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QUANTUM THEORY WITHOUT THE SUPERPOSITION PRINCIPLE

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It follows from the experiment of Clark and co-workers [1] that the probability of the $K_L \to 2\mu$ decay is approximately one-third the theoretical limit. One of the explanations of this fact, not yet refuted by experiment, might be the assumption that the superposition principle is violated (this possibility, to which Kobzarev called attention, was discussed in a review by Dolgov, Zakharov, and Okun' [2]).

Quantum equations that violate the superposition principle were proposed earlier by Laurent and Roos [3] to explain the CP-violating decay $K_L \to 2\pi$. In the cited papers, however, the question of the applicability of nonlinear quantum-mechanical equations was analyzed from the physical point of view.

We show in the present paper, first, that a probabilistic interpretation of the wave function and of the transition amplitudes (S matrix) is nontheless possible in a theory that violates the superposition principle. Second, we show that in such a theory there exist real conserved physical quantities (energy, momentum, angular momentum), which are connected as usual with the symmetry properties of space-time. In this sense one can visualize the existence of quantum physics in which the superposition principle does not hold. All the foregoing is illustrated by an example of systems with a finite number of degrees of freedom, i.e., with a nonlinear generalization of quantum mechanics.

We denote by $\psi(x, t)$ the wave function of a system of several particles, x standing for the aggregate of all the coordinates (for simplicity we assume all particles to be spinless). The nonlinear equation for the wave function can be written in the form ($h = 1$)

$$[H + F(x, \psi, \psi^*)]\psi = i\frac{\partial \psi}{\partial t}. \qquad (1)$$

Here H is a linear Hermitian operator that includes the kinetic-energy operator H_0 and the particle interaction potentials V(x), and F is a nonlinear operator. We shall regard F as a real function of x, ψ, and ψ^* (nonlinearities of a different type will be considered in another paper). We assume also that all the forces are short-range, i.e., when the interparticle distances are increased the values of V and F decrease sufficiently rapidly. Let ψ_1 and ψ_2 be two solutions of Eq. (1). Then, using the standard procedure for deriving the continuity equation, we obtain

$$\frac{\partial}{\partial t} < \psi_1 | \psi_2 > = i < \psi | F(1) - F(2) | \psi_2 > . \qquad (2)$$

In (2) we have put for brevity

$$F(a) = F(x, \psi_a, \psi_a^*),$$

and the scalar product $\langle\psi_1|\psi_2\rangle$ is defined in the usual manner:

$$\langle \psi_1 | \psi_2 \rangle = \int \psi_1^* \psi_2 \, dx \, .$$

If $\psi_1 = \psi_2 = \psi$, then (2) leads to conservation of the norm:

$$\frac{\partial}{\partial t} \langle \psi | \psi \rangle = 0 \, . \tag{4}$$

This equation ensures the possibility of interpreting $|\psi(x, t)|^2$ as a probability density, if it is assumed that

$$\langle \psi | \psi \rangle = 1 \, . \tag{5}$$

It must be emphasized that an arbitrary normalization of the ψ functions is not permissible, since the properties of the solutions of (1) depend significantly on the value of $\langle\psi|\psi\rangle$. The probabalistic interpretation fixes the normalization (5) which must henceforth be regarded as obligatory. From (3) it follows also, when $\psi_1 \neq \psi_2$, that in general

$$\frac{\partial}{\partial t} \langle \psi_1 | \psi_2 \rangle \neq 0 \, . \tag{6}$$

The inequality (6) distinguishes the quantum theory without the superposition principle from the usual linear quantum mechanics. It signifies, in particular, that the ψ functions, which are orthogonal at the initial instant of time, may turn out in the future not to be orthogonal. This circumstance affects quite strongly the properties of the S martix. The latter is introduced in the same manner as in the usual theory, with the interaction turned on adiabatically at $t = -\infty$. We denote by $\psi_i(x, t)$ the solution of (1) under the initial condition for $\phi_i(x)$:

$$\psi_i(x, -\infty) = \psi_i^{(-)} = \phi_i(x).$$

The functions $\phi_i(x)$ are the solutions of the wave equation for the noninteracting particles and are assumed to be orthonormal and comprising a complete system. Then, at $t = +\infty$, when the interaction is again turned off, we should have

$$\psi_i(x, +\infty) = \psi_i^{(+)} = \sum_j S_{ij} \phi_j \tag{7}$$

from which it follows that

$$S_{ij} = \langle \phi_j | \psi_i^{(+)} \rangle \tag{8}$$

The quantities S_{ij} can be identified with the amplitudes of the transitions $i \to j$. In fact, using the completeness of the system ϕ_i and Eqs. (4) and (5), we readily obtain

$$\sum_j |S_{ij}|^2 = \langle \psi_i^{(+)} | \psi_i^{(+)} \rangle = 1 \, . \tag{9}$$

Thus, it is permissible to regard $|S_{ij}|^2$ as transition probabilities. On the other hand, taking (6) into account, we conclude that

$$\sum_i S_{ij} S_{kj}^* = \langle \psi_k^{(+)} | \psi_i^{(+)} \rangle \neq 0, \quad k \neq i . \tag{10}$$

in spite of the orthogonality of the ψ functions at $t = -\infty$. Formula (10) means that in the quantum theory without the superposition principle the S matrix is not unitary, but satisfies the relation

$$SS^+ = 1 + \eta ,$$

where the Hermitian matrix η has, by virtue of (9), zero diagonal elements. We note that the matrix η can be diagonalized by means of the unitary transformation $\eta \to U\eta U^+$, but in this case the S matrix will not transform canonically ($S \to USU^+$) owing to the nonlinear dependence of the solutions ψ_i on the initial basis functions ϕ_i.

We consider now the question of the conserved physical quantities. It is easy to show that the mean value $\langle H + F \rangle$ of the nonlinear "Hamiltonian" H + F is not conserved in time (with the exception of the case of stationary solutions), and therefore cannot be identified with the energy of the system. To obtain conserved quantities we can use a variational principle. Then, choosing the Lagrangian in the form

$$L = \psi^* H \psi + \psi^* R(x, \psi, \psi^*)\, \psi - \frac{i}{2} (\psi^* \dot{\psi} - \dot{\psi}^* \psi) \tag{11}$$

we obtain the equation of motion (1) in which

$$F = R + \psi^* \frac{\partial R}{\partial \psi^*} . \tag{12}$$

Further, assuming that H is invariant against all transformations of the reference frames, including time shifts, and R varies in this case only as a result of the change of ψ, we obtain in the usual manner the conserved values of the energy E and of the momentum p:

$$E = \langle H + R \rangle , \quad \mathrm{p} = - i \langle \sum \frac{\partial}{\partial \mathrm{x}} \rangle \tag{13}$$

(summation is carried out over all the particles). Since the energy E should be real and furthermore, F must be real in order to conserve the norm, it follows from (12) and (13) that

$$R^* = R = \rho(x, |\psi|) , \qquad F = f(x, |\psi|). \tag{14}$$

The dependence of R and F on $|\psi|$ but not on ψ and ψ^* separately, ensures the existence (for the nonlinearity of the type under consideration) of stationary states.

The foregoing construction of nonlinear quantum mechanics can be extended also to include field theory with the aid of the formalism of Fock functionals. The formulation of quantum theory without the superposition principle will be considered in a more detailed article.

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ELECTROMAGNETIC CONTRIBUTIONS OF THE DIFFERENCE OF THE TOTAL CROSS SECTIONS FOR THE SCATTERING OF PARTICLES AND ANTIPARTICLES AT HIGH ENERGIES

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Many recent papers have been devoted to the electromagnetic contributions to hadron scattering cross sections [1 - 7]. We calculate here the differences, due to the interference between the electromagnetic and strong interactions at energies $E \gtrsim 300$ GeV, between the total cross sections for the scattering of particles and antiparticles. We show that the main contribution to the difference of the total cross sections of charged particles and antiparticles decreases logarithmically with increasing energy ($\sim 1/\ln s$) if the scattering cone narrows down logarithmically.

As is well known [3, 5], the radiative correction to the amplitude of elastic scattering of charged particles (we shall speak, for concreteness, of $\pi^+ p$ scattering), which takes correct account of the contribution of soft photons, is given at high energies by

$$A^{rad}_{\pi^+ p}(s, \vec{\Delta}^2) = -\frac{i\alpha}{\pi} \int \frac{d^2 q_\perp}{q_\perp^2 + \lambda^2} A_{\pi^+ p}(s, \vec{\Delta} - \vec{q}_\perp), \tag{1}$$

where λ is the photon mass and $\vec{\Delta}$ is the momentum transfer.

Expression (1) leads to the so-called Bethe phase [8], $\phi_B = -\alpha \ln (1/\lambda^{e\ell} t)$, where $t = \vec{\Delta}^2$, and $\lambda^{e\ell} = \alpha'_P \ln s + R_P^2$ is the slope in the amplitude of elastic scattering.

To obtain the contribution made to the difference between the total cross sections it is necessary to employ in (1) the optical theorem. At not too high energies (up to several dozen GeV), neglecting as a result the dependence of the phase of the amplitude on the momentum transfer, we thus obtain

$$r_{em} = 2\frac{\sigma^- - \sigma^+}{\sigma^- + \sigma^+} = 2\alpha \left.\frac{\mathrm{Re}(A_{\pi^+ p} + A_{\pi^- p})}{\mathrm{Im}(A_{\pi^+ p} + A_{\pi^- p})}\right|_{t=0} \left(\ln \frac{1}{\lambda^{e\ell} t_{min}} - \gamma\right), \tag{2}$$

where t_{min} is the experimental resolution with respect to the momentum transfer and $\gamma = 0.58$ is Euler's constant.

We assume that the amplitude of the πN interaction has a Regge-like behavior. Then expression (2) corresponds to a $\gamma P'$ branch cut and decreases with increasing energy like $(1/s)^{1/2}$. Other contributions to the difference between the total cross sections also exist and decrease in power-law fashion with increasing s. These are the contributions of the beams to the $\gamma P'$ branch cuts (see Fig. a), and also the electromagnetic renormalization of the residue of