega_c$ take part in the scattering. The probability of phonon emission is proportional to the density of the unoccupied states at the lower level and, at a fixed magnetic field, they oscillate as functions of p_z . These oscillations are analogous to the giant quantum oscillations of ultrasound absorption. The main contribution to the resonance is made

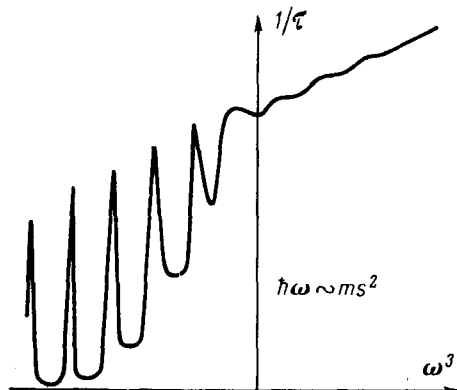


Fig. 2

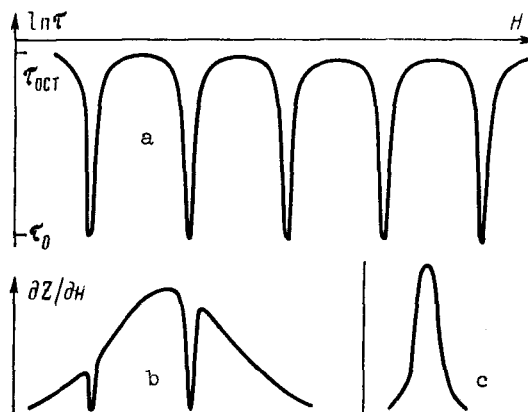


Fig. 3

Fig. 2. Approximate form of the frequency dependence of the relaxation time of the electrons of the central section. (To clarify the figure, the period of the quantum oscillations has been greatly magnified.)

Fig. 3. Possible cases of modulation of the cyclotron-resonance lines (b and c) by quantum oscillations of τ (a).

by electrons having maximum τ and executing transitions between Landau levels symmetrically disposed relative to the Fermi level. The dependence of the relaxation time on the temperature for these electrons is obviously exponential: $\tau \sim \tau_0 \exp(\hbar\omega/kT)$, where τ_0 is the value for the continuous spectrum. Allowance for the dependence of the cyclotron mass on p_z , which is usually made in the

theory of cyclotron resonance, is unnecessary here, for under realizable experimental conditions one can expect the condition (*) to be satisfied for only one or two pairs of levels directly on the extremal section. The character of the $\tau(\omega_c)$ dependence for this case is shown in Fig. 2. In spite of the smallness of the number of electrons, the resonance can be strong, owing to the large relaxation time. On the other hand, if it is possible to lower the temperature and the residual frequency of the collisions in such a way as to satisfy the condition (*) for a large number of levels, then, the conditions of quantum cyclotron resonance are apparently satisfied simultaneously [5], and the lines corresponding to the transitions of electrons from different levels will be observed. If we consider one pair of levels, then the cyclotron-resonance line turns out to be modulated by a system of giant quantum oscillations. If the period of the quantum oscillations is smaller than the line width, the observed line will be cut up, and in the opposite case the line will depend periodically on the frequency of the electromagnetic field (Fig. 3).

The described line narrowing can probably be observed in mercury at ~ 5 GHz at $T \leq 0.1^\circ\text{K}$. Indeed, the frequency of mercury is at present higher by one order of magnitude than the frequencies of the other even purest metals and gives grounds for hoping to obtain a residual relaxation time > 10 nsec, while the relaxation time determined by the scattering from the phonons is 0.1 nsec at $T = 1.5^\circ\text{K}$ and $\omega = 2 \times 10^{11}$.

The author is grateful to P.L. Kapitza for interest in the work, and to M.S. Khaikin, I.M. Lifshitz, M.I. Kaganov, and V.S. Edel'man for a useful discussion.

- [1] S.M. Cheremisin, V.S. Edel'man, and M.S. Khaikin, Zh. Eksp. Teor. Fiz. 61, 112 (1971) [Sov. Phys.-JETP 34, 594 (1972)].
- [2] I.Ya. Krasnopolin and M.S. Khaikin. ZhETF Pis. Red. 12, 76 (1970) [JETP Lett. 12, 54 (1970)].
- [3] P.Goy and G. Weisbuch, Phys. Rev. Lett. 25, 225 (1970).

- [4] H. Sher and T. Holstein, Phys. Rev. 148, 598 (1966).
 [5] I.M. Lifshitz, M.Ya. Azbel', and A.A. Slutskin, Zh. Eksp. Teor. Fiz. 43, 1464 (1962) [Sov. Phys.-JETP 16, 1035 (1963)].

SCATTERING OF LIGHT BY LIGHT

P.S. Isaev and V.I. Khilaskov

Joint Institute for Nuclear Research

Submitted 23 June 1972

ZhETF Pis. Red. 16, No. 3, 190 - 193 (5 August 1972)

Scattering of light by light is a fundamental effect in quantum electrodynamics. This process has been investigated so far principally within the framework of quantum electrodynamics without allowance for strong interactions [1, 2]. In [3] they considered resonant scattering of light by light via single-particle hadron states (π^0 , η^0 , etc.). Such scattering is significant only in narrow energy intervals corresponding to the positions of the intermediate-meson masses.

It is of great interest to investigate the contributions of two-particle hadronic states to the process of scattering of light by light (e.g., $\pi\pi$ and $K\bar{K}$ states). The expressions obtained in [4] for the amplitudes of the reaction $\gamma + \gamma \rightarrow \pi + \pi$ have enabled us to calculate the s and d waves of scattering of light by light with the aid of the method of dispersion relations. The imaginary part of the amplitude of the process $\gamma\gamma \rightarrow \gamma\gamma$ was expressed with the aid of the unitarity condition in terms of the amplitude of the process $\gamma\gamma \rightarrow \pi\pi$, and the real part was reconstructed with the aid of the imaginary part from the dispersion equation with one subtraction. The calculations were performed for white light. The cross section of the s waves of $\gamma\gamma \rightarrow \gamma\gamma$, corresponding to "down" and "down-up" s waves of the process $\gamma\gamma \rightarrow \pi\pi$ [4], are shown in Fig. 1 (the cross sections of the d waves are smaller than the cross sections of the s waves at the threshold and are comparable with them in the region of the f meson). Comparing the obtained curves with the results of [1, 2], we can see that scattering of light by light via a two-pion state dominates in a wide range of energies (approximately from 300 to 900 MeV). We have calculated the amplitudes of the process $\gamma\gamma \rightarrow K\bar{K}$ and their contribution to the process $\gamma\gamma \rightarrow \gamma\gamma$. In the energy region under consideration, the contribution of the $K\bar{K}$ two-particle state to the process $\gamma\gamma \rightarrow \gamma\gamma$ is negligibly small. The cross section for the scattering of light by light $\gamma\gamma \rightarrow \gamma\gamma$ is shown qualitatively in Fig. 2.

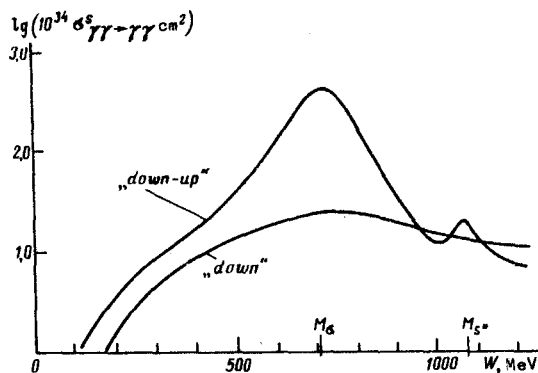


Fig. 1

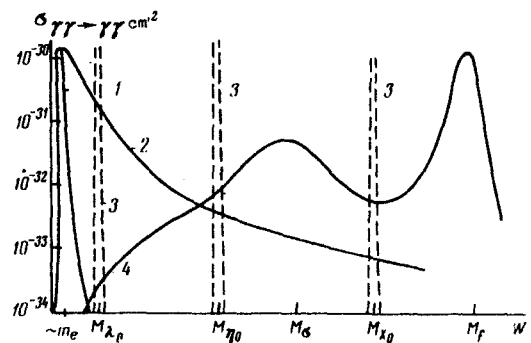


Fig. 2. 1 - $4\pi(d\sigma_{\gamma\gamma}/d\Omega)$ ($W, \theta \approx \pi/2$), 2 - $4\pi(d\sigma_{\gamma\gamma}/d\Omega)$ ($W, \theta = 0$) [1]; 3 - resonant scattering of light [3]; 4 - scattering via two-pion state ($\sigma_{\text{down-up}}^s + \sigma^d$).