$$\frac{\text{cross section (2)}}{\text{cross section (1)}} \lesssim 1\%.$$

Thus, by recording photons with specified energies at sufficiently large angles ($\theta \geq 8 - 10°$), and simultaneously recording the forward scattering of one of the leptons, we can separate the process $ee \to ee\gamma\gamma$ with intermediate two-photon interaction. From the known kinematic relation (1) we can obtain in such a case the cross section for the scattering of light by light.

We note that the $ee \to ee\gamma\gamma$ processes were observed experimentally [7], but with photon production along the direction of the initial beams. Figure 4 shows the cross sections of the processes $ee \to ee\gamma\gamma$, calculated from the s waves of scattering of light by light (Fig. 1) with the aid of the equivalent-photon method. The cross sections of the reaction $ee \to ee\gamma\gamma$ are quite small and lie at the borderline of the present-day experimental capabilities.

In conclusion, we are deeply grateful to N.N. Bogolyubov for interest in the work and valuable hints, and to D.V. Shirkov and R.M. Muradyan for a discussion of the results.

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ANOMALOUS ABSORPTION OF ELECTROMAGNETIC RADIATION AT DOUBLE THE PLASMA FREQUENCY

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The possibility of anomalous absorption of energy of a high-frequency electric field

$$E_z(x,t) = E_0 \cos(\omega_0 t - k_0 x) \tag{1}$$

is most frequently attributed to the decay of the pump wave into a plasmon and a phonon near the critical density of the plasma n_c [1 - 4]. We wish to call attention in this note to the fact that in a hot plasma it is necessary to take into account the presence of one more absorption band at densities one-quarter the critical value. This absorption is due to the process of the decay of the pump wave into two plasma oscillations (ω_1, $\vec{k}_1$) and (ω_2, $\vec{k}_2$). The instability increment can be expressed in terms of the interaction matrix $V_{k_1,k_0,-k_2}$ [5], which connects the probability amplitudes of the three waves:

$$\nu_d(k) = \sqrt{\left(V_{k_1,k_0,-k_2}\right)^2 \frac{E_0^2}{8\pi|\omega_0|} - \frac{1}{4}(\omega_0 - \omega_1 - \omega_2)^2}. \tag{2}$$

Here the matrix element $V_{k_1,k_0,-k_2}$ differs from the element $V_{k_0 k_1 k_2}$ calculated is [6] only in sign. In the approximation $k_{1,2} >> k_0$ we have

$$V_{k_1, k_o, -k_2} = \frac{\omega_p}{\sqrt{4 n_o m |\omega_o|}} k_o \sin 2\theta \cos \phi , \tag{3}$$

where θ and ϕ are spherical coordinates with a polar axis along z.

The mechanism of anomalous absorption of a high-frequency field consists in an effective increase of the collisions between the electrons and the developing plasma waves. The problem of finding the effective collision frequency reduces to a quantitative description of the nonlinear stage of the instability, on the basis of the theory of a weakly-turbulent plasma (an example of such an approach is described in [7]). Since no analytic solution can be obtained for the problem of interest to us, we shall use simpler physical considerations for the estimate of ν_{eff}. The point is that the effective collisions between the particles and the turbulent pulsations, like the Coulomb collisions, should increase the instability threshold. It is natural to assume that the development of the instability ceases when the instability threshold is reached. This threshold is calculated from the effective collision frequency:

$$\{ 0{,}5\, \omega_p (e E_o^2 / 8\pi n_o mc^2)^{1/2} - \nu_{eff}^{\ell} \} W_k^{\ell} = 0 \tag{4}$$

where W_k^ℓ is the spectral energy density of the plasma oscillations. This expression gives us the effective collision frequency describing the dissipation of the plasmon energy. We shall assume that, just as in a Coulomb plasma, the same frequency determines the dissipation of the electromagnetic wave[1)].

On the other hand, it must be borne in mind that in the lowest order of the expansion in the oscillation energy, the main nonlinear process in the kinetic equation for the plasmons is induced scattering of plasmons by ions, in which the number of plasmons is conserved. Therefore any consistent theory should provide a real sink for the number of plasmons. A natural sink appears when account is taken of the Coulomb collisions of the particles. However, in a rarefied plasma, a weak sink is the cause of the production of a plasmon condensate:

$$\int \frac{dk}{2\pi} W_k^{\ell} = \frac{\nu_{eff}}{\nu_{eff}} \frac{E_o^2}{8\pi} . \tag{5}$$

A natural way out of the resultant situation is to assume the presence, for the electrons, of a non-Maxwellian tail capable of absorbing plasmons. The mechanism for the development of such a tail appears in an inhomogeneous plasma. When an electromagnetic wave is incident at 45° to the direction of the inhomogeneity, we can confine ourselves to a one-dimensional model of the turbulence and obtain a simple analytic solution. Such a model is described by a system of two equations for the spectral density of the plasmon energy W_k^ℓ and for the electron distribution function $F_e(v)$:

$$\frac{\omega_p}{L} \frac{dW_k^{\ell}}{dk} = 2 \left| \nu_d(k) + \frac{\pi \omega_p^3}{2k^2} \frac{dF_e}{dv} (\omega_p / k) \right| W_k^{\ell} , \tag{6}$$

[1)] An analytic solution describing the nonlinear stage of the decay into plasma and acoustic waves [7] confirms the validity of such a simple estimate.

$$\frac{v}{L}(F_e - F_{m,e}) = \frac{d}{dv}\frac{2\pi e^2}{m^2} W_k^{\ell}(k = \omega_p/v)\frac{d}{dv} F_e , \tag{7}$$

where we have assumed that the slow electrons have time to become maxwellized, and the fast ones leave the plasma; L is the inhomogeneity scale, and $F_{m,e}$ is the Maxwellian distribution function of the electrons. The plasmon drift in k-space carries the oscillations out of the unstable region. The largest gain is possessed by the pair of perturbations in which the frequency of one is constant:

$$K = (2\pi\omega_p L/48c)E_0^2/8\pi n_0 T_e . \tag{8}$$

The instability threshold is determined by the condition for reaching the nonlinear level K > ln Λ (ln Λ is the Coulomb logarithm)[2]. The complete solution of (6) can be represented in the simple form

$$W_k^{\ell} = \overline{W}\exp\{-\pi\omega_p L F_e(\omega_p/k)\}, \quad k^2\lambda_D^2 > \frac{\omega_0 - 2\omega_p}{3\omega_p}, \tag{9}$$

where the average value of the spectral energy density W is estimated either from the linear-gain law or is determined with allowance for the nonlinear saturation as a result of the induced scattering of the plasmons by the ions. The electrons are drawn out from the main distribution into the tail because of the presence of oscillations with small phase velocity. With the aid of (9), the solution of the quasilinear equation (7) is represented in terms of the probability integral $\Phi(\alpha)$ (the approximation $\overline{W}/nT \gg \lambda_D^2/L$):

$$\Phi[\pi\omega_p L F_{me}(v_*)] - \Phi[\pi\omega_p L F_e(v)] = \sqrt{\frac{\pi}{32}}\left[\frac{L}{\lambda_D^2}\frac{\overline{W}}{n_0 T}\right]^{-1/2}\frac{v^2 - v_*^2}{v_{Te}^2}, \tag{10}$$

where v_* is determined from the condition that the derivatives of the distribution function be continuous at the point $v = v_*$.

The obtained solution is not suitable at small values of L, when the number of the electrons in the tail is sufficient to maintain the instability near the threshold:

$$F_e(v) = K/\pi\omega_p L, \quad v/v_{Te} < \sqrt{3\omega_p/(\omega_0 - 2\omega_p)} . \tag{11}$$

If the values of L are too large, a plasmon condensate is produced. The bunching of the plasmons in the packet as a result of modulation instability [8] can lead to an increase of the inhomogeneity of the plasma as a result of the inhomogeneity of the high-frequency pressure.

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QUANTUM THEORY WITHOUT THE SUPERPOSITION PRINCIPLE

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It follows from the experiment of Clark and co-workers [1] that the probability of the $K_L \to 2\mu$ decay is approximately one-third the theoretical limit. One of the explanations of this fact, not yet refuted by experiment, might be the assumption that the superposition principle is violated (this possibility, to which Kobzarev called attention, was discussed in a review by Dolgov, Zakharov, and Okun' [2]).

Quantum equations that violate the superposition principle were proposed earlier by Laurent and Roos [3] to explain the CP-violating decay $K_L \to 2\pi$. In the cited papers, however, the question of the applicability of nonlinear quantum-mechanical equations was analyzed from the physical point of view.

We show in the present paper, first, that a probabilistic interpretation of the wave function and of the transition amplitudes (S matrix) is nontheless possible in a theory that violates the superposition principle. Second, we show that in such a theory there exist real conserved physical quantities (energy, momentum, angular momentum), which are connected as usual with the symmetry properties of space-time. In this sense one can visualize the existence of quantum physics in which the superposition principle does not hold. All the foregoing is illustrated by an example of systems with a finite number of degrees of freedom, i.e., with a nonlinear generalization of quantum mechanics.

We denote by $\psi(x, t)$ the wave function of a system of several particles, x standing for the aggregate of all the coordinates (for simplicity we assume all particles to be spinless). The nonlinear equation for the wave function can be written in the form ($h = 1$)

$$[H + F(x, \psi, \psi^*)]\psi = i\frac{\partial \psi}{\partial t}. \tag{1}$$

Here H is a linear Hermitian operator that includes the kinetic-energy operator H_0 and the particle interaction potentials $V(x)$, and F is a nonlinear operator. We shall regard F as a real function of x, ψ, and ψ^* (nonlinearities of a different type will be considered in another paper). We assume also that all the forces are short-range, i.e., when the interparticle distances are increased the values of V and F decrease sufficiently rapidly. Let ψ_1 and ψ_2 be two solutions of Eq. (1). Then, using the standard procedure for deriving the continuity equation, we obtain

$$\frac{\partial}{\partial t} < \psi_1 | \psi_2 > = i < \psi | F(1) - F(2) | \psi_2 > . \tag{2}$$